Wojciech P. HUNEK, Rafał STANISŁAWSKI, Krzysztof. J. LATAWIEC
OPOLE UNIVERSITY OF TECHNOLOGY, INSTITUTE OF CONTROL AND COMPUTER ENGINEERING
ul. Sosnkowskiego 31, 45-272 Opole, Poland

Pole-free control designs for nonsquare LTI MIMO systems

Dr inż. Wojciech P. HUNEK

He received his M.Sc. and Ph.D. degrees in electrical engineering from the Department of Electrical, Control and Computer Engineering, Opole University of Technology, in 1999 and 2003, respectively. He is an assistant professor at the Department's Institute of Control and Computer Engineering. He has authored or co-authored some 50 papers, mainly concerning modern ideas in multivariable control theory.

e-mail: w.hunek@po.opole.pl

Prof. dr hab. inż. Krzysztof J. LATAWIEC

He received his habilitation degree in 1999. He is a Professor of Control and Robotics and Head of Group of Control Systems and Electronics of the Department of Electrical, Control and Computer Engineering, Opole University of Technology. His research interests are focused on various aspects of identification and control of linear and nonlinear systems, the area in which he has authored or co-authored some 150 papers.

e-mail: lata@po.opole.pl

Dr inż. Rafał STANISŁAWSKI

He received his M.Sc. and Ph.D. degrees in electrical engineering from the Department of Electrical, Control and Computer Engineering, Opole University of Technology. He is an assistant professor at the Department's Institute of Control and Computer Engineering. His research interests concentrate on modeling and identification of complex dynamic systems.

e-mail: r.stanislawski@po.opole.pl

Abstract

This paper presents (structurally stable) pole-free control designs for minimum variance control (MVC) and generalized minimum variance control (GMVC). For MVC, the authors' general approach to pole-free design, that is the Smith-factorization approach, is advocated. For GMVC, a numerical optimization procedure is used in order to minimize the sum of modules of all poles of the closed-loop GMVC system. As a result of the optimization, pole-free GMVC converges to pole-free MVC.

Keywords: minimum variance control, generalized minimum variance control, pole-free design, guaranteed stability.

Projektowanie bezbiegunowych układów sterowania dla niekwadratowych liniowych obiektów stacjonarnych

Streszczenie

W artykule przedstawiono autorskie metody projektowania bezbiegunowych, strukturalnie stabilnych, niekwadratowych układów sterowania minimalno-wariancyjnego (MVC) oraz uogólnionego sterowania minimalno-wariancyjnego (GMVC). W szczególności skoncentrowano się na podejściu opartym na faktoryzacji Smitha. Pokazano, że wynikiem zadania optymalizacji, w którym minimalizuje się sumę modułów biegunów układu zamkniętego GMVC, jest otrzymane bezbiegunowe GMVC, sprowadzające się do bezbiegunowego MVC dla macierzy wagowej sterowań $Q(q^{-1})$ dążącej do zera. Wskazano na rolę bezbiegunowych metod, w tym podejścia bazującego na punktach i kierunkach ekstremalnych, w projektowaniu dwóch wspomnianych powyżej, strukturalnie stabilnych strategii sterowania. Przedstawiono także problematykę tzw. zer sterowniczych oraz kreujących je nowych τ -, σ - i S -inwersji macierzy wielomianowych oraz podkreślono wkład zer sterowniczych typu 2 w projektowaniu odpornych układów regulacji 'okołominimalnowariacyjnej'. Badania symulacyjne przeprowadzono w środowisku Matlab/Simulink.

Słowa kluczowe: Sterowanie minimalnowariancyjne, uogólnione sterowanie minimalnowariancyjne, projektowanie bezbiegunowe, gwarantowana stabilność.

1. Introduction

Possible instability of inverse/internal model control (IMC) as well as of perfect control, or more generally of minimum variance control (MVC) for nonsquare LTI MIMO systems [1, 3, 4, 6-8] has led the authors to questioning the celebrated statement that nonsquare systems generically have no (transmission) zeros [8] and to introducing a new class of multivariable zeros, called control zeros [1-4, 6-8]. Control zeros are an intriguing extension of transmission zeros for nonsquare LTI MIMO systems under IMC or MVC. Like for SISO and square MIMO systems, control zeros are related to the stabilizing potential of IMC/MVC, and, in the input-output modeling framework considered, they are generated by (poles of) a generalized inverse of the 'numerator' polynomial matrix $B(.)$. Originally, the unique, so-called T -inverse, being the minimum-norm right or least-squares left inverse involving the regular (rather than conjugate) transpose of the polynomial matrix, has been employed in the specific case of full normal rank systems [8]. The associated (unique) control zeros have later been called by the authors 'control zeros type 1' [1, 3, 6-8], as opposed to a plethora of 'control zeros type 2' generated by new right/left polynomial matrix inverses called τ -, σ - and S -inverses [1, 3, 4, 6, 7], with the two latter producing an infinite number of control zeros type 2.

Selection of stable control zeros from amongst of the myriads offered by the inverses, or placing control zeros to desirable locations, is a difficult problem that has been solved only recently [3, 4]. An attractive alternative could be to get rid of control zeros, except of, nongenerically, (stable) transmission zeros, so that the closed-loop MVC system would have no poles, implying that MVC would be guaranteed to be (structurally) stable. In such a pole-free MVC design to be tackled here, right/left polynomial matrix inverses would be selected not to have any poles. In this paper, the state-of-the-art in a pole-free MVC design is briefly reported. Then, a deterministic version of MVC, that is perfect control, is used in simulations to compare its pole-free design with that for a deterministic version of Generalized MVC (DGMVC). Based on specific simulation results we can conclude rather surprisingly that the latter pole-free design converges to that for perfect control, that is pole-free design for DGMVC is only possible with its control weighting matrix tending to the zero matrix.

2. Pole-free minimum variance control

2.1. Discrete-time minimum variance control

In the MVC framework, we consider the ARMAX system description of a right-invertible LTI MIMO system

$$\underline{A}(q^{-1})y(t) = q^{-d}\underline{B}(q^{-1})u(t) + \underline{C}(q^{-1})v(t), \qquad (1)$$

where $u(t)\in\Re^{n_u}$ and $y(t)\in\Re^{n_y}$ are the input and output vectors at discrete time t , respectively, d is the time delay, $\underline{A}(q^{-1})$ and

$\underline{B}(q^{-1})$ are the matrix polynomials of orders n and m, respectively, and the $\underline{C}(q^{-1})$ matrix polynomial of order k is assumed stable. (Note: Similar results can be obtained for left-invertible systems.)

Then the general MVC law, minimizing the performance index

$$\min_{u(t)} E\left\{\left[y(t+d)-y_{ref}(t+d)\right]^{\mathrm{T}}\left[y(t+d)-y_{ref}(t+d)\right]\right\}, \quad (2)$$

where $y_{ref}(t+d)$ and $y(t+d)=\underline{\widetilde{C}}^{-1}(q^{-1})[\underline{\widetilde{F}}(q^{-1})\underline{B}(q^{-1})u(t)+\underline{\widetilde{H}}(q^{-1})y(t)]+\underline{F}(q^{-1})v(t)$ are the output reference/setpoint and the stochastic output predictor, respectively, is of form [1-4, 6, 7]

$$u(t)=\underline{B}^{\mathrm{R}}(q^{-1})\underline{y}(t), \quad (3)$$

where $\underline{y}=\underline{\widetilde{F}}^{-1}(q^{-1})\left[\underline{\widetilde{C}}(q^{-1})y_{ref}(t+d)-\underline{\widetilde{H}}(q^{-1})y(t)\right]$.

The appropriate polynomial ($n_y \times n_y$)-matrices $\underline{\widetilde{F}}(q^{-1})=I_{n_y}+\underline{\widetilde{f}}_1 q^{-1}+\ldots+\underline{\widetilde{f}}_{d-1}q^{-d+1}$ and $\underline{\widetilde{H}}(q^{-1})=\underline{\widetilde{h}}_0+\underline{\widetilde{h}}_1 q^{-1}+\ldots+\underline{\widetilde{h}}_{n-1}q^{-n+1}$ are computed from the polynomial matrix identity (called Diophantine equation)

$$\underline{\widetilde{C}}(q^{-1})=\underline{\widetilde{F}}(q^{-1})\underline{A}(q^{-1})+q^{-d}\,\underline{\widetilde{H}}(q^{-1}), \quad (4)$$

with

$$\underline{\widetilde{C}}(q^{-1})\underline{F}(q^{-1})=\underline{\widetilde{F}}(q^{-1})\underline{C}(q^{-1}), \quad (5)$$

where $\underline{F}(q^{-1})=I_{n_y}+\underline{f}_1 q^{-1}+\ldots+\underline{f}_{d-1}q^{-d+1}$, $\underline{\widetilde{C}}(q^{-1})=\underline{\widetilde{c}}_0+\underline{\widetilde{c}}_1 q^{-1}+\ldots+\underline{\widetilde{c}}_k q^{-k}$. For right-invertible systems, the symbol $\underline{B}^{\mathrm{R}}(q^{-1})$ indicates, in general, an infinite number of right inverses of the numerator polynomial matrix $\underline{B}(q^{-1})$ [1-4, 6-8].

2.2. Extreme points and extreme directions method

Our first attempt at designing of pole-free MVC has been via the stimulating Extreme Points and Extreme Directions (EPED) method [5]. Our pole-free MVC solution by the EPED method [1, 3] has, however, suffered from two disadvantages: 1) high computational effort and 2) the method is invalid in the (nongeneric) case of the presence of transmission zero(s) in the system. The two drawbacks are eliminated in our.

2.3. Smith-factorization approach

Consider an n_u-input n_y-output LTI system described by the ARMAX model (1). Put $w=z^{-1}$ and factorize $\underline{B}(w)$ to the Smith form $\underline{B}(w)=U(w)S(w)V(w)$, where $U(w)$ and $V(w)$ are (nonunique) unimodular matrices. Now, $\underline{B}^{\mathrm{R}}(w)=V^{-1}(w)S^{\mathrm{R}}(w)U^{-1}(w)$, with determinants of $U(w)$ and $V(w)$ being independent of w, that is possible instability of an inverse polynomial matrix $\underline{B}^{\mathrm{R}}(w)$ being related to $S^{\mathrm{R}}(w)$ only.

Remark 1.

We use the operator $w=z^{-1}$ (or $w=q^{-1}$, depending on the context), whose correspondence to the s operator for continuous-time systems has pioneeringly been explored in Ref. [2].

Theorem 3.1. [3]

Consider a right-invertible polynomial matrix $\underline{B}(z^{-1})$ of dimension $n_y \times n_u$. Use the Smith factorization and obtain the inverse polynomial matrix $\underline{B}^{\mathrm{R}}(w)=V^{-1}(w)S^{\mathrm{R}}(w)U^{-1}(w)$, with $w=z^{-1}$ and $U(w)$ and $V(w)$ being unimodular. Then applying the minimum-norm right T-inverse $S_0^{\mathrm{R}}(w)=S^{\mathrm{T}}(w)[S(w)S^{\mathrm{T}}(w)]^{-1}$ guarantees a stable pole-free design of $\underline{B}^{\mathrm{R}}(w)$ for $\underline{B}(w)$ without transmission zeros and a stable design of $\underline{B}^{\mathrm{R}}(w)$ for $\underline{B}(w)$ with stable transmission zeros.

Remark 2.

Obviously, the stability of an inverse polynomial matrix with respect to w is translated to the requirement for all its poles to lie outside the unit disk.

Remark 3.

MVC applications of the above Theorem 3.1 are immediate [3, 4].

Remark 4.

The above specific selection of $S^{\mathrm{R}}(w)=S_0^{\mathrm{R}}(w)$ can be generalized by using the so-called S-inverse [3, 4], in which case we have $S^{\mathrm{R}}(w)=\underline{S}^{\mathrm{R}}(w)S_{tz}^{-1}(w)$, where $\underline{S}^{\mathrm{R}}(w)=\begin{bmatrix} I_{n_y} \\ L(w)\end{bmatrix}$ and $S_{tz}^{-1}(w)=[diag(1/\varepsilon_1,\ldots,1/\varepsilon_{n_y})]$, with $L(w)$ being an arbitrary polynomial $(n_u-n_y)\times n_y$-matrix and ε_i, $i=1,\ldots,n_y$, are the invariant factors containing transmission zeros. Now, $L(w)$ provides arbitrary degrees of freedom to the pole-free design, in that zeros of the closed-loop MVC system can be assigned to arbitrary locations, which can be of interest in a robust MVC design [1, 3, 4].

2.4. Simulation example

Consider a three-input two-output unstable system described by (noise-free) deterministic part of the ARMAX model (1), with $\underline{B}(q^{-1})=\begin{bmatrix} 2+3.8q^{-1} & 2q^{-1} & 1 \\ 4q^{-1} & 1+1.9q^{-1} & 1\end{bmatrix}$, $\underline{A}(q^{-1})=\begin{bmatrix} 1+1.1q^{-1} & 0.02q^{-1} \\ 0.3q^{-1} & 1+0.04q^{-1}\end{bmatrix}$ and $d=2$. The unstable control zeros type 1, obtained on a basis of the T-inverse of $\underline{B}(q^{-1})$, determine unstable MVC or perfect control of the system. Right the same holds for all the unstable control zeros type 2 generated by τ-inverses. On the other hand, it is very difficult to find stable control zeros type 2 on a basis of σ-inverses. Since the system has one (stable) transmission zero at $z=0.1$, it is impossible to employ the EPED method for a stable MVC design. Therefore, we apply our method of Section 2.3. Now, after substitution

$w = q^{-1}$ and Smith factorization we obtain $U(w) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $S(w) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & w-10 & 0 \end{bmatrix}$, $V(w) = \begin{bmatrix} 3.8w+2 & 2w & 1 \\ 0.2 & -0.1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ and finally

$$u(t) = \begin{bmatrix} 0 & 5 & -0.5 \\ 0 & 0 & -1 \\ 1 & -10-19q^{-1} & 1+3.9q^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1}{q^{-1}-10} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \cdot \underline{F}^{-1}(q^{-1})\left[y_{ref}(t+d) - \underline{H}(q^{-1})y(t)\right], \quad (6)$$

with specific forms of $\underline{F}(q^{-1}) = \begin{bmatrix} 1-1.1q^{-1} & -0.02q^{-1} \\ -0.3q^{-1} & 1-0.04q^{-1} \end{bmatrix}$ and $\underline{H}(q^{-1}) = \begin{bmatrix} 1.2160 & 0.0228 \\ 0.3420 & 0.0076 \end{bmatrix}$. Now, for $y_{1ref}(t) = 1$ and $y_{2ref}(t) = 1.5$, the outputs remain at the reference/setpoint for $t \ge d = 2$ under the stabilizing perfect control. We refer to the performance of (noise-free) perfect control rather than MVC, so that output plots would not be blurred by disturbances.

3. Pole-free generalized minimum variance control

3.1. Discrete-time generalized minimum variance control

Let an LTI discrete-time system be described by the ARX model

$$\underline{A}(q^{-1})y(t) = q^{-d}\underline{B}(q^{-1})u(t) + v(t). \quad (7)$$

Then the GMVC law, minimizing the performance index

$$\min_{u(t)} E\left\{\left\|y(t+d) - y_{ref}(t+d)\right\|_P^2 + \left\|u(t)\right\|_Q^2\right\}, \quad (8)$$

is of form [4]

$$u(t) = \{Q(q^{-1}) + \underline{b}_0^{\mathrm{T}} P(q^{-1})\widetilde{\underline{F}}(q^{-1})\underline{B}(q^{-1})\}^{-1}\underline{b}_0^{\mathrm{T}} P(q^{-1}) \cdot [y_{ref}(t+d) - \widetilde{\underline{H}}(q^{-1})y(t)], \quad (9)$$

where $\widetilde{\underline{F}}(q^{-1})$ and $\widetilde{\underline{H}}(q^{-1})$ are computed, as before, from the polynomial matrix identity (4) and $\underline{b}_0$ is the leading coefficient of $\underline{B}(q^{-1})$. For the correspondence with MVC, we put $P(q^{-1}) = I$.

Remark 5.

In order to provide steady-state error-free control, a simple integration modification can be introduced, that is the increment $\Delta u(t)$ can be included in Eqs. (7) to (9) as well as the $\underline{A}'(q^{-1}) = \underline{A}(q^{-1})(1-q^{-1})$ polynomial matrix is substituted for the $\underline{A}(q^{-1})$ one both in Eq. (7) and the polynomial matrix identity.

3.2. Simulation example

Consider a three-input two-output unstable system without transmission zeros, described by the deterministic part of the ARMAX model (1), with $\underline{B}(q^{-1}) = \begin{bmatrix} 1 & 1-0.7q^{-1} & 1-0.5q^{-1} \\ 1-0.3q^{-1} & 2 & 1-0.1q^{-1} \end{bmatrix}$, $\underline{A}(q^{-1}) = \begin{bmatrix} 1-1.1q^{-1} & 0.02q^{-1} \\ 0.3q^{-1} & 1-0.04q^{-1} \end{bmatrix}$ and $d = 1$. Again, for clarity of output plots, we assume that the system disturbance is zero and we consider a deterministic version of GMVC (DGMVC), with Q being the parameter matrix. We start with a numerical optimization procedure in which the sum of modules of all zeros of the characteristic equation of a closed-loop control system $\det(Q + b_0^{\mathrm{T}} PA^{-1}B) = 0$ is minimized with respect to the parameters of Q. Now, for $y_{1ref}(t) = 5$, $y_{2ref}(t) = 1$ and $P = I_2$ we arrive at pole-free DGMVC, with the minimum $Q = 1\mathrm{e}\text{-}14 \begin{bmatrix} 0.0814 & -0.2808 & -0.1281 \\ 0.0744 & -0.0204 & 0.0556 \\ 0.0969 & 0.0623 & 0.0661 \end{bmatrix}$ very close to zero, which in fact provides pole-free *perfect control*, that is $y(t) = y_{ref}(t)$ for $t \ge d = 1$. It is not surprising that in this case we do not have to arrange for an integration action.

4. Conclusions

The paper has compared pole-free control designs for MVC and GMVC, thus seeking for conditions to obtain the guaranteed closed-loop stability of the two control strategies. Rather surprisingly, a pole-free GMVC design has in simulations been shown to converge to the pole-free MVC one, that is when the control weighting matrix for pole-free GMVC is tending to the zero matrix.

5. References

[1] Hunek W. P.: Towards robust minimum variance control of nonsquare LTI MIMO systems, Archives of Control Sciences 18 (2008), no. 1, 59–71.

[2] Hunek W. P. and Latawiec K. J.: Minimum variance control of discrete-time and continuous-time LTI MIMO systems – a new unified framework, Control and Cybernetics 38 (2009), no. 3, 609–624.

[3] Hunek W. P. and Latawiec K. J.: A study on new right/left inverses of nonsquare polynomial matrices, International Journal of Applied Mathematics and Computer Science 21 (2011), no. 2.

[4] Hunek W. P., Latawiec K. J., Łukaniszyn M. and Gawdzik A.: Constrained minimum variance control of nonsquare LTI MIMO systems, Proceedings of the 15th IEEE International Conference on Methods and Models in Automation and Robotics (MMAR'2010), Międzyzdroje, Poland, 2010, Proceedings CD, DOI: 10.1109/MMAR.2010. 5587205, pp. 365–370.

[5] Kaczorek T. and Łopatka R.: Existence and computation of the set of positive solutions to polynomial matrix equations, International Journal of Applied Mathematics and Computer Science 10 (2000), no. 2, 309–320.

[6] Latawiec K. J.: The power of inverse systems in linear and nonlinear modeling and control, Opole University of Technology Press, Opole, Poland, 2004.

[7] Latawiec K. J.: Control zeros and maximum-accuracy/maximum-speed control of LTI MIMO discrete-time systems, Control and Cybernetics 34 (2005), no. 2, 453–475.

[8] Latawiec K. J., Bańka S. and Tokarzewski J.: Control zeros and nonminimum phase LTI MIMO systems, Annual Reviews in Control 24 (2000), no. 1, 105–112, DOI: 10.1016/S1367-5788(00)90021-X.

otrzymano / received: 28.11.2010
przyjęto do druku / accepted: 02.02.2011 *artykuł recenzowany*