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Selected aspects of determining static characteristic of optical ultra-rapid movement sensor

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Abstract

An application of very dynamic sensors is a key point for registration of ultra-rapid displacements. The most common ones are optimeters. The paper describes a method for determining the static characteristic of the optimeter based on a photodiode system. The output characteristic was supsmoothed with use of the mean square approximation. Only precise filtering would allow further processing of the obtained function of displacement, especially in twice differentiation case. Hence, the authors' proposal is focused on the procedure of approximation in the Mathcad environment. A core of this procedure is based on the Gram orthogonal functions for a set of measurement data. The measurement data set was created by adding a set of stochastic interferences to a known theoretical function. Calculation results are the basis for selection of a measurement data number. The number of data is very important when measurement results of the characteristic are obtained point by point.

Keywords: optical sensor, ultra rapid displacement, Gram function, mean square error.

Wybrane aspekty wyznaczania charakterystyki wyjściowej optycznego czujnika ultraszybkich przemieszczeń

Streszczenie

W artykule przedstawiono metodę wyznaczania charakterystyki wyjściowej optycznego czujnika ultraszybkich przemieszczeń prostoliniowych. Zasada działania czujnika polega na rejestracji sygnału generowanego przez układ składający się z szeregu połączonych fotodiod odsłanianych przysłoną połączoną z dyskiem napędu elektrodynamicznego [3]. W celu konwersji sygnału $u(t)$ na funkcję przemieszczenia $x(t)$ konieczne jest precyzyjne wyznaczenie charakterystyki wyjściowej czujnika. Ponieważ praca fotoelementu w układzie fotodiody generuje między innymi szum śrutowy, konieczna jest filtracja uzyskanego zbioru danych. Również sygnał czujnika rejestrowany w dynamicznej pracy napędu wymaga dokładnej filtracji, jeżeli chcemy dokonać dalszej obróbki sygnału, zwłaszcza przez różniczkowanie. W artykule przedstawiono zarys stworzonej w środowisku Mathcad procedury aproksymacji w oparciu o ortogonalne funkcje Grama. Procedura ta posłużyła do badań symulacyjnych, określających wpływ liczby punktów pomiarowych na błąd średniokwadratowy i jednocześnie błędy wyznaczone względem znanej funkcji teoretycznej. Stąd, symulowany zbiór danych stanowił sumę wartości znanej funkcji analitycznej i generowanych zakłóceń. Celem tych symulacji była próba odpowiedzi na pytanie, ile punktów pomiarowych, zwłaszcza przy wyznaczaniu charakterystyki wyjściowej (napięcie w funkcji położenia) z użyciem śruby mikrometrycznej wystarczy do uzyskania satysfakcjonującej funkcji aproksymującej.

Słowa kluczowe: czujnik optyczny, szybkie przemieszczenie, funkcje Grama, błąd średniokwadratowy.

1. Introduction

An important part of testing ultra-rapid electrodynamic actuators is registration of the drive mobile element displacement (Fig. 1). Due to significant acceleration, when instantaneous values reach even 50000g, displacement sensors must be characterized by such features as: the lack of inertia, low weight and contactlessness of the moving parts as well as vibration resistance. In addition, due to the strong magnetic field generated by the actuator coil, the sensor must be resistant to influence of this field. The time-function of the rectilinear displacement can be determined by an indirect method with use of acceleration registration by means of ultra dynamic piezoelectric sensors [8], and then double integration of the obtained function. For the direct methods very expensive ultra-fast digital cameras with sophisticated software are used. They make it possible to determine the movable object trajectory and, consequently, displacement, velocity and acceleration as a time function.

Currently, the Phantom V-12 camera is able to do one million frames per second [8]. An alternative method is to use much cheaper optical sensors. At the Faculty of Marine Electrical Engineering of Gdynia Maritime University there was built an optical sensor of rectilinear displacement, whose main component is the system of sequentially connected photoelements so-called "photosensitive ruler". The photosensitive ruler is covered with a lightweight diaphragm, Fig.1. The diaphragm is joined with a disc drive by means of a lightweight aluminum pull-rod. During the drive motion, the moving diaphragm reveals the photosensitive ruler. The increase of the illuminated area in the optical sensor increases the voltage output signal at its terminals.

In order to convert the voltage signal which is registered by the oscilloscope into function of a displacement, the sensor output characteristic must be very precisely determined.

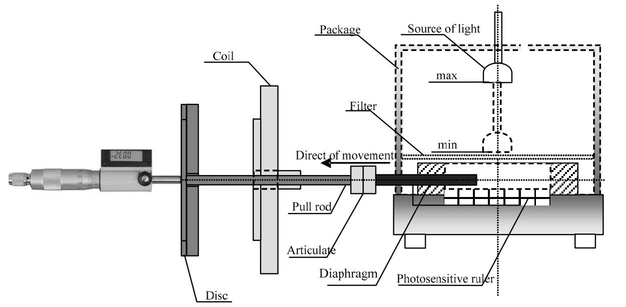

Fig. 1. The measuring system for disc displacement
Rys. 1. Układ pomiarowy rejestracji przemieszczenia dysku

Characteristics of the optical sensor can be obtained in one of the two systems: the photovoltaic cells (Fig. 2a), or in the photodiode system supplied from an external source (Fig. 2b).

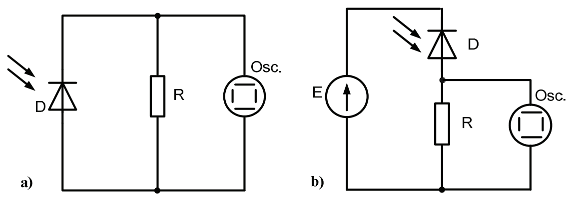

Fig. 2. a) Photovoltaic cell system b) photodiode system
Rys. 2. a) Układ fotoogniwa, b) układ fotodiody

Investigations aiming at obtaining the optimal characteristic were carried out in the photodiode system (Fig. 2), because the photovoltaic cell system did not have sufficient sensitivity and dynamics.

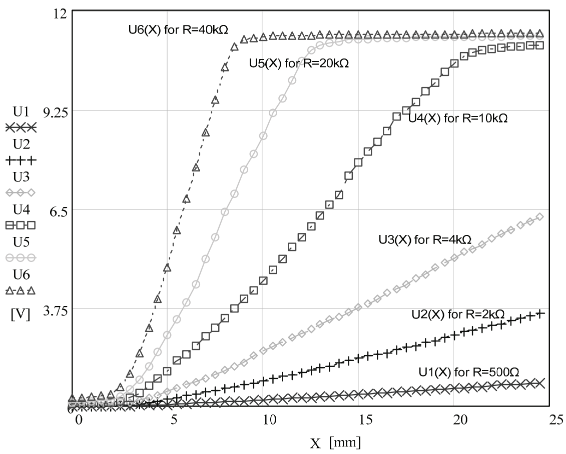

Fig. 3. Characteristics of the optimeter for different *R*
Rys. 3. Rodzina charakterystyk czujnika dla różnych wartości *R*

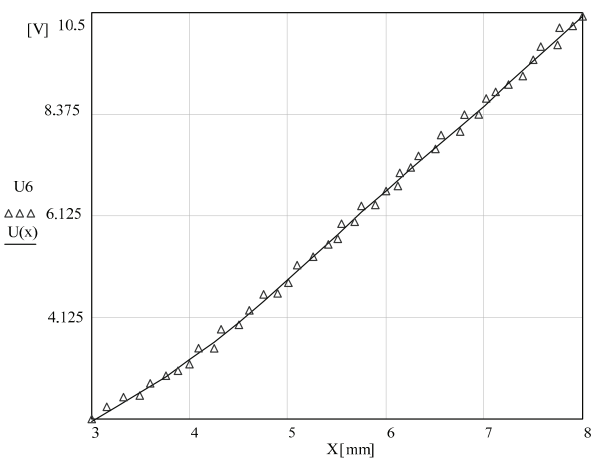

Fig. 4. The output aproximated characteristic (U6) for Δx=5 mm
Rys. 4. Zaproksymowana charakterystyka (U6) dla Δx =5 mm

Research of electrodynamic actuators is associated with their use in hybrid circuit-breakers, where the essential displacement, significant for work of the circuit-breaker does not exceed 5 mm [3]. Therefore, the external characteristic determined in the range between deadband and zone of saturation is sufficient. Fig. 4 presents a set of measurement points and output characteristic based on those points in the range between 3 and 8 mm of the photosensitive ruler. At the reduced Δx, (for which the characteristic was determined) one can observe the distortions with which measurement points are burdened.

The observed distortions can be caused by both the measuring system and the photoelement which generates a photodiode shot noise and low frequency noise in the photodiode system [1]. As a light source lamp uses LED, we cannot exclude the impact of the ambient temperature changes on the photodiode current in determining the characteristics [1]. Due to the distortion of both deterministic and stochastic nature, it was necessary to make numerical filtering ensuring a repetitive and precise output characteristic of the sensor.

2. Data set approximation

Determination of the precise function on the basis of measurement points burdened with interference is essential to obtain a final result with the satisfactory accuracy. Approximation, due to its filtering properties, is one of the most important processes of data processing performed by an engineer.

In practice, we do not know the true theoretical function. Therefore we need to evaluate whether the function obtained by the approximation describes sufficiently a theoretical function mostly on the basis of the mean square error:

$$\Delta st_n := \sqrt{\frac{1}{n+1}\cdot\sum_{i=0}^{n}(y_i - gL(x_i))^2}\ , \tag{1}$$

where:

- y_i - values of measurement points,
- $gL(x_i)$ - values of approximation function,
- n+1 - number of measurement points.

Among the decisions that we must make while performing the approximation of data measurement points the most important are: selection of approximation depending on the definition of measurement deviation, the type of basic functions used in the polynomial and the degree of the applied polynomial. Another important issue needs to be solved is means to decide about the number of points taken into account in order to obtain the best approximation.

This is especially important in determining the static characteristics in measuring systems, where taking into consideration a large number of points is time consuming (contrary to the oscilloscope measurement). It seems that the latter problem is not sufficiently emphasized in the classic textbooks of numerical methods [2, 4, 7].

Therefore, the authors attempted to develop a procedure to compare the influence of the number of data on the mean square deviation. This procedure also determines the characteristics of the absolute and relative error as a time function in relation to the theoretical curve.

We can keep an equal distance to the arguments, both by determining an output characteristic of the sensor and displacement function. Hence to approximate the data set, the orthogonal Gram functions in Mathcad environment were applied. In Mathcad the Gram functions can be easily formulated as follows:

$$\mathrm{Pol\_Grama}(x,k,n) := \sum_{s=0}^{k}\left[(-1)^s\cdot\frac{k!}{s!\cdot(k-s)!}\cdot\frac{(k+s)!}{s!\cdot k!}\cdot \mathrm{if}\left[s>0,\frac{\prod_{i=0}^{s-1}(x-i)}{\prod_{i=0}^{s-1}(n-i)},1\right]\right] \tag{2}$$

The above formula generates automatically the successive orthogonal Gram polynomials depending on the argument *k*. The coefficients wsp_j of the polynomial were calculated as follows:

$$s_j := \sum_{i=0}^{n}\left[Pol\_Grama(i,j,n)\right]^2$$

$$c_j := \sum_{i=0}^{n} \left[y_i \cdot Pol\_Grama(i,j,n) \right], \qquad wsp_j := \frac{(c_j)}{(s_j)} \tag{3}$$

Finally, the polynomial which approximates the data set and applies above wsp_j coefficients may be written as:

$$gL(t) := \left[\sum_{j=0}^{m} \left(wsp_j \cdot wielomian\_Grama(getQ(t), j, n) \right) \right] \tag{4}$$

By the approximation of the set of points it is assumed a priori that they were determined with unspecified precision. At the beginning we approximated the data being a sum of the given $h1(t)$ function and the disturbances. The aim of these simulations was to form the conclusions which could be applied to the procedure of determining the output characteristic of the presented optical sensor.

For this above reason, in a separate file named the *Data Generator* the interferences were simulated by means of a number generator *(rnd)*. It is worth noting that the sign of the d_i disturbance was randomly chosen.

$$d_i := (-1)^{trunc(rnd(9))} \cdot rnd(0.5) \qquad h1(t) := 0.01e^{0.6t} + 2 \tag{5}$$

In the file named *Gram* the approximation of data was created. The Gram procedure can choose the data from the *Data Generator* file depending on the declared number. The data set created consisted of 2500 points, therefore the simulations were conducted for n being the divisor of 2500 (n = 10, 20, 25, 50, 100, 200, 500, 1250, 2500). This guaranteed each time the choice of the equidistant points from the data set. The simulations were performed using a 6 degree polynomial satisfying the condition $m < 2\sqrt{n}$ which defined the polynomial degree in relation to the number of equidistant points [7]. The exact description of the *Gram* procedure is presented in [5]. The simulation results for different subsets, depending on n are shown in Figs. 5-8.

Fig. 5 shows the simulated data set by the procedure *Data Generator* for the number of points n = 25. Fig. 6 shows the polynomial function obtained by approximation of the data set for n = 25 and 2500. While observing Fig. 6, in which there is the function approximating the data set defined for a smaller number of measurement points (n = 25), it can be stated that its compliance with the function $h1(t)$ (compared to $gL2500$) is unsatisfactory. On the other hand, from the course of the mean square deviation (Fig. 7) one can notice that this error increases and is established for large n (the mean square error from the Fig. 7 was determined on the basis of formula (1)).

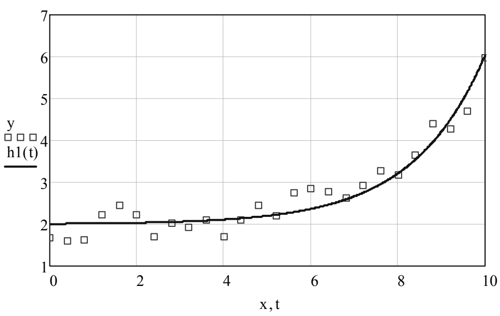

Fig. 5. Generated data set (n=25) and function $h1(t)$

Rys. 5. Zbiór danych pomiarowych (n=25) i funkcja $h1(t)$

For all test cases the maximum absolute (Δmax) and relative errors (δmax) were specified (Fig. 8) on the basis of trajectories of those errors as a function of time.

By observing the course in Fig. 7 it can be concluded that the smallest value of the mean square deviation was obtained for the approximation with n = 10 points, although both absolute and relative error for this course proved to be substantial (Fig. 8). This means that the deviation should not be the only indicator of selection of the best approximation, in spite of the smaller mean square deviation. Similar simulations were performed for the function $h1(t)$ which is similar to the output characteristic from Fig. 4. In this case fifty measurement points (n=50) were the sufficient number to obtain the satisfactory approximation.

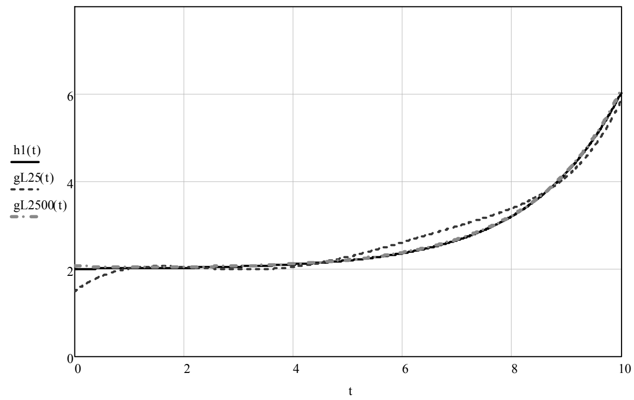

Fig. 6. Approximation for n=25 and 2500 in comparison to $h1(t)$

Rys. 6. Aproksymacja dla n=25 i 2500 na tle funkcji $h1(t)$

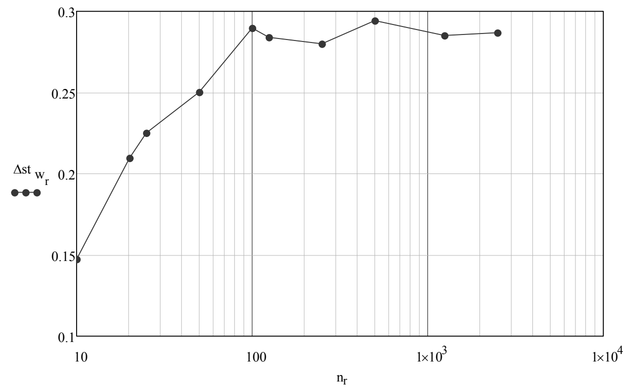

Fig. 7. Mean square deviation as n function

Rys. 7. Odchylenie średniokwadratowe w funkcji n

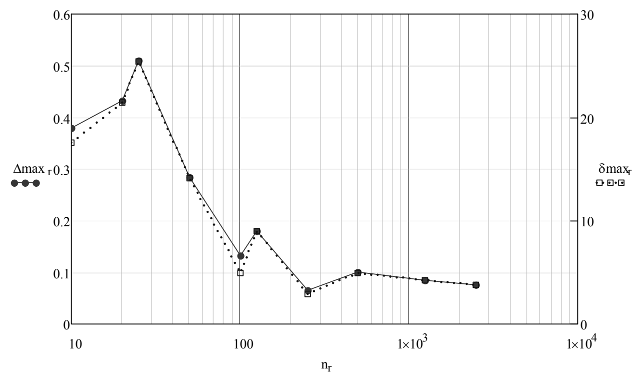

Fig. 8. Maximal absolute and relative error as n function

Rys. 8. Maksymalny błąd bezwzględny i względny w funkcji n

3. Description of the method

To approximate the measurements points obtained while determining the output characteristic of the optical sensor, the Mathcad procedure presented above was used. To determine this output characteristic, the sufficient number of measurement points (for 5 mm displacement) was defined as n=50. This procedure was also used to approximate the results of the oscilloscope measurement before converting the disc displacement (Fig. 10). The conversion into the disc displacement (Fig. 11) was performed with use of the output characteristic (Fig. 4). The block diagram of the method for determining the displacement is presented in Fig. 9.

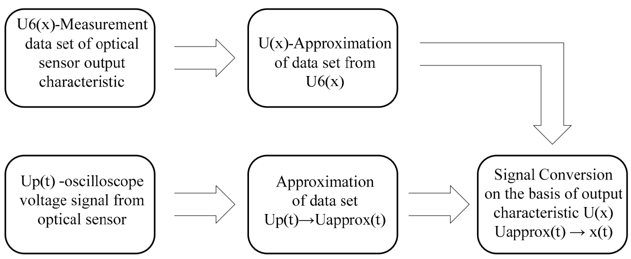

Fig. 9. Block diagram of determining the displacement as a time function
Rys. 9. Schemat blokowy wyznaczania przebiegu przemieszczenia

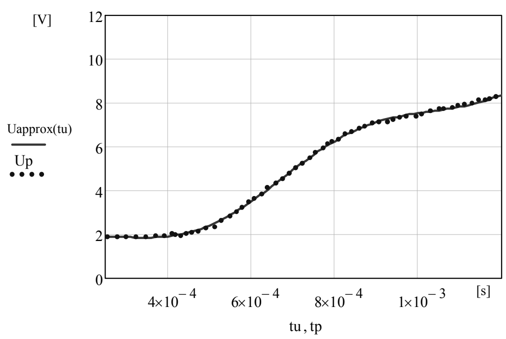

Fig. 10. The voltage trajectory on the sensor terminals and its approximation
Rys. 10. Przebieg napięcia na zaciskach czujnika i jego aproksymacja

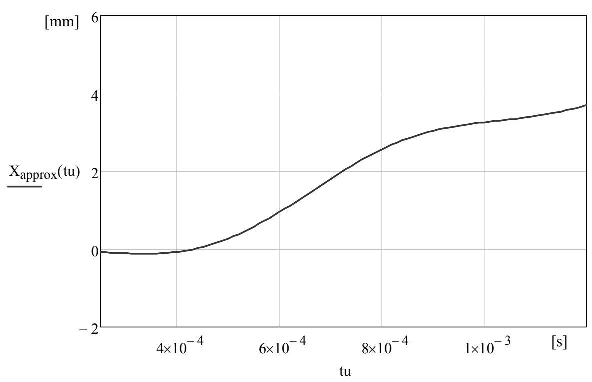

Fig. 11. Displacement after conversion on the basis of sensor output characteristic
Rys. 11. Przebieg przemieszczenia uzyskany po konwersji na podstawie charakterystyki wyjściowej czujnika

Thanks to precise designation of the output characteristic and fine filtration of the measurement trajectories it was possible to provide further data processing enabling to obtain the course of speed and acceleration (Fig. 12).

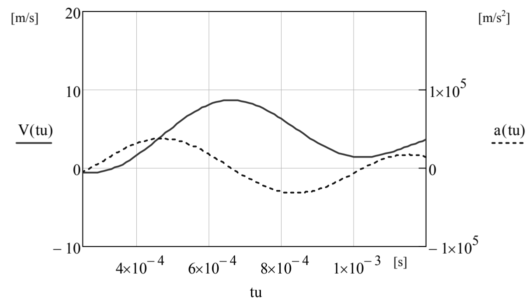

Fig. 12. Trajectories $v(t)$ and $a(t)$ after differentiation $x(t)$
Rys. 12. Przebieg $v(t)$ i $a(t)$ po zróżniczkowaniu $x(t)$

4. Conclusion

In [7] we can find a statement that with increase in the number of measurement points we approach the expected value (assuming no systematic errors). However, observing the course of the errors and approximations it can be concluded that the best approximation in the analyzed example is a case of n = 250 points (Fig. 7). However, the problem seems to be open because in engineering practice we do not know the theoretical function and we cannot make such comparisons. A separate issue (which was not discussed in this paper) is choice of the basis functions and polynomial degree. Summing up the results obtained from the filtration procedure of measurement points by means of the mean square approximation method, it can be stated that:

- Starting from a given number of measuring points a mean square error does not change its value,
- A course of the mean square error in the function of the number of measurement points should not be the sole criterion for determining their sufficient number for determining the approximating function,
- Mean square error also cannot be the sole indicator defining the approximating polynomial degree.

The simulations enabled us to define a sufficient number of measurement points (n = 50) for determining the output characteristic step by step. In the oscilloscope registration of $U(t)$ signal of the optical sensor all 2500 points were used.

5. References

[1] Boot K.: Optoelektronika. WKiŁ, Warszawa 2001.
[2] Dahlquist G., Bjorck A.: Numerical methods. PWN Warszawa 1983.
[3] Czucha J., Woloszyn J., Woloszyn M.: The comparision of ultra fast A.C. hybrid circuit breakers with GTO and IGBT. 35th Universities Power Engineering Conference UPEC'2000, Belfast, 6-8 September 2000.
[4] Fortuna A. and others: Numerical methods. WNT Warszawa 1982.
[5] Jankowski P., Młyński A., Wołoszyn J.: Data Approximation in Mathcad Environment. International conference on fundamentals of electrotechnics and circuit theory IC-SPETO Ustroń 2010.
[6] Jankowski P., Dudojć B., Mindykowski J.: Simple method of dynamic displacement record of contacts driven by inductive dynamic drive. Metrology and measurement systems, PAN, 2009,(p.5-18).
[7] Ralston A.: Introduction to numerical analysis. PWN Warszawa 1983.
[8] www.visionresearch.com

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