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Zone Selection in Image Compression Using Piecewise-Linear Transforms

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Abstract

Experimental results of retained zone shape selection in zonal transform compression of greyscale images with the use of piecewise-linear transforms have been presented in the paper. The Walsh and PWL transforms used in the initial stage of the compression algorithm have been described. The zonal compression method, based on retaining of the spectral coefficients belonging to the specified zone, has been presented. Besides the commonly used square zone, various shapes of the retained spectral zone have been considered. Computational results have been compared in terms of the Mean-Square Error (*MSE*) and the Peak Signal-to-Noise Ratio (*PSNR*) versus the Compression Ratio (*CR*). Visual inspection of the reconstructed images has been performed. Results of experiments with the use of the described method have been enclosed in the form of plots and images.

Keywords: image compression, lossy compression methods, zonal compression, piecewise-linear transforms, transform compression.

Dobór strefy w kompresji obrazów z zastosowaniem transformat odcinkowo-liniowych

Streszczenie

W artykule przedstawiono rezultaty badań nad wyborem kształtu zachowanej strefy w transformacyjnej kompresji strefowej obrazów ze skalą szarości z zastosowaniem transformat odcinkowo-liniowych. Opisano transformaty Walsha i PWL używane w początkowym stadium algorytmu kompresji. Przestawiono metodę kompresji strefowej, opartą o zachowywanie współczynników widmowych leżących wewnątrz wyspecyfikowanej strefy (rys. 2). Obok powszechnie stosowanej strefy kwadratowej rozważono różne kształty zachowanej strefy widmowej (rys. 3). Kształty stref dobrano na podstawie analizy obrazów widma współczynników stosowanych transformat (rys. 1). Przeanalizowano wpływ rozmiaru każdej z badanych stref na efektywność kompresji. W celu uniezależnienia analizy od parametrów wewnętrznych poszczególnych stref, określających bezpośrednio ich wymiary, wprowadzono obiektywny parametr, nazwany współczynnikiem kompresji (ang. *Compression Ratio, CR*), zależny wprost od liczby zer występujących w widmie zdegradowanym. Efektywność kompresji porównano w sensie błędu średniokwadratowego (ang. *Mean-Square Error, MSE*) oraz maksymalnego współczynnika sygnał-szum (ang. *Peak Signal-to-Noise Ratio, PSNR*) w funkcji współczynnika kompresji. Dokonano także wizualnej oceny jakości rekonstruowanych obrazów. Rezultaty badań z użyciem opisanej metody zamieszczono w postaci wykresów (rys. 4 – 7) i obrazów (rys. 8). Na podstawie przeprowadzonych badań wybrano strefy o najlepszym kształcie dla każdej z analizowanych transformat.

Słowa kluczowe: kompresja obrazów, metody kompresji stratnej, kompresja strefowa, transformaty odcinkowo-liniowe, kompresja transformacyjna.

1. Introduction

The problem of image compression is one of the main problems emerging in various branches of contemporary telecommunication and other sciences. Continuously growing numbers of processed images accompanied by demand of their high quality forces the rapid development of modern image compression methods. All the methods are focused on the same goal: how to lessen the file size with no evident loss of the quality. The more effective the compression method, the better the quality of the reconstructed image at simultaneously smaller size of the file representing it.

In general, compression methods are classified in two categories: the lossless and the lossy ones [1, 2]. The lossless (reversible) compression methods are recommended for very demanding applications, where the reconstructed image has to be identical with the original one. This implies low compression factor, with simultaneous high reconstruction quality. In more common applications, the lossy (irreversible) compression, allowing for a certain loss of information, is widely used. Higher compression factor is achieved at the expense of the quality of the reconstructed image.

To evaluate the compression effectiveness in an objective manner following quality measures have been used: Mean Square Error (*MSE*), Normalised Mean Square Error (*NMSE*), Signal-to-Noise Ratio (*SNR*) and Peak Signal-to-Noise Ratio (*PSNR*). However in case of image compression the visual quality of the reconstructed image is also taken into account. Human eye can be sensitive in a particular way that the objective quality measures do not cover, so the visual inspection, usually performed by a representative group of viewers, is one of the commonly accepted non-quantitative quality measures.

2. The Transformations

The piecewise-linear transforms, similarly as other transforms, can be presented as a projection of a signal onto the appropriate set of basis functions [3].

The Walsh Transform represents mapping of the signal onto the set of piecewise-constant Walsh basis functions

$$x(n) = \sum_{i=0}^{N-1} c_i \cdot Wal_i(n)\ , \ n = 0, 1, 2, ..., N\text{-}1, \qquad (1)$$

where: c_i - expansion coefficients (spectrum); $Wal_i(n)$ - set of the discrete Walsh functions, constituting the transformation kernel.

The expansion coefficients are defined as follows:

$$c_i = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot Wal_i(n)\ , \ n = 0, 1, 2, ..., N\text{-}1. \qquad (2)$$

The PWL transform (Periodic Walsh Piecewise-Linear) represents mapping of the signal onto the set of piecewise-linear PWL basis functions, introduced by Dziech [4]:

$$x(n) = \sum_{i=0}^{N-1} c_i \cdot PWL_i(n)\ , \ n = 0, 1, 2, ..., N\text{-}1, \qquad (3)$$

where: c_i - expansion coefficients (spectrum); $PWL_i(n)$ - set of the discrete PWL functions, defined as the result of integration of the Walsh functions and supplementing the obtained set of functions with the constant function $PWL_0(n)$ = 1, constituting the transformation kernel.

The expansion coefficients are defined as follows:

$$c_i = -\frac{1}{2^{k+1}} \sum_{n=0}^{N-1} x(n) \cdot Wal'_i(n)\ , n = 0, 1, 2, ..., N\text{-}1, \qquad (4)$$

where: $Wal'_i(n)$ – derivatives of the Walsh functions, k – PWL group index, $k = 1,2, ..., \log_2 N$.

In the matrix notation the forward and inverse piecewise-linear transforms can be expressed as follows [5]:

$$\mathbf{c} = \mathbf{D} \cdot \mathbf{PL} \cdot \mathbf{x}$$
$$\mathbf{x} = \mathbf{IPL} \cdot \mathbf{c}\ , \tag{5}$$

where: **c** - piecewise-linear spectrum, **x** - signal vector, **D** - diagonal matrix of normalising coefficients, **PL** and **IPL** - matrix of the forward and inverse piecewise-linear transform, respectively.

Expansion of the above transformations onto the two-dimensional case is performed by application of the one-dimensional transforms to the rows and columns of the 2D discrete signal, representing the image [6]. First, the transformation is applied to the rows of the original image matrix, according to (6):

$$F(i,m) = \frac{1}{N}\sum_{n=0}^{N-1} I(n,m) \cdot PL\left(\frac{i \cdot n}{N}\right), \tag{6}$$

where: $F(i,m)$ – result of 1D transformation of rows of the original image (intermediate representation), $I(n,m)$ – original image matrix, $PL\left(\frac{i \cdot n}{N}\right)$ - basis functions (Walsh or PWL), constituting the transformation kernel.

Then the result of the above calculation is a subject of further transformation, on the columns of the obtained matrix, according to (7):

$$P(i,j) = \frac{1}{M}\sum_{m=0}^{M-1} F(i,m) \cdot PL\left(\frac{j \cdot m}{M}\right), \tag{7}$$

where: $P(i,j)$ – result of 2D transformation of the original image, calculated as a 1D transformation of columns of the intermediate representation from the preceding stage; the spectrum, $PL\left(\frac{j \cdot m}{M}\right)$ - basis functions (Walsh or PWL), constituting the transformation kernel.

The matrix form of the transformation is then especially useful. The resulting image spectra show specific coefficient distribution, which may suggest the shape of the zone selected during the compression process. It is obvious that the retained zone should contain coefficients of largest values, which are of major importance during the image reconstruction process. Exemplary Walsh and PWL spectra of the test image *Lena* are presented in Fig. 1.

a) original image b) Walsh spectrum c) PWL spectrum

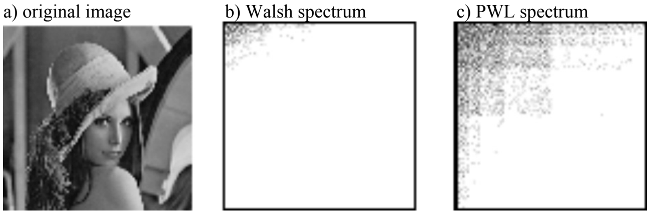

Fig. 1. Original image and its spectra
Rys. 1. Obraz oryginalny i jego widma

It is easy to be noticed that the large-amplitude coefficients, marked black, are cumulated in the upper left corner of the spectrum matrix. Their farther distribution is dependent on the type of the transform. In such a case the zonal compression method is especially desired and recommended.

3. The Zonal Compression Method

The block diagram of the zonal compression method is presented in Fig. 2 [7, 8].

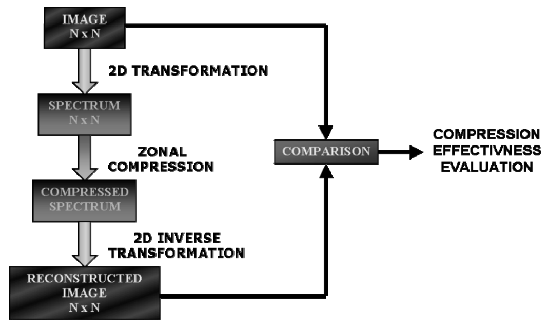

Fig. 2. The zonal compression method – block diagram
Rys. 2. Metoda kompresji strefowej – schemat blokowy

In the initial stage of the compression algorithm the two-dimensional transformation of the original image is performed. The obtained spectrum, showing specific features dependent on the transformation used, is then the subject of the actual compression. The main step of the compression process starts from assuming the shape of the selected zone. The coefficients belonging to the selected zone are then retained while others are set to zero. The resulting degraded spectrum contains many zero-valued coefficients. In the last stage of the algorithm the compressed spectrum is inverse transformed giving a reconstructed image. The procedure is repeated for changing sizes of the retained zones.

Evaluation of the reconstructed image is performed by a comparison with the original. Besides the objective, quantitative measures, as the *MSE* and *PSNR*, the visual inspection is also taken into account.

In the presented compression method the attention was focused mainly on the shape of the zone selected to be retained in the spectrum domain. The considered zone shapes are depicted in Fig. 3.

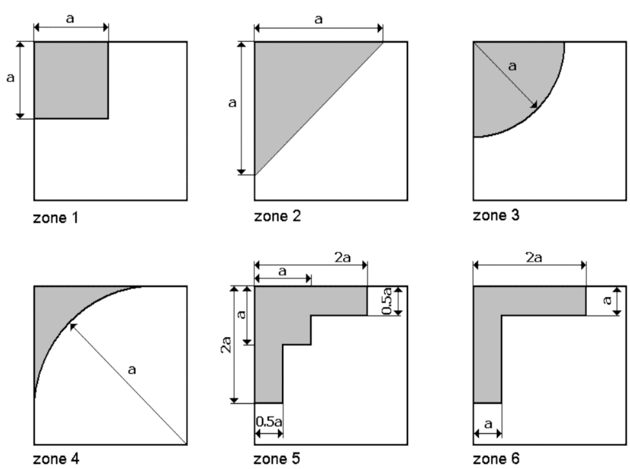

Fig. 3. Shapes of the zones determining the area of retained coefficients in the spectrum domain
Rys. 3. Kształty stref określających obszar zachowywanych współczynników w dziedzinie widmowej

Having selected the retained zone shape, the compression efficiency has been evaluated. Zone shapes have been compared for the same transform and image. On the other hand, for a given zone and image, the comparison has been done from the transform point of view. Finally, the image has been taken as a main criterion, with the same zone and transform.

4. Results of Experiments

According to the above algorithm, zonal compression has been performed over a set of greyscale images. All the images are of 256x256 pixels size, with 256 grey levels. The Walsh and PWL transforms have been used in the initial stage of the algorithm. The retained zones have been shaped according to the assumptions given in Fig. 3.

To evaluate the compression effectiveness, the reconstructed image has been compared with the original. Visual inspection has been carried out, but the quantitative quality measures, as more objective, have been calculated. Typical measures have been used [8]:

a) Mean-Square Error, *MSE*, given by (8):

$$MSE(I,\tilde{I}) = \frac{1}{M \cdot N} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \left(I(i,j) - \tilde{I}(i,j) \right)^2 , \qquad (8)$$

where: $I(i,j)$ – original image; $\tilde{I}(i,j)$ – reconstructed image; $i = 0, 1, ..., M-1$, $j = 0, 1, ..., N-1$; M, N – dimensions of the image;

b) Signal-to-Noise Ratio, *PSNR*, given by (9):

$$PSNR(I,\tilde{I}) = 10 \cdot \log_{10} \frac{(L-1)^2}{MSE(I,\tilde{I})} \text{ [dB]}, \qquad (9)$$

where: $I(i,j)$ – original image; $\tilde{I}(i,j)$ – reconstructed image; $i = 0, 1, ..., M-1$, $j = 0, 1, ..., N-1$; M, N – dimensions of the image, L – number of grey levels in the original image.

To compare the compression effectiveness in various cases, the above defined measures are referred to another objective coefficient, called Compression Ratio, *CR*, defined by (10):

$$CR(I) = \frac{z \cdot 100\%}{M \cdot N} , \qquad (10)$$

where: z – number of zero coefficients in the image spectrum; M, N – dimensions of the image spectrum (the same as of the image).

The *CR* can be also calculated analytically, dependently on the size of the retained zone, according to (11):

$$CR(I) = \left(1 - \frac{S(a)}{S} \right) \cdot 100\% , \qquad (11)$$

where: $S(a)$ – size of the retained zone, $S = M \cdot N$ - like in (10). In this paper *CR* has been calculated empirically, according to the number of zeroes determined numerically.

The presented method has been applied to each of test images, with the considered six zones and two transforms. The compression quality can be evaluated from various points of view, i.e. zone, transform or image type. In this paper the attention was focused on the zone shape. For each of the considered images, the influence of the zone shape for a given transform on compression quality has been evaluated. The results are presented for one of the considered images, called *Lena*, which is one of the most popular test images used in image compression procedures. Besides the charts presenting the computational results, exemplary reconstructed images have been included for visual inspection.

Figures 4 and 5 present the comparison of *MSE* and *PSNR* values versus *CR*, for the Walsh transform and all the considered zones.

Analysing the waveforms depicted in Figs. 4 and 5 it is easy to notice that for the Walsh transform best results, i.e. smallest *MSE* and largest *PSNR* values have been obtained for zone 4, marked by solid grey line. For small *CR* values worst results have been obtained for zone 1, and for larger *CR*s – for zone 6.

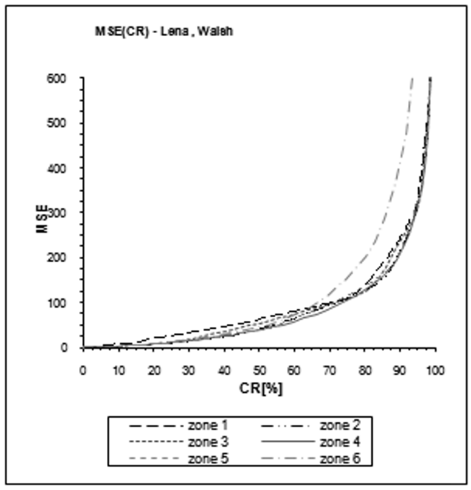

Fig. 4. Mean-Square-Error versus Compression Ratio, image *Lena*, Walsh transform
Rys. 4. Błąd średniokwadratowy w funkcji współczynnika kompresji, obraz *Lena*, transformata Walsha

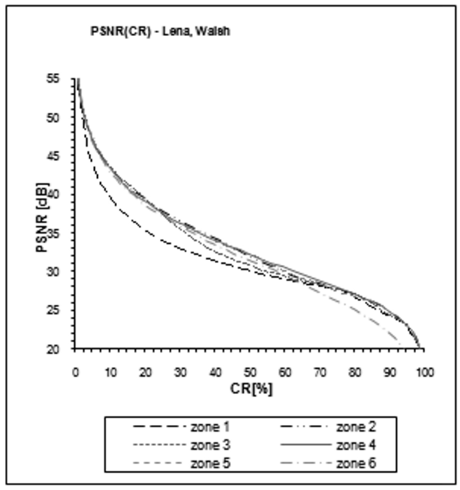

Fig. 5. Peak-Signal-to-Noise Ratio versus Compression Ratio, image *Lena*, Walsh transform
Rys. 5. Maksymalny współczynnik sygnał-szum w funkcji współczynnika kompresji, obraz *Lena*, transformata Walsha

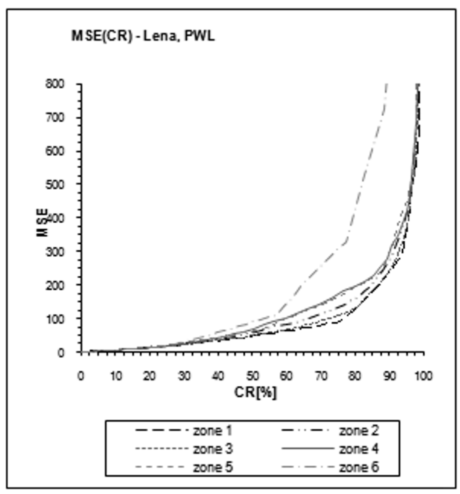

Fig. 6. Mean-Square-Error versus Compression Ratio, image *Lena*, PWL transform
Rys. 6. Błąd średniokwadratowy w funkcji współczynnika kompresji, obraz *Lena*, transformata PWL

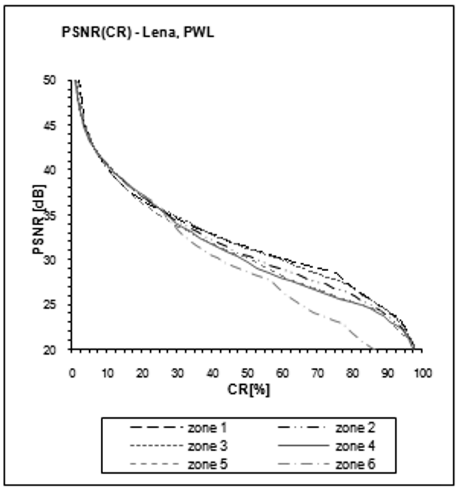

Fig. 7. Peak-Signal-to-Noise Ratio versus Compression Ratio, image *Lena*, PWL transform
Rys. 7. Maksymalny współczynnik sygnał-szum w funkcji współczynnika kompresji, obraz *Lena*, transformata PWL

The comparison of *MSE* and *PSNR* values versus *CR* for the PWL transform and all the considered zones are depicted in Fig. 6 and 7, respectively.

Comparing the waveforms presented above it is obvious that zone 1, marked by dashed black line, gives best results for the PWL transform. In a large range of *CR* values, zone 6 shows worst results, similarly to the Walsh transform considered former.

Concluding from the above zone 6 should be rejected from further analysis as the one giving worst results in case of each transform.

Examples of reconstructed images, for best zone shape selected for each transform, are presented in Fig. 8.

a) original image *Lena*

b) Walsh , zone 4, *CR*=50%; *MSE*=39,23; *PSNR*=32,19 dB;

c) PWL, zone 1, *CR*=50%; *MSE*=47,83; *PSNR*=31,33 dB;

Fig. 8. Compression results for *CR* = 50%: a) original image; b) Walsh transform and zone 4; c) PWL transform and zone 1
Rys. 8. Rezultaty kompresji dla *CR* = 50%: a) obraz oryginalny; b) transformata Walsha i strefa 4; b) transformata PWL i strefa 1

Visual inspection of the reconstructed images presented above confirms the conclusions formulated on the base of the calculated quality measures. For the assumed Compression Ratio equal to 50% the reconstructed images are of very high visual quality. The differences between the original and the reconstruction are invisible for a typical human viewer.

5. Conclusions

The zonal compression method with the use of Walsh and PWL transforms and various shapes of the retained zone has been investigated in the paper. The transformation has been initially applied to the processed image. The resulting spectrum has been compressed by rejection of coefficients placed outside of the selected zone. Various zone shapes have been taken into account. The obtained degraded spectrum has been finally inverse transformed, giving the resulting reconstructed image. Selection of the zone shape is a significant factor of the compression effectiveness, but it is firmly connected with the transformation type. For the Walsh transform best results have been obtained for zone 4 of a rounded shape. For the PWL transform zone 1 of a squared shape has given best results. In both cases zone 6 containing two rectangles showed worst effectiveness, so it shoud not be recommended for further investigations. The conclusions formulated on the base of quantitative quality measures are confirmed by the visual inpection of reconstructed images. Comparison of the experimental results leads to the conclusion that the zonal method with the use of Walsh and PWL transform is an effective tool for image compression. Comparison of the compression effectiveness of the selected the best zone shape for a given transform with other methods is a subject of further research.

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