COMPUTER METHODS IN OPTIMALIZATION OF FIBER COMPOSITES STRUCTURE

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Summary

Composite materials are currently the most dynamic area of the modern engineering industry. Engineers around the world are trying to improve the composites performance to fulfill the requirements of modern constructing. Due to the existing optimization algorithms and computer tools, there is a huge potential in this area. The following article covers a study of the fiber composites optimization using the computer methods both in micro and macro scale. The results of computer analysis have been compared with the well-known analytical models.

1. INTRODUCTION

Due to coupling of FEM (Finite Element Modeling) and optimalization algorithms, it is possible to perform calculations in order to determine the best construction, and not only to analyze. Composites mechanics [11] is one of the most interesting fields, in which such method can be applied. Optimalization designing can refer to the composites in both scales – micro (representative volume element's structure) or macro (the structure of laminate, the shape of constructing element).

The homogenization of a layer's features is a part of the computer assisted designing. The material properties of a layer – considered as homogeneous orthotrophy material – result from the parameters of each components shapes and features. The number of layers, their thickness and the way they are combined influence mechanical properties of the multilayer constructing elements. Design variables, such as parameters determining the composite's structure or material properties, are part of the data used in the optimalization process. The most desirable combination of design variables results in the minimal value of the function of goal. The function of goal represents the features of particular structure. It is defined in the way that its reduction results with an improved quality.

The optimalization analysis has been performed on a laminate shell and multilayer cylindrical shell examples. The results show that the FEM simulation models are useful for designing of composites structures. The results of the optimalization process depend on many factors and require multiple analyses with varying start parameters and convergence criteria. Verifying the obtained solutions is also crucial.

The fiber reinforced metal matrix composites are becoming more and more popular theses days [8, 9]. It is due to their particular physical and strenght properties. The following features: good resistance to axial tension, modulus of rigidity, electrica and thermal conductivity, determine possible functionnality/application of the compositès. However, due to relatively high production costs, the composites are only being used in construction elements, when financially viable.

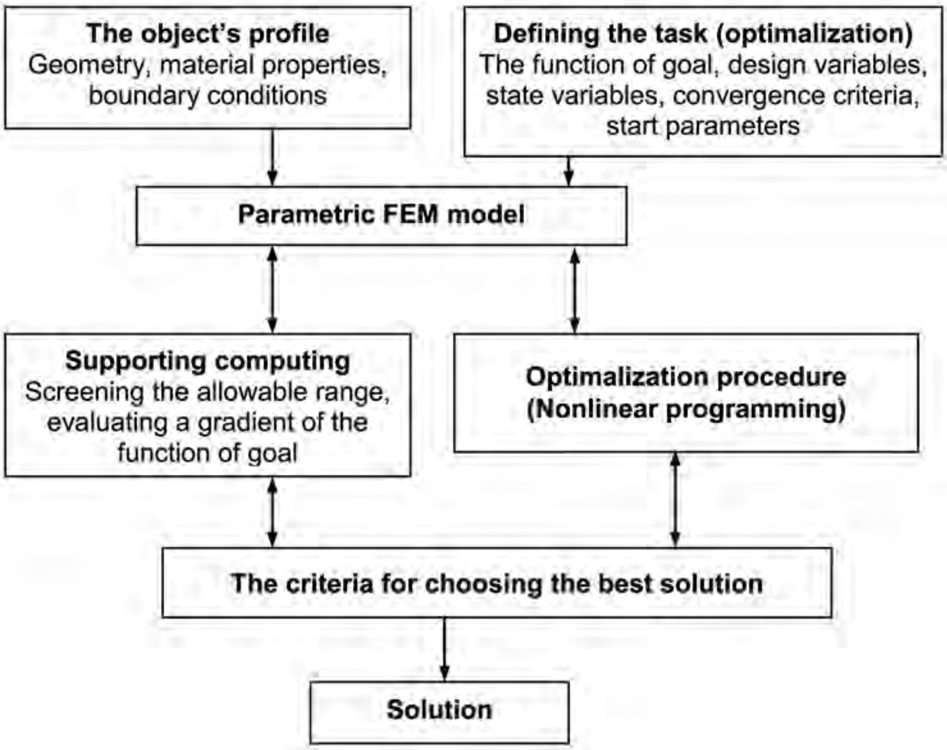


Fig. 1. Final element modeling and optimalization algorithms in the computer assisted designing of composites

Final element modeling method [3] is widely implemented for the analysis of properties and resistance of the composites' structures. Fields of stresses, strains, heat transfers, electric and magnetic field, as well as coupled fields are analyzed [2, 4, 5, 6, 7].

It is highly probable, that computer methods will be commonly used in engineering, not only to analyze particular structures, but also to design them.

2. PARAMETRICAL OPTIMALIZATION ALGORITHMS

The implementation of both FEM and numeric optimalization techniques is possible on the condition of building a parametric FEM model. The parametric process consists of three stages: preprocessing, solution and post processing. Such procedure is an iterating process. The design variables are different for each iteration; however they are being determined according to one optimalization algorithm. All possible parameters defining the shape of the object, material properties, constrain or loads conditions can make for the design variables. Design variables' vector is expressed as: $x=[x_1,x_2,...,x_n]$. Possible criteria of the function of goal $f=f(x)$, could be as follows: minimum of stresses, minimum of volume or minimum of weight etc.

The allowable range X can be defined by determining the minimal and maximal values of design variables or by physical parameters of the modeled object.

$$x_i^* \le x_i \le x_i^{**} \quad (i=1,2,...,k), \quad w_i^* \le w_i(x) \le w_i^{**} \quad (i=1,2,...,m) \tag{1}$$

The allowable range of solutions is a set of points in n-dimensional Euklidean space, corresponding with the above- mentioned conditions.

$$x \in X, \quad X \subset R^n$$

The function of goal is becoming close to the minimum for the subsequent iterations. The process of iteration calculations stops, when the vector of current design variables $x^{(j)}$ is within allowable range and all predefined conditions are met.

$$\left|f^{(j)} - f^{(j-1)}\right| \le \tau, \quad \left|x_i^{(j)} - x_i^{(j-1)}\right| \le \rho_i \quad (i=1,2,3,...,n) \tag{2}$$

In order to find the minimum of the function of goal, gradient and non gradient methods of the nonlinear programming are being applied [10]. The optimalization process of a construction, performed with the use of complex FEM models made of huge number of finite elements, requires significant expenditure of numeric computing/is related with serious numeric costs. Therefore in order to increase effectiveness of the analysis, FEM equations are being used for fast evaluation of the gradient of function of goal (by examining in what way the solution changes, depending of variations of design variables). Parametric FEM models be effective tools for designers (Fig. 1). The supporting computing includes:

- A separate analysis of a specified set of design variables. Such computing can be performed for any allowable x vector.
- Evaluation of the gradient. There is a special procedure to calculate the gradient of function of goal for a given x vector. The procedure shows to what extend the function is sensitive to the alteration of x_i parameter.
- Screening the allowable range, either randomly or by altering one of the variables. Assuming the rest of variables is predefined.

All of the above techniques are useful for optimalization method and its particular parameters (start point, primal step, criteria of convergence). They are also helpful, when optimalization is particularly difficult. The best solution is being chosen from/within the results of supporting computing. The analysis presented in this thesis was performed, using Ansys FEM program (in Ansys Parametric Design Language).

3. PARAMETRIC MODELS OF REPRESENTATIVE VOLUME ELEMENT OF THE COMPOSITE

Resultant coefficient of rigidity for the complex and non homogeneous structures can be estimated using theortical formulas or numerical computing. The computing can be performed for RVE, loaded with subsequent component of stress. Representative volume is the smallest hording fragment of the composite. The fragment, analysed under the right boundering conditions (load, constrain), allows to evaluate the stress in micro scale, as well as the composite's properties in a macro scale.

Examples of models used for evaluation of composite's resultant ellasticity coefficients are shown on the Fig. 2.

Knowing the components of stress, the element is exposed to and having obtained components of strain, it is possible to evaluate elements of the matrix of elasticity coefficients from constitutive equation:

$$\varepsilon_i = S_{ij}\sigma_j \quad (i, j = 1, 2, \ldots, 6) \tag{3}$$

where: ε_i – stands for components of strain, σ_j – stands for components of stress, S_{ij} – matrix of vulnerability [1].

The following start values of geometrical parameters were assumed (Fig. 2):

$X_1 = r = 0.7$[mm], $X_2 = a = 0.15$[mm], coordinate axe y: $X_3 = b = 0.15$[mm], $c = 2a = 0.3$[mm].

The volume contribution of fiber in this case is:

$$V_f = \frac{0.5\pi r^2}{ab} = \frac{0.5\pi \cdot 0.07^2}{0.15 \cdot 0.15} = 0.34208$$

The following properties of material were assumed:

Copper: $E_m = 126\,086$[MPa], $v_m = 0.32$;

Silicon and carbide fiber: $E_f = 413807$[MPa], $v_f = 0.16$.

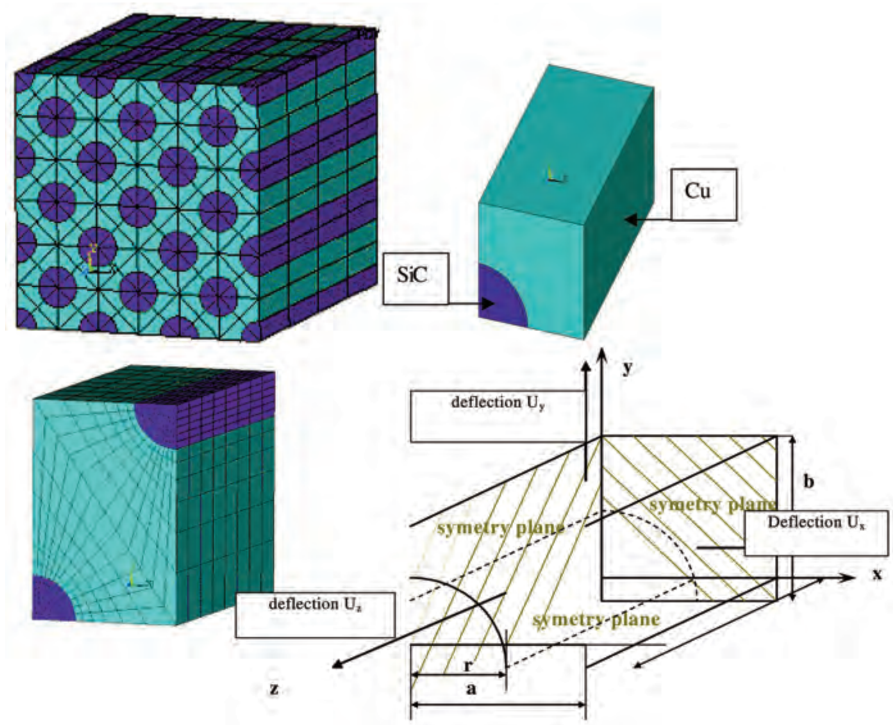


Fig. 2. Hording RVE and the bounding condition in FEM analysis

Subsequent simple loads, corresponding to the stretching σ = 100[MPa] on the three coordinate axes x, y and z, and to the shearing in three perpendicular planes, allow to identify stress and strain fields (Fig. 3, 4).

Table 1 shows the resultant coefficients estimated for the assumed values. The resultant coefficients can be evaluated by using theoretical formulas, however the accuracy of results may not be precise enough.

Building a parametrical model helps to estimate the sample's performance, contingent on particular parameters of the composite's structure. (Fig. 5, 6, 7). The results show (Fig. 6, 7) that increased fiber volume contribution leads to reduction of stress in both, the fibers and the matrix.

Table 1. Calculated resultant elasticity coefficients

	Theoretical model	FEM model
E_z[GPa]	224.5	225.6
E_x[GPa]	165.4	177.7
E_y[GPa]	165.4	177.7
$\nu_{zx}=\nu_{zy}$	0.27	0.26
$\nu_{xz}=\nu_{yz}$	0.19	0.20
$\nu_{xy}=\nu_{yx}$	0.24	0.35
G_{xy}[GPa]	63.7	71.4

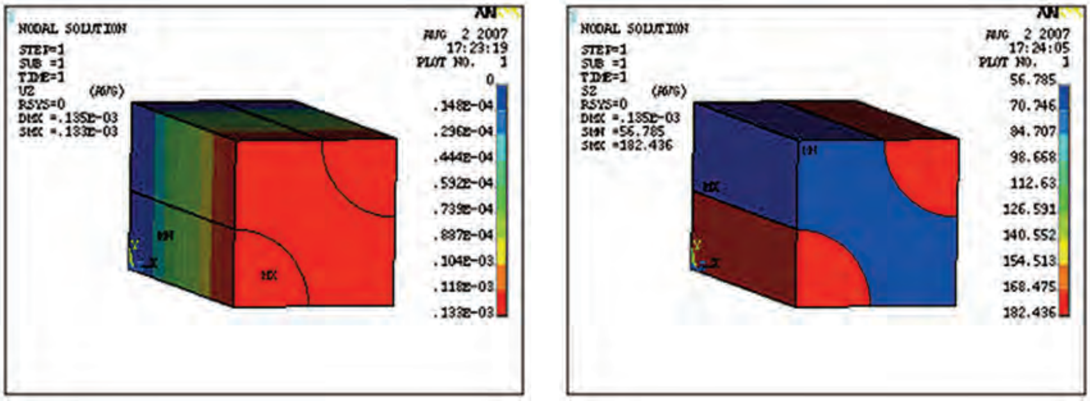


Fig. 3. Axial tension test (fiber direction) σ_z = 100[MPa] – distribution u_z[mm] and σ_z[MPa]

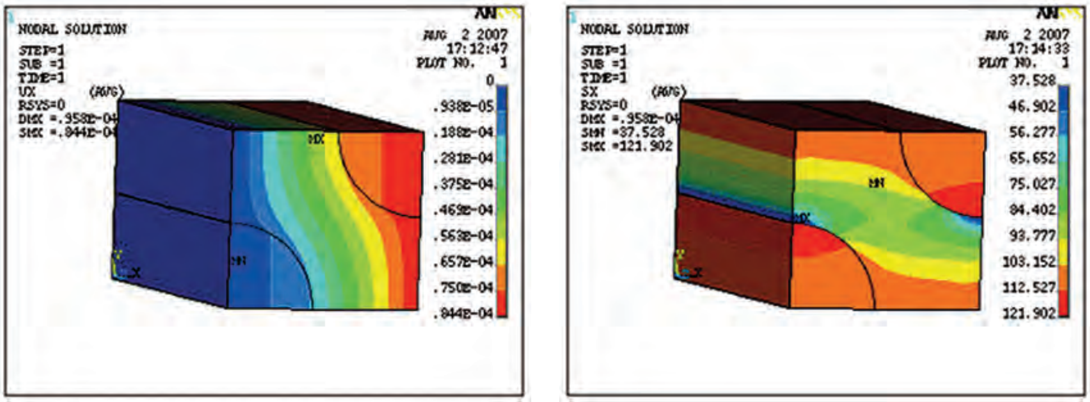


Fig. 4. Axial tension test (fiber direction) σ_x = 100[MPa] – distribution u_x[mm] and σ_x[MPa]

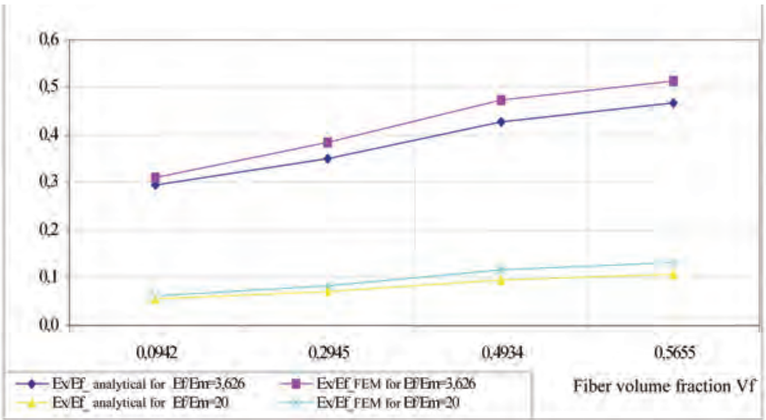


Fig. 5. Function of E_x/E_f to the fiber volume contribution for different proportions E_f/E_m

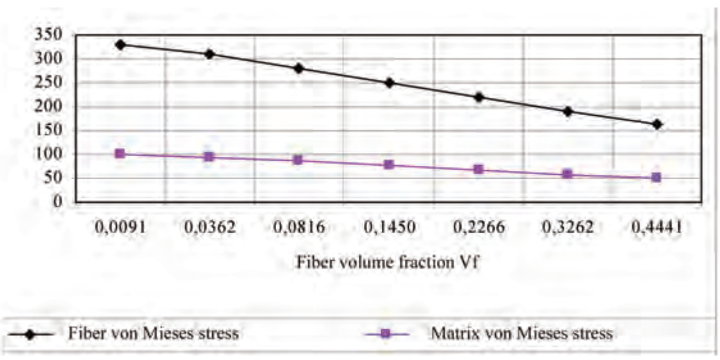


Fig. 6. Function of von Mieses stress in fiber and matrix to fiber volume contribution – Axial tension 100[MPa]

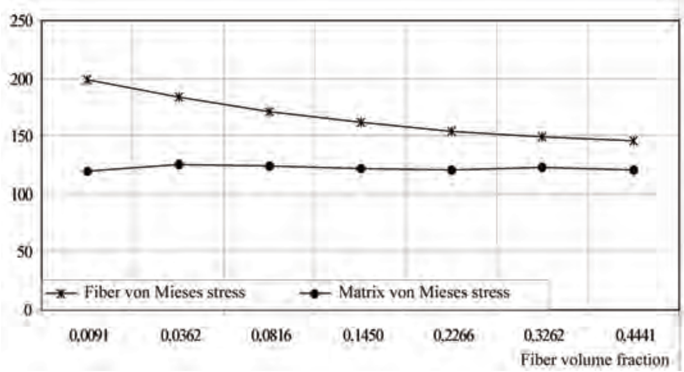


Fig. 7. Function of von Mieses stress in fiber and matrix to fiber volume contribution – diagonal tension 100[MPa]

4. THE EXAMPLES OF APPLIANCE OF NON LINEAR PROGRAMMING FOR DESIGNING OF THE COMPOSITE'S STRUCTURE

4.1. Validation of the numerical model of a laminate

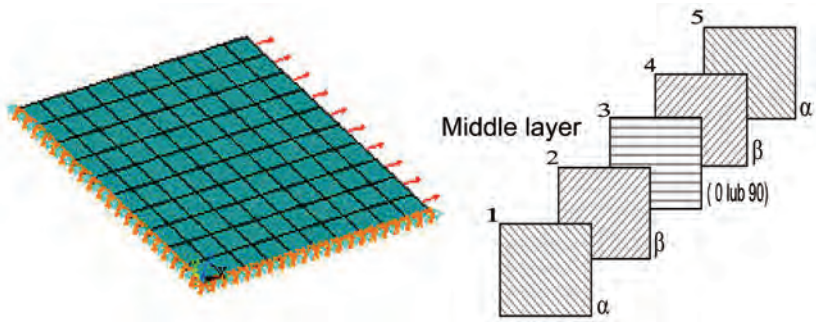


Fig. 8. Model of a 5-layers plate

The following geometrical parameters and material properties of the multilayer plate were assumed for computing (Fig. 8):

width: a = 20[mm],
height: b = 20[mm],
tightness of the laminate: t = 5[mm],
number of layers: n = 5,
longitudinal Young modulus: E_x = 225.6[GPa],
diagonal Young modulus: E_y = 177.7[GPa],
Poisson's coefficient: v_{xy} = 0.26,
coefficient of rigidity: G_{xy} = 71.4[GPa].

The laminate plate was loaded with continuous loads q = 100[kN/m] in direction of x axial. The properties of laminate can be determined, basing on the previously evaluated properties of a single layer.

Table 2. The table shows the summary of results from analytical and FEM analysis – 2D laminate model (layers' sequence 90°, 0°, 90°, 0°, 90°)

	Theoretical stress values		Numerical stress values	
Layer	0°	90°	0°	90°
σ_x[MPa]	23.07	17.95	23.08	17.95
σ_y[MPa]	0.67083	–0.44732	0.6889	–0.4593

4.2. The optimalization of layers' sequence in a multilayer plate

The angles of fibers in a single layer were variables in the parametric model of a laminate plate:

1. $x_1 = \alpha$,
2. $x_2 = \beta$,
3. 0,
4. $x_2 = \beta$,
5. $x_1 = \alpha$.

For this kind of model, each element of the solution can be considered a function of two arguments $f(x_1,x_2)$. A series of optimalization computing was performed, using gradient and non gradient methods. (respectively: Subproblem Approximation Method and First Order Method). The results were being analyzed, with regard to both: design variables and optimalization method. Minimal deflections were the function of goal. The Fig. 9 and 10 show the course and the results of optimalization computing of a simple tension of the plate.

The numeric computing results correspond with the assumption (fiber packing in the direction of tension). The function of goal (the deflection u_x) was reduced by 18% approx.

Table 3. The values of parameters before and after optimalization process of the stretched plate (Fig. 8)

Method	BEFORE OPTIMALIZATION			AFTER OPTIMALIZATION			Improvement [%]
	$X_1=\alpha$	$X_2=\beta$	$f=UX$ [mm]	$X_1=\alpha$	$X_2=\beta$	UX [mm]	
Gradient (Fig. 9)	80°	85°	0.00225	0°	0°	0.00185	17.65
Non gradient (Fig. 10)	80°	85°	0.00225	0°	0°	0.00185	17.61

The computing was repeated for different loads. The results corresponding with the two-directional loads (Fig. 11) are shown in Fig. 12, 13 and the Table 4. In this case both results are convergent; however the accuracy of the minimum of the function of goal is worst.

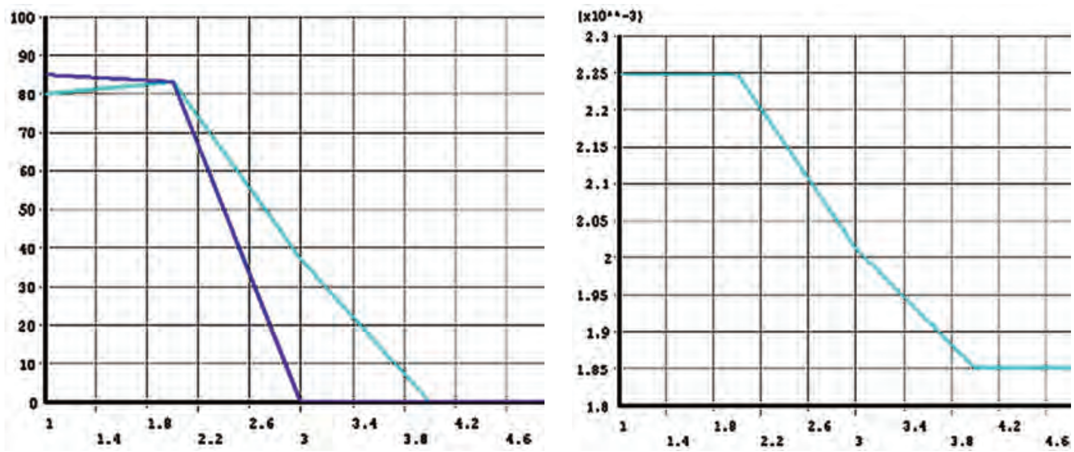


Fig. 9. The course of search for the minimum of deflection in the stretched laminate plate (Fig. 8): changes in angles of fibers in a single layer x_1, x_2 and corresponding alternations of the function of goal $f(x_1,x_2)$ = max u_x. Gradient method

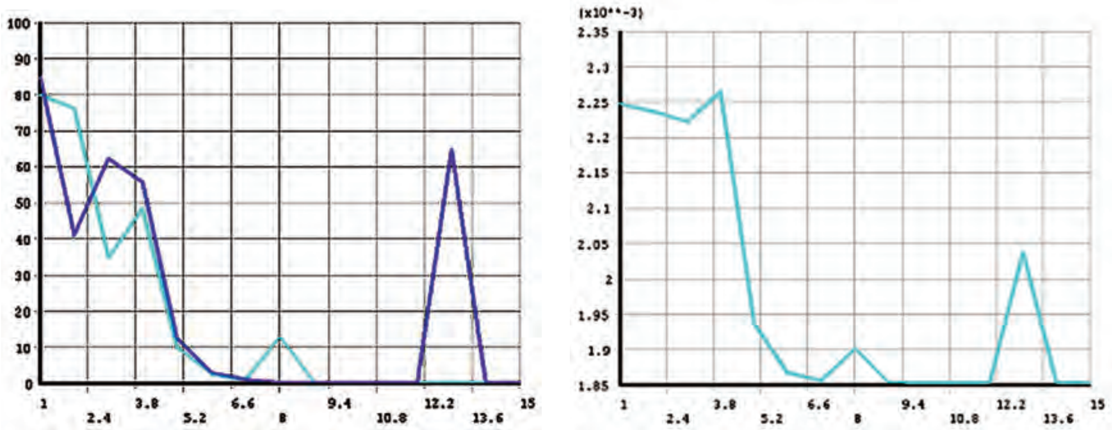


Fig. 10. The course of search for the minimum of deflection in the stretched laminate plate (Fig. 8): changes in angles of fibers in a single layer x_1, x_2 and corresponding alternations of the function of goal $f(x_1,x_2)$ = max u_x. Non gradient method

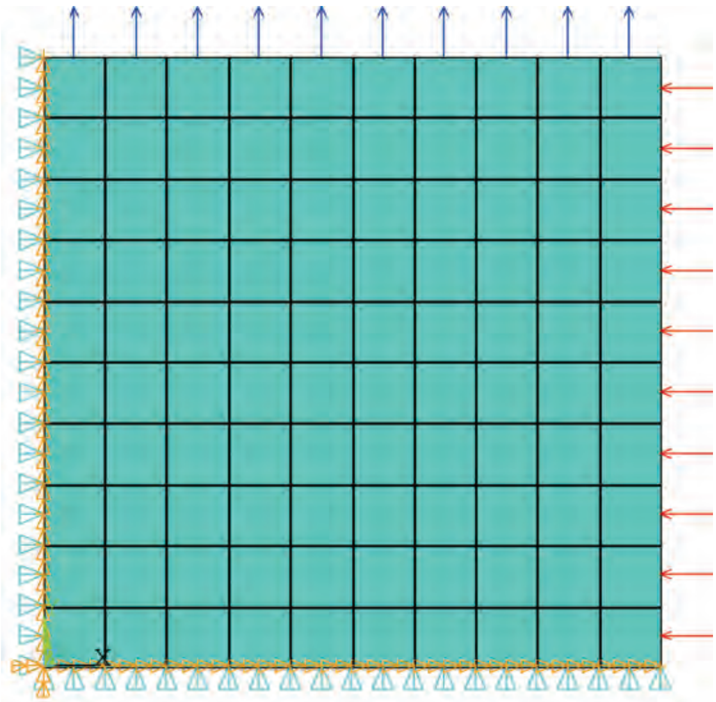


Fig. 11. Two-directionally loaded multilayer plate ($q_x = -100$[kN/m], $q_y = 100$[kN/m])

Table 4. The values of parameters before and after the optimalization process of two-directionally loaded plate (Fig. 11)

Method	BEFORE OPTIMALIZATION			AFTER OPTIMALIZATION			Improvement [%]
	$X_1=\alpha$	$X_2=\beta$	$f=UX$ [mm]	$X_1=\alpha$	$X_2=\beta$	UY [mm]	
Gradient (Fig. 12)	5°	85°	0.005	53°	53°	0.00037	25.18
Non gradient (Fig. 13)	5°	85°	0.005	56°	55°	0.00037	25.01

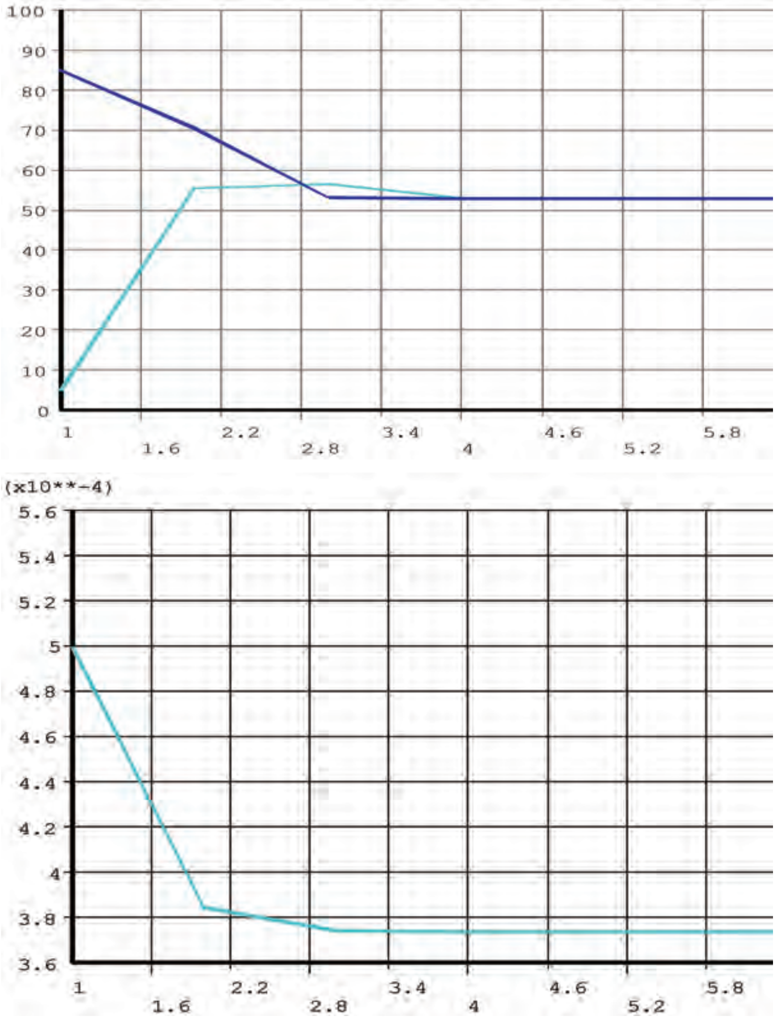


Fig. 12. The course of searching for the minimum of deflection in the stretched laminate plate (Fig. 11): changes in angles of fibers in a single layer x_1, x_2 and corresponding alternations of the function of goal $f(x_1,x_2)$ = max u_x. Gradient method

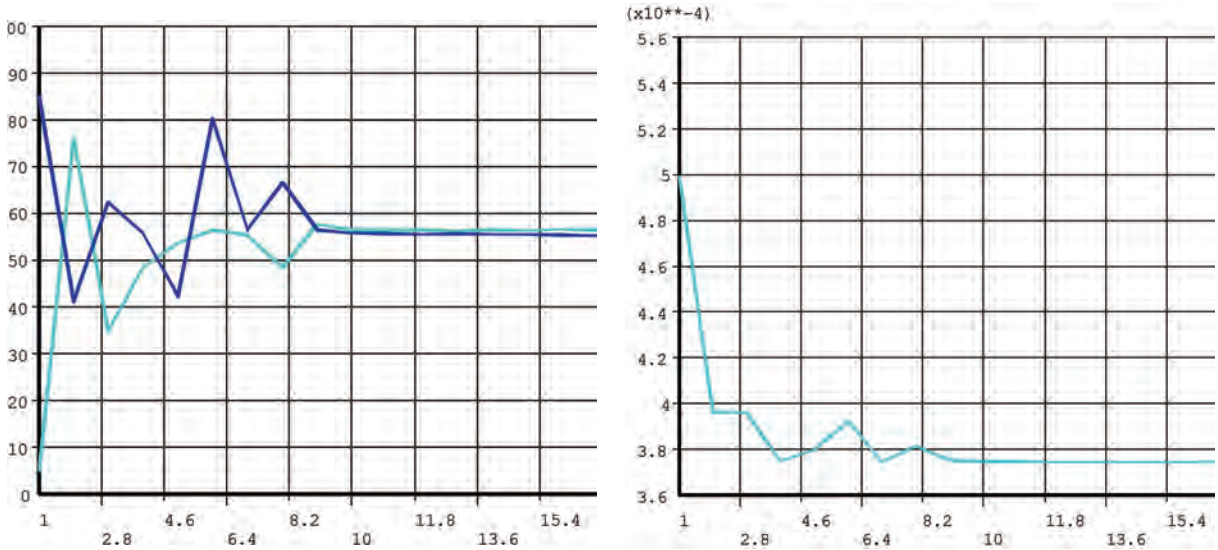


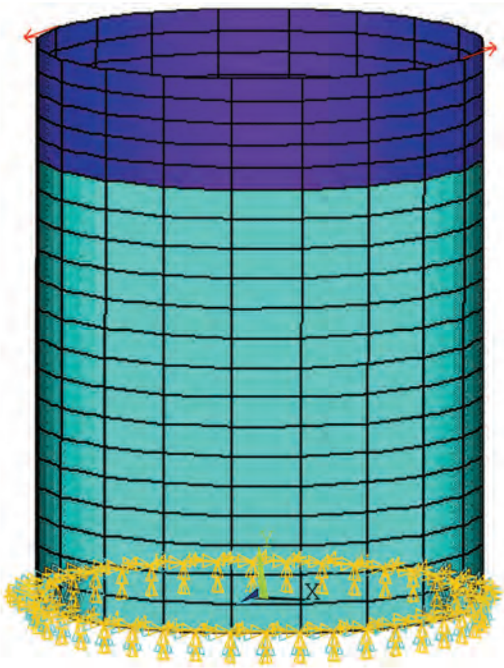
Fig. 13. The course of search for the minimum of deflection in the stretched laminate plate (Fig. 11): changes in angles of fibers in a single layer x_1, x_2 and corresponding alternations of the function of goal $f(x_1,x_2)$ = max u_x. Non gradient method

4.3. The optimalization of the layers sequence in cylindrical shell

The computing was repeated for different loads. In the following section, the authors present results of search for the minimal value of twist angle. The structure and properties of the laminate are the same as in case of other examples.

The results corresponding with the two-directional loads (Fig. 11) are shown in Fig. 12, 13 and the Table 4. In this case both results are convergent; however the accuracy of the minimum of the function of goal is worst.

For the balanced distribution of the twist moment in a form of coupled forces, a "stiff rib" was joint to the model (Fig. 14).



Radius: r = 20[mm]
Height: b = 20[mm]
Fig. 14. FEM model of multilayer model of a twisted cylindrical shell

The results are shown in Table 5 and in the Fig. 15, 16. In this case the vulnerability of function of goal (twist angle) to the variation of design variables was small. The differences in optimal results are due to lack of accuracy in numerical computing (different x_i after optimalization). The values of the function of goal are close to equal in both cases.

Table 5. The values of parameters before and after optimalization process of twisted cylindrical shell (Fig. 14)

Method	BEFORE OPTIMALIZATION			AFTER OPTIMALIZATION			Improvement [%]
	$X_1=\alpha$	$X_2=\beta$	$f=\varphi$ [mm]	$X_1=\alpha$	$X_2=\beta$	$f=\varphi$ [mm]	
Gradient (Fig. 15)	5	85	0.0581	36	0	0.05580	3.98
Non gradient (Fig. 16)	5	85	0.0581	37	18	0.05585	3.89

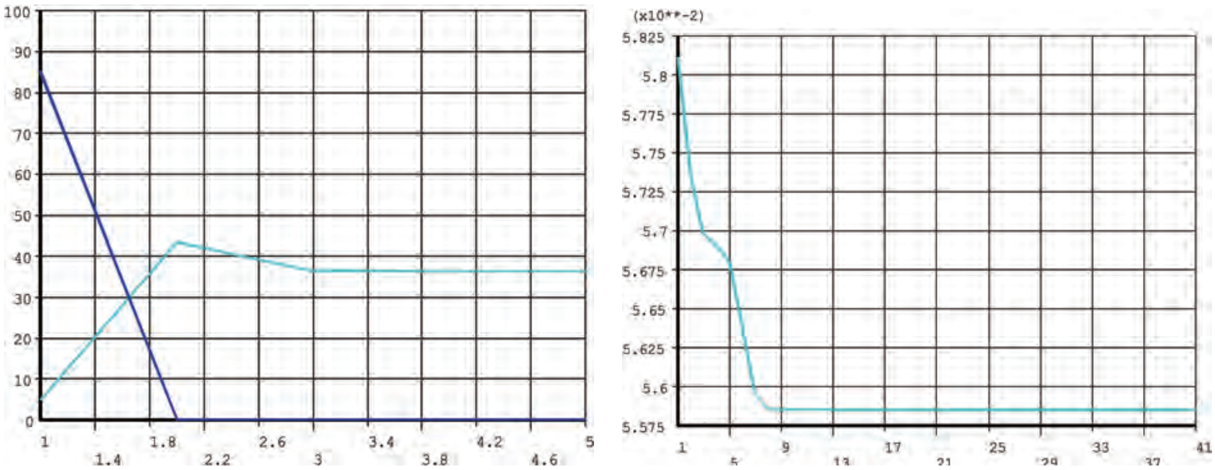


Fig. 15. The course of searching for the minimal angle of twist in cylindrical shell: changes in angles of fibers in a single layer x_1, x_2 and corresponding alternations of the function of goal $f(x_1,x_2) = \varphi = max\ u_z/R$. Non gradient method

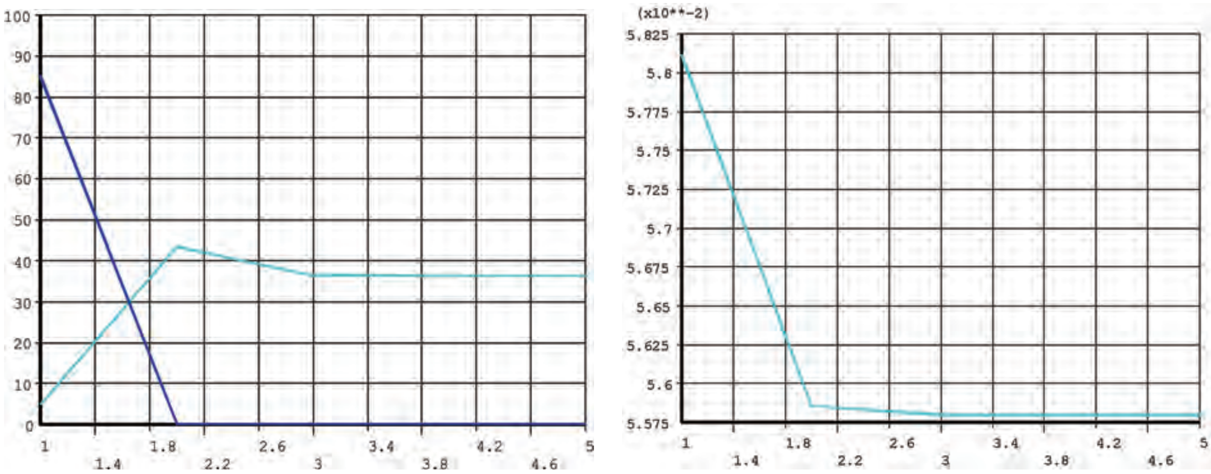


Fig. 16. The course of searching for the minimal angle of twist in cylindrical shell: changes in angles of fibers in a single layer x_1, x_2 and corresponding alternations of the function of goal $f(x_1,x_2) = \varphi = max\ u_z/R$. Gradient method

5. CONCLUSIONS

The way of designing the composites is quite simple, in practice. It possible however, to obtain better results, using numeric methods for designing.

The FEM analysis method offers reliable results, in terms of both micro and macro scales. The examples presented in this thesis show significant utility of parametric FEM models.

The potential of analytical methods in terms of composites mechanics is limited to simple models. In such cases FEM results are similar to theoretical formulas. Numeric methods are irreplaceable when solving an engineering issue is necessary. Almost all of these issues are very complex cases.

It was proven, the theoretical formulas allow accurate evaluation of the average elasticity properties (e.g. resultant longitudinal Young modulus for the continuous fiber composite) and only approximate evaluation of other properties (resultant diagonal Young modulus, stress values). Parametric FEM models allow evaluate mechanical parameters with significant precision if refers the composite material's structure, as an element of construction.

The building of parametric FEM models offers more algorithmic approach to designing. An optimal structure of a composite element is defined by common procedures of non linear programming [10]. Designing with the use of numeric methods requires a critical approach to obtained results. The computing is vulnerable to streamlines in a model and to FEM desecrating. Moreover iteration techniques of optimalization lead to finding local minima for the function of goal, instead of global minima, within the whole allowable range.

Therefore in order to make sure the obtained solution is correct, the optimalization process is repeated several times, for different values of parameters of optimalization process (Fig. 1).

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METODY KOMPUTEROWE W OPTYMALIZACJI STRUKTUR KOMPOZYTOWYCH

Streszczenie

Materiały kompozytowe to obecnie bardzo dynamicznie rozwijająca sie branża. Inżynierowie na całym świecie pracują nad ich udoskonalaniem pod kątem wymagań stawianych przez współczesne konstrukcje. Rozwój metod i narzędzi do analizy komputerowej daje olbrzymie możliwości działań na tym polu. Prezentowany tu artykuł zawiera przykłady optymalizacji materiałów kompozytowych z włóknem węglowym zarówno w skali mikro jak i makro. . Wyniki komputerowej analizy zostały porównane z dobrze znanymi modelami analitycznymi.