USE OF PIEZOELECTRIC ELEMENTS IN COMPOSITE TORSIONAL SYSTEM WITH DYNAMIC VIBRATION ELIMINATOR

Michal Stepkowski
Institute of Aviation

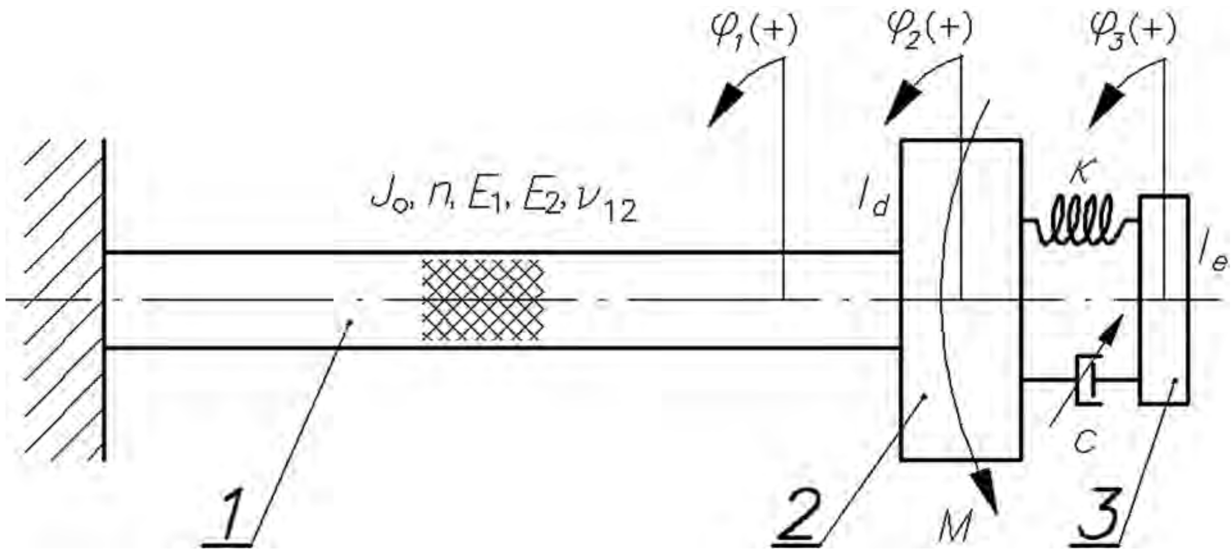
Abstract

This article describes the outcome of author's Master Thesis written in the Institute of Machine Design Fundamentals of Warsaw University of Technology.

One of development directions of automotive industry is mass reduction. In many cases this is brought to change of material from conventional steel/aluminum to composite structures. This kind of change may have major influence on dynamic response of the system, resulting sometimes with higher vibration amplitude and/or moving system's natural frequencies into machine's operational speed range.

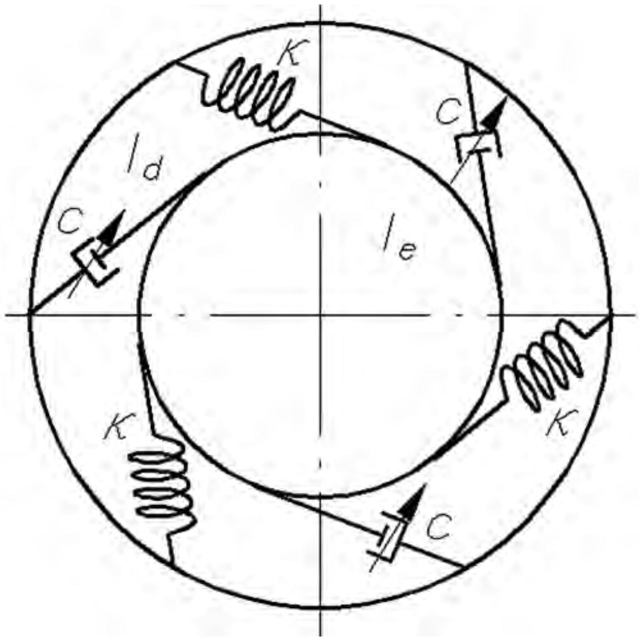
One of most common parts uprated this way is car's drive shaft. Following study concerns the problem of torsional vibration acting on composite shaft system and dynamic elimination of these vibrations. Starting with the physical model and its translation to mathematical model, results, system dynamics improvement, and benefits of chosen method are presented.

1. GENERAL DESCRIPTION



Picture 1. Physical model of considered system

Considered is n-layer laminated shaft treated as continuous system, and described by following physical properties: length l, Young's Modulus E_1 (in fibers direction) and E_2 (in direction perpendicular to fibers), and Poisson's Ratio v_{12}. Shear Modulus in this case is a function of lamination angle and laminate's physical properties. Shaft's section is described with geometrical moment of inertia J_O. Shaft is fixed on one side and has a plate (discrete system) representing running gear properties attached to the other side. Plate is characterized by mass moment of inertia I_d. Sinusoidal moment *M* attached to the plate describes the reaction to loads that are taken by the engine. Eliminator described by mass moment of inertia I_e is attached to the plate by torsional spring (characterized by stiffness), and torsional damper (characterized by damping coefficient *c*). Connections between plate and eliminator are shown on picture 2.



Picture 2. Joints between plate and eliminator

2. FORCED TORSIONAL VIBRATION OF LAMINATED SHAFT TREATED AS CONTINUOUS SYSTEM

Vibratory motion of considered system can be modeled with following equation [1]:

$$\rho J_o \frac{\partial^2 \phi}{\partial t^2} - G_{12} J_o \cdot \frac{\partial^2 \phi}{\partial x^2} - G_{12} \gamma \cdot \frac{\partial^3 \phi}{\partial x^2 \partial t} = M(x,t) \tag{1}$$

where: - shaft's density, - eigenfunction, $M(x,t) = M_O \sin(vt)$, M_O – moment's amplitude, - inner damping.

Given inhomogeneous differential equation describes forced vibratory motion of continuous system with inner damping taken into account. In this case eigenfunction is a function of displacement (shaft's length) and time: (*x,t*).

Solution for equation (1) is amplitude given by formula (2):

$$\left|\tilde{A}\right| = \left| -\frac{M_o \cdot X_1}{X_2 \cdot (X_{e1} - X_{e2}) + X_1 \cdot (X_{e4} - X_{e3})} \cdot \left(e^{-i\frac{v}{a}\frac{x}{\sqrt{1+\gamma i v}}} - e^{+i\frac{v}{a}\frac{x}{\sqrt{1+\gamma i v}}} \right) \right| \tag{2}$$

where:

$$X_1 = \kappa + icv + I_e \cdot v^2 \tag{3}$$

$$X_2 = \kappa - icv \tag{4}$$

$$X_{e1}=-\kappa\cdot e^{-i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}-icv\cdot e^{-i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}} \tag{5}$$

$$X_{e2}=-\kappa\cdot e^{+i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}-icv\cdot e^{+i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}} \tag{6}$$

$$X_{e3}=-\kappa\cdot e^{-i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}+icv\cdot e^{-i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}-I_d v^2\cdot e^{-i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}-\frac{iG_{12}J_o v\cdot e^{-i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}}{a\cdot\sqrt{1+i\gamma v}} \tag{7}$$

$$X_{e4}=-\kappa\cdot e^{+i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}+icv\cdot e^{+i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}-I_d v^2\cdot e^{+i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}-\frac{iG_{12}J_o v\cdot e^{+i\frac{v}{a}\cdot\frac{l}{\sqrt{1+i\gamma v}}}}{a\cdot\sqrt{1+i\gamma v}} \tag{8}$$

3. DATA USED FOR SIMULATION

For simulation of composite structure, epoxy/graphite laminate was chosen. This laminate is very popular in the industry because of its physical and strength properties. Following table briefly describes most important of these properties:

Table 1. Physical properties of epoxy/graphite laminate

Parameter	Value
E_1 [10^{11} Pa]	2.11
E_2 [10^{11} Pa]	0.053
G_{12} [10^{11} Pa]	0.026
ν_{12}	0.25
ρ [kg/m^3]	1560

Plate representing car's drive train, and eliminator's plate were modeled with steel. They are characterized with following properties:

Table 2. Physical properties of steel

Parameter	Value
ρ_s [kg/m^3]	7850
E_s [Pa]	$2.06\cdot10^{11}$
G_s [Pa]	$7.92\cdot10^{10}$

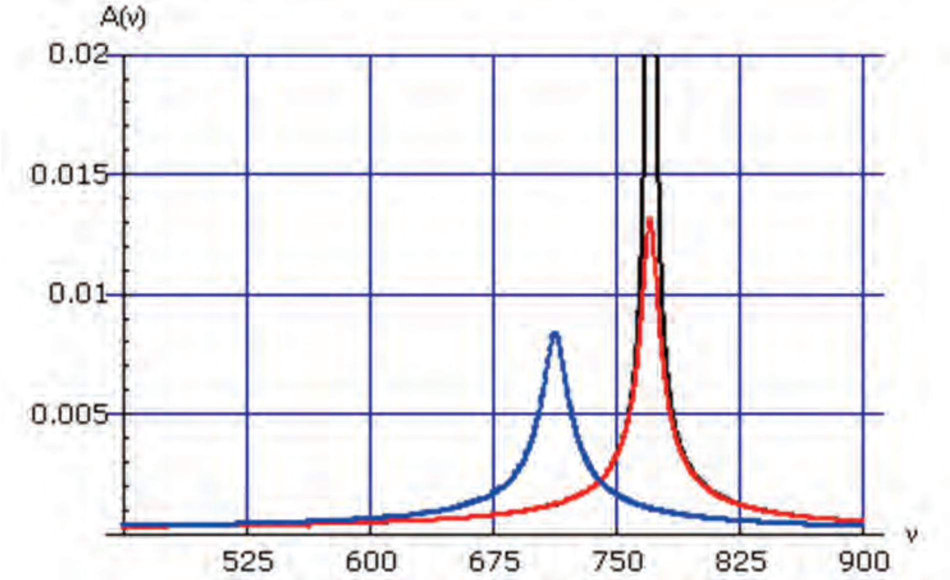
Considered shaft is treated as laminated structure with symmetrical layers. Number of orthotropic layers in this case is uneven (middle layer creates the plane of symmetry).
Following data was taken for simulation:
- shaft length l = 2 m. Shafts of this length appear in cars [2],
- shaft section described with outer radius r_z = 0.04 m, thickness g_W = 0.8 mm, and number of layers n = 9;
- lamination angle = 45;
- orthotropic layer delay time describing inner damping: $_{11}$ = 0 (in fibers direction), $_{22}$ = 0 (perpendicular to fibers direction), $_{12}$ = 0.01 s (in shear direction);
- material data for composite and steel in accordance to tables 1 and 2;

- steel plate thickness l_d = 0,5 m (to calculate mass moment of inertia);
- moment's amplitude M_o = 10 Nm. Because moment is proportional to torsional vibration amplitude, and the result is analyzed in relative scale (amplitude of one case is compared with amplitude of another case), moment's amplitude has no impact on the system, considering performed simulations.

Considered case is a study of vibration reduction, that's why shaft properties will remain unchanged. Variables in this system are: torsional damping coefficient *c*, torsional spring stiffness and eliminator's mass moment of inertia I_e. Because first natural frequency of the system is close to 750 rad/s, analysis is focused to that work area.

4. NUMERICAL SIMULATIONS RESULTS

Analysis of amplitude vs frequency charts for different values of *c*, , and I_e, allowed to choose following data for eliminator dimensions simulations: = 0.5S (where S is shaft's torsional stiffness $\frac{G_{12} \cdot J_o}{2 \cdot l}$), *c* = 50 Ns/rad, $I_e = I_d/5$. Semi-active change of damping is made by change of dimensions of rings made from zircon and titan oxidants (PZT ceramics). Rings are installed in torsional damper. In addition to that feature, spring will be switched on to the system using the same rule.



Picture 3. System's resonance characteristic: black curve is showing the system without eliminator, red curve – system with eliminator and with spring switched off ($_1$ = 0, c_1 = 1 Ns/rad, $I_e = I_d/5$), blue curve – system with eliminator and with spring switched on ($_2$ = S/2, c_2 = 50 Ns/rad, $I_e = I_d/5$).

System's parameters change strategy is brought to change of voltage that drives piezoelectric rings radial dimension change. This voltage change is made when red curve crosses the blue curve in the area of first natural frequency (picture 3).

What should be mentioned is the eliminator's moment of inertia effect on system's first natural frequency when spring is not connected. It is negligible. Using small values of damping coefficient (below *c* = 20 Ns/rad), curves representing system with and without eliminator are overlaid. Increase in damping coefficient result only with decrease in amplitude. Change of natural frequency can be seen only for higher values of *c* (above *c* = 20 Ns/rad). Because of that, one may assume that with relatively small damping, and spring not attached to the system, whole model in the area of first natural frequency acts like there is no eliminator at all.

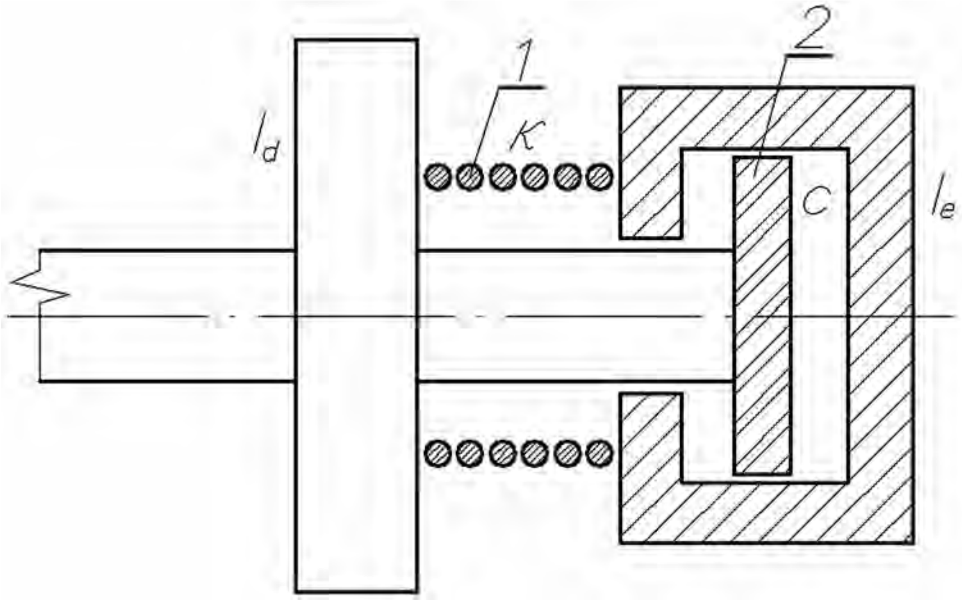
5. CONCEPTUAL DESIGN OF SEMI-ACTIVE VIBRATION ELIMINATOR

Knowing the characteristic on which the amplitude of torsional vibration will be kept, one can try to estimate eliminator's dimensions and driving parameter changes strategy. As was mentioned, to allow the designed eliminator to semi-actively tune to the conditions dictated by system's dynamics, PZT ceramics will be used. Following table briefly describes this material:

Table 3. Physical properties of PZT (lead zirconate titanate) ceramics

Parameter	Symbol	Value
Density	ρ_{PZT} [kg/m^3]	7500
Young's Modulus	E_{PZT} [Pa]	$5 \cdot 10^{10}$
Piezoelectric Charge Constant	d_{31} [m/V]	$-170 \cdot 10^{-12}$
	d_{33} [m/V]	$400 \cdot 10^{-12}$
	d_{15} [m/V]	$500 \cdot 10^{-12}$

Picture 4 shows the concept of the eliminator – torsional dampener with spring attached.



Picture 4. Conceptual design of eliminator

Torsional spring (1) which in the lower range of frequencies (below the limiting frequency for which the change of piezoelectric electrode polarization will be performed) would not be attached to the assembly, will be switched on also by piezoelectric components (frictional joint). Because of low inertia of the spring, one can assume that while being not attached to the system, spring don't have any influence to the overall assembly.

Change of damping will be performed by change of radial dimensions of ceramic rings (2), and in result by decreasing the gap between eliminator casing's inner diameter and PZT rings outer diameter.

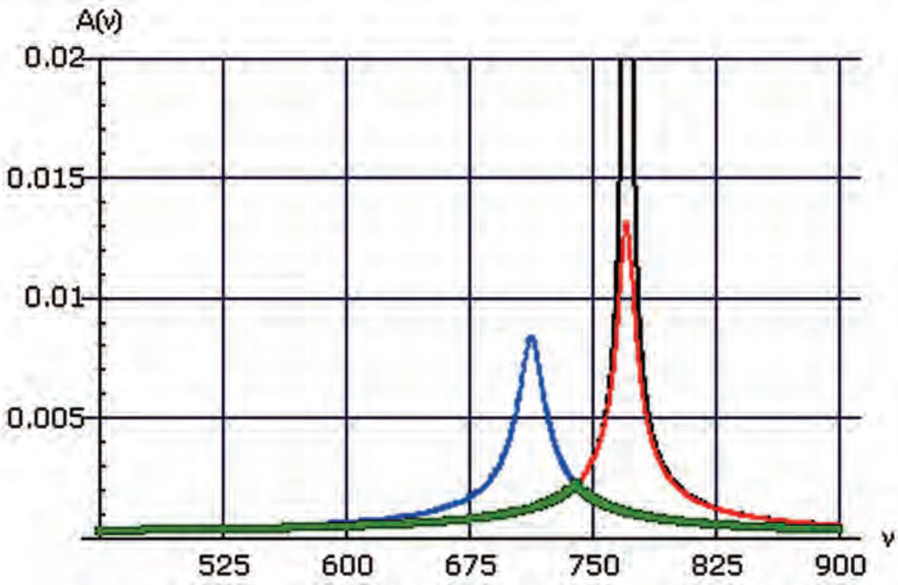
6. DATA USED FOR SIMULATION

Following table holds the data used for simulation of designed eliminator's dimensions. Limiting frequency, at which we want to change PZT rings dimensions is a frequency, at which curves for parameters: ${}_1 = 0$, $c_1 = 1$ Ns/rad, and ${}_2 = 0$, $c_2 = 1$ Ns/rad are crossing.

Table 4. Data used for eliminator dimensions simulation

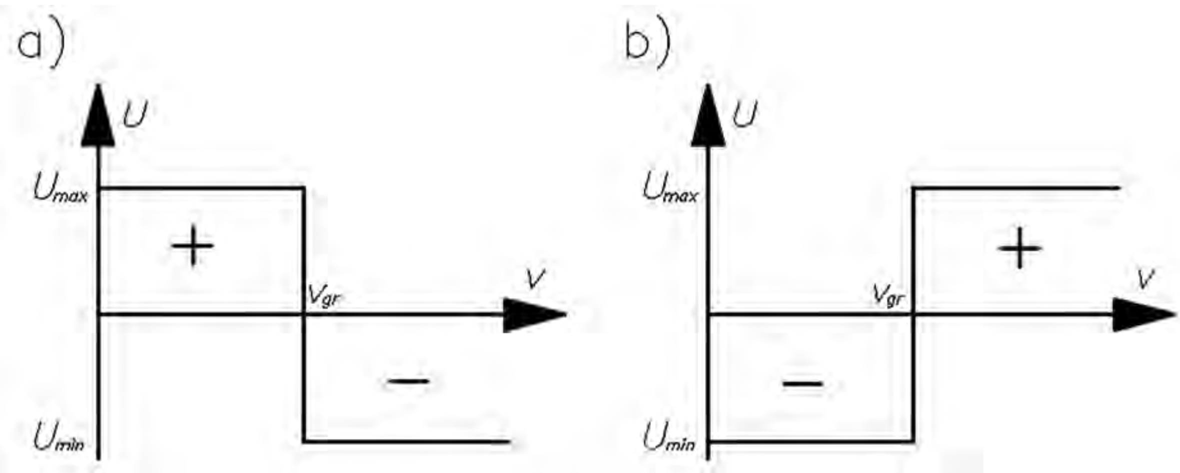
Parameter	Symbol	Value
Plate's Mass Moment of Inertia	I_d [kg/m²]	0.0945
Eliminator's Mass Moment of Inertia	$I_e = I_d/5$ [kg/m²]	0.0189
Spring Stiffness	$\kappa_2 = \frac{G_{12} \cdot J_o}{2 \cdot l}$ [N/rad]	14400
Damping Coefficient For Bigger Gap	c_1 [Ns/rad]	1
Damping Coefficient For Smaller Gap	c_2 [Ns/rad]	50

Below, one can find the chart showing the result amplitude and the curves crossing point, which is amplitude's maximum point.



Picture 5. System's parameters change strategy: black curve is showing the system without eliminator, red curve - system with eliminator and with spring switched off, blue curve – system with eliminator and with spring switched on. Green color indicated resulting amplitude.

Searched frequency value is v_{gr} = 740 rad/s. Based on this value, scheme representing PZT Ceramics electrodes polarization change strategy can be shown.



Picture 6. Piezoelectric elements electrodes polarization change strategy: a) for case of rings assembled on the shaft, b) for case of rings assembled on the casing's inner surface.

7. SUMMARY

Use of semi-active dynamic vibration elimination method allowed to obtain very gentle change of amplitude in studied frequency range (very sharp resonance was eliminated). Comparing to conventional dynamic elimination, following results were obtained:

three times smaller amplitude comparing to conventional eliminator characterized by following parameters: c = 50 Ns/rad, = $\frac{G_{12} \cdot J_o}{2 \cdot l}$ = 14400 N/rad,

five times smaller amplitude comparing to conventional eliminator using c = 1 Ns/rad, = 0.

What should be mentioned again is the must to switch the spring off the system when it's working below limiting frequency. If one would like to count only on inertia influence (that can be visualized by moving the curve created with eliminator taken into account, away from the curve created with eliminator not taken into account), obtained results would be unsatisfactory – results are almost the same as for conventional eliminator using the same parameters. Further increase in eliminator's inertia would result in bigger size of the overall assembly, that's why other solution was needed. Satisfactory results were obtained by periodic elimination of the spring from the system.

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Michal Stepkowski

WYKORZYSTANIE ELEMENTÓW PIEZOELEKTRYCZNYCH W ZŁOŻONYM SYSTEMIE SKRĘTNYM Z DYNAMICZNYM ELIMINATOREM DRGAŃ

Streszczenie

Poniższy artykuł opisuje wynik Pracy Magisterskiej napisanej w Instytucie Podstaw Budowy Maszyn Politechniki Warszawskiej.

Jedną z celów przemysłu samochodowego jest redukcja masy. W wielu przypadkach sprowadza się ona do zmiany materiału z konwencjonalnej stali bądź aluminium na kompozyt. Tego typu zmiana może mieć bardzo duży wpływ na na odpowiedź dynamiczną układu, co w wyniku może się objawić wyższą amplitudą drgań oraz/lub przesunięciem częstości własnych układu do zakresu prędkości pracy maszyny.

Jedną z najczęściej przeprojektowywanych w ten sposób części jest wał napędowy samochodu. Poniższa praca opisuje problem drgań skrętnych działających na układ z wałem kompozytowym oraz dynamicznej eliminacji tych drgań. Począwszy od modelu fizycznego i jego przełożenia na model matemetyczny, przedstawione są wyniki pracy, sposób poprawy dynamiki układu oraz zalety wybranej metody.