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Improved stability and robust stability conditions for a general model of scalar continuous-discrete linear systems

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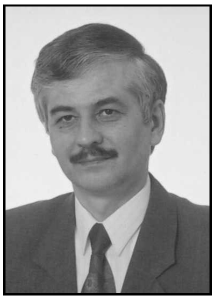

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Abstract

The paper improves the main result of the previous authors paper [1]. First it is shown that the conditions for asymptotic stability and for robust stability of a general model of scalar continuous-discrete linear systems given in this paper are only necessary. Next, the necessary and sufficient conditions are established. The conditions are expressed in terms of coefficients of the model.

Keywords: continuous-discrete system, positive system, scalar system, stability, robust stability.

Poprawione warunki stabilności i odpornej stabilności modelu ogólnego skalarnych liniowych układów ciągło-dyskretnych

Streszczenie

W pracy podano poprawione warunki stabilności oraz odpornej stabilności modelu ogólnego (1) skalarnych liniowych układów ciągło-dyskretnych, standardowych oraz dodatnich. Pokazano, że podane w pracy [1] warunki są tylko konieczne. Bazują one bowiem na warunku stabilności (7), który jest słuszny dla klasy (5) wielomianów dwóch zmiennych niezależnych. Wielomian charakterystyczny (4) rozpatrywanego układu nie należy do klasy (5), ale do klasy (8) wielomianów. Wobec tego do badania stabilności modelu (1) należy wykorzystać warunki (7) i (9), które są konieczne i wystarczające dla asymptotycznej stabilności klasy (8) wielomianów. Bazując na tych warunkach w twierdzeniu 1 sformułowano kryterium asymptotycznej stabilności analizowanej klasy układów. Warunki asymptotycznej stabilności oraz odpornej stabilności standardowego układu ciągło-dyskretnego podano w twierdzeniu 2 oraz w twierdzeniu 4, odpowiednio. Natomiast warunki asymptotycznej stabilności oraz odpornej stabilności dodatniego układu ciągło-dyskretnego podano w twierdzeniach 3 i 5. Wszystkie warunki są wyrażone w terminach współczynników modelu (1) (lub wartości krańcowych przedziałów (18), z których te współczynniki mogą przyjmować swoje wartości).

Słowa kluczowe: układ ciągło-dyskretny, dodatni, skalarny, stabilność, odporna stabilność.

1. Preliminaries

The following notation will be used: $\Re$ - the set of real numbers, Z_+ - the set of non-negative integers, $\Re_+ = [0, \infty]$.

Consider the state equation of general model of scalar continuous-discrete linear system (for $i \in Z_+$ and $t \in \Re_+$)

$$\dot{x}(t,i+1) = a_0 x(t,i) + a_1 \dot{x}(t,i) + a_2 x(t,i+1) + bu(t,i), \tag{1}$$

where $\dot{x}(t,i) = \partial x(t,i)/\partial t$, $x(t,i) \in \Re$, $u(t,i) \in \Re$ and a_0, a_1, a_2, b are real constant coefficients.

The scalar general model (1) is:

- positive (i.e. $x(t,i) \geq 0$ for all non-negative boundary conditions and non-negative inputs) if and only if [2, 1]

$$a_0 \geq 0,\ a_1 \geq 0,\ a_2 \in \Re,\ b \geq 0 \text{ and } a = a_0 + a_1 a_2 \geq 0, \tag{2}$$

- asymptotically stable if and only if [1]

$$w(s,z) \neq 0,\ \ \mathrm{Re}\, s \geq 0,\ \ |z| \geq 1, \tag{3}$$

where

$$w(s,z) = sz - a_0 - sa_1 - za_2 \tag{4}$$

is the characteristic function of equation (1) (polynomial in two independent variables *s* and *z*).

Polynomial (4) satisfying condition (3) is called continuous-discrete stable (C-D stable) or Hurwitz-Schur stable [3].

In Theorem 2 of paper [4] it has been shown that in the case of polynomials

$$w(s,z) = s^n + \sum_{p=0}^{n-1} \sum_{q=0}^{m} a_{pq} s^p z^q \tag{5}$$

condition (3) is equivalent to

$$w(s,z) \neq 0,\ \ \mathrm{Re}\, s \geq 0,\ \ |z| = 1, \tag{6}$$

which can be written in the form

$$w(s, \exp(j\omega)) \neq 0,\ \ \mathrm{Re}\, s \geq 0,\ \ \forall \omega \in [0, 2\pi]. \tag{7}$$

From condition 2) of Assertion given in [5] we have that for the more general class of polynomials, of the form

$$w(s,z) = \sum_{p=0}^{n} \sum_{q=0}^{m} a_{pq} s^p z^q, \tag{8}$$

condition (3) is equivalent to two conditions: (6) and

$$w(s,z) \neq 0,\ \ \mathrm{Re}\, s = 0,\ \ |z| \geq 1. \tag{9}$$

Characteristic polynomial (4) belongs to class (8). Therefore, condition (6) (equivalently (7)) is a necessary condition (not sufficient) for stability.

The main result of paper [1] is based on condition (7) and therefore, the conditions for stability and for robust stability of model (1) given in [1] are only necessary.

In this paper, based on conditions (7) and (9), we give simple analytical conditions for stability and for robust stability of a general model of scalar continuous-discrete linear systems, standard and positive.

2. The main result

Condition (9) can be written in the form

$$w(jy, z) \neq 0,\ \ |z| \geq 1,\ \ \forall y \in [0, \infty). \tag{10}$$

From the above considerations we the following theorem.

Theorem 1. General scalar model (1) is asymptotically stable if and only if conditions (7) and (10) hold.

The following lemma has been proved in [1].

Lemma 1. Condition (7) holds for polynomial (4) if and only if

$$\max\left\{\frac{a_0+a_2}{1-a_1},\ \frac{a_2-a_0}{1+a_1}\right\}<0,\quad a_1\neq\pm1, \tag{11}$$

or equivalently, if and only if one of the following conditions holds:

$$a_1>1,\quad -a_0<a_2<a_0, \tag{12a}$$

$$-1<a_1<1,\quad a_2<a_0,\quad a_2<-a_0, \tag{12b}$$

$$a_1<-1,\quad a_0<a_2<-a_0. \tag{12c}$$

Lemma 2. Condition (10) holds for polynomial (4) if and only if

$$a_1^2<1 \text{ and } a_2^2-a_0^2>0. \tag{13}$$

Proof. From (4) for $s=jy$ we have that the root of equation $w(jy,z)=0$ has the form

$$z(jy)=\frac{a_0+jya_1}{jy-a_2}. \tag{14}$$

Hence, condition (10) holds if and only if $|z(jy)|<1,\ \forall y\in\Re$, i.e.

$$a_2^2-a_0^2+y^2(1-a_1^2)>0,\quad \forall y\in\Re. \tag{15}$$

It is easy to see that (15) is equivalent to (13). ■

From Theorem 1 and Lemmas 1 and 2 we have the following theorem.

Theorem 2. General scalar model (1) is asymptotically stable if and only if

$$-1<a_1<1,\quad a_2<a_0,\quad a_2<-a_0. \tag{16}$$

Moreover, this system is unstable if $a_2\geq -a_0$ or $a_2\geq a_0$.

From Theorem 2 it follows that from the region shown in Figure 1 of paper [1] only the region corresponding to $-1<a_1<1$ is the stability region of system (1) in the plane (a_0,a_2).

Consider the positive scalar general model (1). In this case conditions (2) hold.

From (2) and Theorem 2 we have the following theorem.

Theorem 3. General model (1) is positive and asymptotically stable if and only if

$$0\leq a_1<1,\quad a_0\geq 0,\quad -a_0/a_1\leq a_2<-a_0. \tag{17}$$

Moreover, this system is unstable if $a_2\geq -a_0$.

The stability region of positive system (1), determined by inequalities (17), is shown in Figure 3 of paper [1].

Now we consider system (1) with uncertain parameters and assume that [1]

$$a_i\in A_i=[a_i^-,a_i^+],\quad a_i^-<a_i^+,\quad i=1,2,3, \tag{18}$$

where a_i^- and a_i^+ $(i=1,2,3)$ are given real numbers.

General model (1) with interval coefficients (18) is robustly stable if and only if it is asymptotically stable for all $a_i\in A_i$, $i=1,2,3$.

Theorem 4. Standard uncertain model (1), (18) is robustly stable if and only if

$$a_1^->-1,\quad a_1^+<1 \text{ and } a_2^+<a_0^-,\ a_2^+<-a_0^+. \tag{19}$$

Proof. It follows from Theorem 2 and condition (14b) of [1].

Theorem 5. General uncertain model (1), (18) is positive and robustly stable if and only if

$$0\leq a_1^-<a_1^+<1 \text{ and } a_0^-\geq 0,\quad a_2^+<-a_0^+,\ a_2^-\geq -a_0^-/a_1^+. \tag{20}$$

Proof. From (2) it follows that general uncertain model (1), (18) is positive if and only if (see also Theorem 7 in [1])

$$a_0^-\geq 0,\ a_1^-\geq 0 \text{ and } a_2^-\geq -a_0^-/a_1^+,\ a_2^+=\infty. \tag{21}$$

The proof follows directly from (21) and Theorem 3.

3. Concluding remarks

The results of the paper [1] have been improved. It has been shown that the conditions for asymptotic stability and for robust stability of model (1), given in [1], are only necessary and the necessary and sufficient conditions are established.

The work was supported by the Ministry of Science and High Education of Poland under grant N N514 1939 33.

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otrzymano / received: 16.11.2010
przyjęto do druku / accepted: 03.01.2011 *artykuł recenzowany*