Przemysław WŁODARSKI
WEST POMERANIAN UNIVERSITY OF TECHNOLOGY, FACULTY OF ELECTRICAL ENGINEERING,
DEPARTMENT OF SIGNAL PROCESSING AND MULTIMEDIA ENGINEERING, 26 Kwietnia 10, 71-126 Szczecin

Markov Modulated Bernoulli Process in modeling network traffic

Ph.D. eng. Przemysław WŁODARSKI

Is an assistant professor in the Department of Signal Processing and Multimedia Engineering at the West Pomeranian University of Technology in Szczecin. He received his M.Sc. degree in telecommunication systems from the Department of Electrical Engineering and Ph.D. degree in computer networks from the Department of Computer Science at Szczecin University of Technology. His main research interests are in the area of network traffic modeling and analysis, network protocols, queueing systems and statistical signal processing.

e-mail: przemyslaw.wlodarski@zut.edu.pl

Abstract

A big impact of self-similar nature of network traffic on queueing performance and Quality of Service in computer networks involves new modeling techniques to provide low computational complexity and better approximation of the real traffic. A discrete-time Markov modulated Bernoulli process is one of the traffic models that captures fractal behavior and can be used in performance testing of queueing systems such as routers or switches. In this paper the model is presented and evaluated from the point of view of the desired self-similarity level. New parameter values that give better results for estimation of Hurst exponent has been proposed. Simulation results are compared with the literature data.

Keywords: network traffic modeling, self-similarity, network performance.

Proces Bernoulliego modulowany łańcuchem Markova w modelowaniu ruchu sieciowego

Streszczenie

Duży wpływ samopodobnego charakteru natężenia ruchu w sieciach komputerowych na efektywność pracy i poziom usług (Quality of Service) w sieci powoduje powstawanie nowych technik modelowania ruchu o niskiej złożoności obliczeniowej, lepiej dopasowanych do rzeczywistego ruchu. Proces Bernoulliego modulowany łańcuchem Markova będący dyskretnym procesem stochastycznym, ma właściwości które powodują, że przy odpowiednim doborze parametrów może on stanowić dobry model natężenia ruchu w sieci uwzględniający zjawisko samopodobieństwa, a co za tym idzie, może być z powodzeniem stosowany do testowania systemów kolejkowych zarówno teoretycznych jak i rzeczywistych takich jak routery czy switche. W artykule zaprezentowano i sprawdzono model pod kątem możliwości osiągnięcia założonego poziomu samopodobieństwa. Zaproponowano nowe wartości parametrów modelu dające lepsze rezultaty estymacji wykładnika Hursta. Wyniki symulacji porównano z danymi literaturowymi.

Słowa kluczowe: modelowanie natężenia ruchu sieciowego, samopodobieństwo, efektywność sieci.

1. Introduction

Traffic measurements taken at packet computer networks show the presence of a significant phenomena that can be seen at multiple time scales [2, 5, 6, 8]. This phenomena is called Long Range Dependence (LRD) and is connected with self-similarity. The main difference between traditional and modern models of network traffic is that the autocorrelation function of the latter decays very slowly and is not summable, i.e.:

$$\sum_{k=0}^{\infty} r(k,H) \to \infty \text{ ,where for } k = 0, 1, \ldots$$

$$r(k,H) = \frac{\sigma^2}{2}\left[(k+1)^{2H} - 2k^{2H} + |k-1|^{2H}\right]. \quad (1)$$

The larger H value (Hurst exponent) the slower decay of the autocorrelation function. Another property is the variance of aggregated random variable [4]:

$$\mathrm{var}\left(X^{(m)}\right) \sim \alpha \cdot m^{2H-2}, \quad (2)$$

where $X_k^{(m)} = m^{-1}\left(X_{km-m+1} + \ldots + X_{km}\right)$.

Highly correlated traffic involves longer network delays and poor queueing performance [11]. A packet loss probability is higher for the self-similar input traffic since a buffer overflows occur more frequently. It causes a big difficulties in Quality of Service assurance and in congestion control mechanisms. Recent research shows that the classical queueing theory cannot be applied to the network traffic, but on the other side a new fractal queueing theory is not well developed. Thus, it is very important to use the proper model of the traffic in analysis and simulation studies for designing new as well as enhancing existing computer networks.

2. MMBP – evaluation and new values for setting parameters

Markov Modulated Bernoulli Process (MMBP) is based on discrete-time Markov chain modulated by Bernoulli process [1], [9, 10]. It is a discrete-time counterpart of Markov Modulated Poisson Process (MMPP) described in [3]. MMBP is described using only 3 parameters: n, a and q, where n denotes number of states in modulating Markov chain ($i = 0, 1, \ldots, n-1$), and a, q are parameters that determine a self-similar level and the mean of the process. In every discrete time step a transition as well as staying in the state can take place. As one can see in Fig. 1, except for the zero state, two situations are possible: a) return to state 0 b) staying in the state. Only during the stay in state 0, a single packet is generated for every time slot.

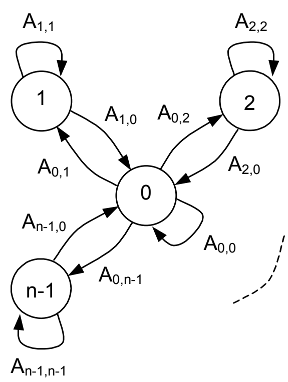

Fig. 1. Modulating Markov chain of MMBP model
Rys. 1. Modulujący łańcuch Markova w modelu MMBP

The matrix of transition probabilities have the following form:

$$A = \begin{bmatrix} 1-\frac{1}{a}-\frac{1}{a^2}-\ldots-\frac{1}{a^{n-1}} & \frac{1}{a} & \frac{1}{a^2} & \cdots & \frac{1}{a^{n-1}} \\ \frac{q}{a} & 1-\frac{q}{a} & 0 & \cdots & 0 \\ \left(\frac{q}{a}\right)^2 & 0 & 1-\left(\frac{q}{a}\right)^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{q}{a}\right)^{n-1} & 0 & 0 & \cdots & 1-\left(\frac{q}{a}\right)^{n-1} \end{bmatrix}. \quad (3)$$

It can easily be seen that the following condition holds:

$$\sum_{j=0}^{n-1} A_{i,j} = 1 \text{ for } i = 0,1,\dots,n-1 . \quad (4)$$

In order to obtain a stationary state distribution (π_i) for any Markov chain, one needs to solve the following set of equations:

$$\pi_i = \sum_{j=0}^{n-1} \pi_j A_{j,i} , \quad i = 0,1,\dots,n-1 \quad (5)$$

that can also be written as: $\vec{\pi} = \vec{\pi} \cdot A$, where $\vec{\pi} = [\pi_0, \pi_1, \dots, \pi_{n-1}]$. In our case, after substituting (3) into (5), one obtains:

$$\begin{cases} \pi_0 = \pi_0\left(1 - \frac{1}{a} - \frac{1}{a^2} - \dots - \frac{1}{a^{n-1}}\right) + \pi_1 \frac{q}{a} + \pi_2\left(\frac{q}{a}\right)^2 + \dots + \pi_{n-1}\left(\frac{q}{a}\right)^{n-1} \\ \pi_1 = \pi_0 \frac{1}{a} + \pi_1\left(1 - \frac{q}{a}\right) \\ \pi_2 = \pi_0 \frac{1}{a^2} + \pi_2\left[1 - \left(\frac{q}{a}\right)^2\right] \\ \dots \\ \pi_{n-1} = \pi_0 \frac{1}{a^{n-1}} + \pi_{n-1}\left[1 - \left(\frac{q}{a}\right)^{n-1}\right] \end{cases} \quad (6)$$

which leads to the following solution:

$$\pi_i = \frac{\pi_0}{q^i} \quad (7)$$

where π_0 is obtained from the condition:

$$\sum_{j=0}^{n-1} \pi_j = 1 \Rightarrow \pi_0 \sum_{j=0}^{n-1} q^{-1} = \pi_0 \frac{1-q^{-n}}{1-q^{-1}} = 1 \Rightarrow \pi_0 = \frac{1-q^{-1}}{1-q^{-n}} \quad (8)$$

Since packets are generated only in state 0, the mean value of the number of packets in unit time equals:

$$E(N) = \sum_{j=0}^{n-1} \pi_j A_{j,0} = \pi_0\left(1 - \frac{1}{a} - \frac{1}{a^2} - \dots - \frac{1}{a^{n-1}}\right) + \\ + \pi_0 \sum_{j=1}^{n-1} \frac{1}{q^j}\left(\frac{q}{a}\right)^j = \pi_0 = \frac{1-q^{-1}}{1-q^{-n}} \quad (9)$$

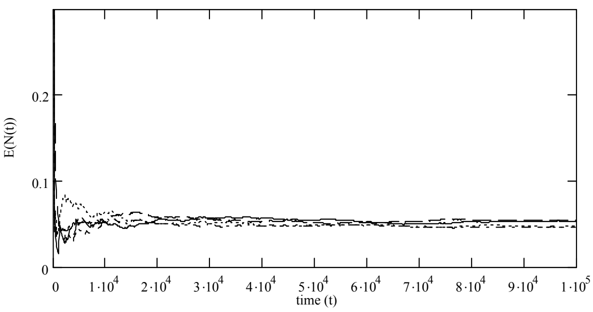

Fig. 2. Four traces of mean number of packets in unit time during time *t* for desired *H*=0.7 and *E*(*N*)=0.05
Rys. 2. 4 przebiegi średniej liczby pakietów w jednostce czasu *t* dla zadanego *H*=0.7 oraz *E*(*N*)=0.05

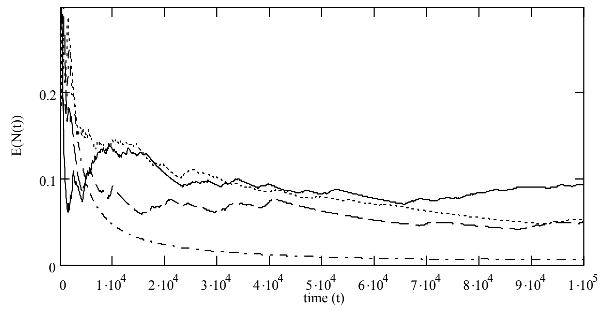

Fig. 2. Four traces of mean number of packets in unit time during time *t* for desired *H*=0.8 and *E*(*N*)=0.05
Rys. 2. 4 przebiegi średniej liczby pakietów w jednostce czasu *t* dla zadanego *H*=0.8 oraz *E*(*N*)=0.05

Mean values for the process are more unstable for higher values of Hurst exponent (fig. 2 and 3). From eq. (9) one can see, that for fixed *n*, mean value, which can be easily obtained from real network traffic, is determined only by *q* parameter. In order to set desired level of self-similarity we have to know a parameter. Since the variance of the aggregated random variable $N^{(m)}$ has the following form:

$$Var\left(N^{(m)}\right) = \frac{E(N)}{m}\left[1 + \frac{2}{m}\sum_{i=1}^{m-1}\left[(m-1)A_{0,0}^i\right]\right] - E(N)^2 . \quad (10)$$

Tab. 1. Literature and recalculated values of *a* and *q* for MMBP model
Tab. 1. Literaturowe i przeliczone wartości *a* oraz *q* dla modelu MMBP

n	H	a		q	
		literature	recalculated	literature	recalculated
4	0.70	2.65	5.35	0.4418	0.4417
	0.75	4.97	6.98		
	0.80	10.27	10.37		
	0.85	28.49	25.16		
5	0.70	2.60	3.73	0.5764	0.5766
	0.75	4.38	4.65		
	0.80	6.70	6.51		
	0.85	12.97	11.79		
	0.90	85.61	82.57		
6	0.70	1.99	3.00	0.6737	0.6739
	0.75	3.22	3.59		
	0.80	4.80	4.69		
	0.85	8.21	7.99		
	0.90	50.02	40.93		

Then, using variance-time method, *a* can be determined numerically for any *H*. In the same way the tabelarized *a* and *q* values for desired H and expected value $E(N)$ can be obtained.

Unfortunately, *a* and *q* values that was presented in [10] do not give good results of estimations of Hurst exponent (Fig. 4).

As it appears, these values differs significantly from the values obtained in this article (Table 1) using methods: variance-time, secants, bisection and linear regression. Recalculated values of parameters for 4-, 5- and 6-state MMBP model and for expected value $E(N)=0.05$ give better results (Table 2), which is caused by fitting to analytical variance for this process (eq. 10) for higher level of aggregation of random variable (above 1000) – Fig. 5.

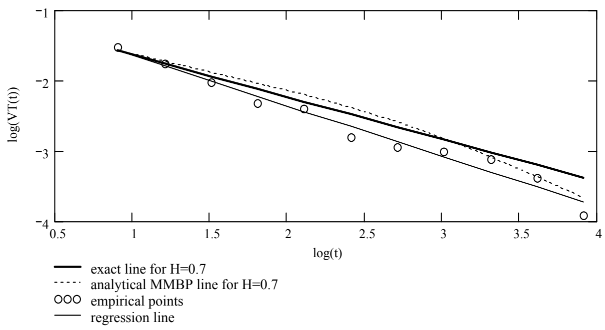

Fig. 4. Example VT analysis for literature *a* and *q*, desired *H*=0.7, estimated $\hat{H}=0.601$

Rys. 4. Przykładowa analiza VT dla literaturowych wartości *a* oraz *q*, zadany poziom *H*=0.7, estymowany $\hat{H}=0.601$

Tab. 2. Estimated Hurst exponent (*H*) for 5-state MMBP, *E*(*N*)=0.05

Tab. 2. Wyniki estymacji wykładnika Hursta (*H*) dla 5-stanowego procesu MMBP, *E*(*N*)=0.05

Desired *H* level	Estimated *H* for literature values of *a* and *q*			Estimated *H* for recalculated values of *a* and *q*		
	VT	IDC	Per	VT	IDC	Per
0.70	0.608	0.619	0.668	0.679	0.692	0.709
0.75	0.635	0.699	0.717	0.732	0.742	0.737
0.80	0.701	0.751	0.750	0.765	0.746	0.759
0.85	0.781	0.777	0.789	0.801	0.793	0.831
0.90	0.822	0.827	0.837	0.864	0.850	0.887

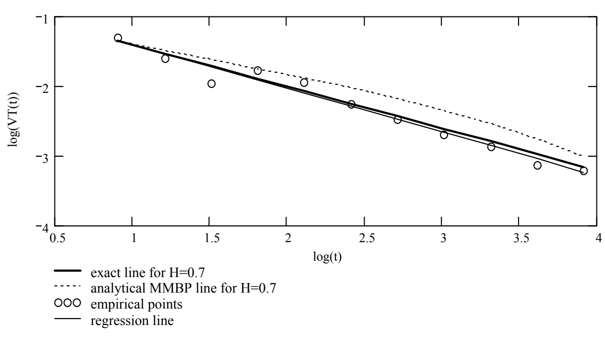

Fig. 5. Example VT analysis for recalculated, proposed *a* and *q* values, desired *H*=0.7, estimated $\hat{H}=0.687$

Rys. 5. Przykładowa analiza VT dla przeliczonych, proponowanych wartości *a* oraz *q*, zadany poziom *H*=0.7, estymowany $\hat{H}=0.687$

In spite of this, an estimated Hurst exponent values for both literature and recalculated *a* and *q* parameters seems to be unsatisfactory. Three methods of estimation have been used: variance-time (VT), Index of Dispersion for Counts (IDC) and periodogram (Per).

3. Conclusions

In this paper the MMBP model of network traffic have been analyzed. Since it has only three parameters that are responsible for fitting both intensity and the self-similarity level of the traffic, it is very tractable. However, it is suggested to use the *a* and *q* values proposed in Table 1, which corresponds to a more accurate estimates of Hurst exponent. Furthermore, one can notice quite big irregularities in the mean for higher *H* values. On the other hand, the MMBP process is very interesting from the point of view of application to queueing analysis because of discrete times, where time units can correspond to single packets, bytes or even bits. In spite of a few drawbacks, it is proposed to use this model in simulation studies, especially in analyzing queueing systems performance.

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otrzymano / received: 10.09.2010
przyjęto do druku / accepted: 01.11.2010 *artykuł recenzowany*