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Fuzzy Reasoning Algorithms for Position Fixing

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Abstract

Mathematical Theory of Evidence (MTE) provides methods for reasoning on certain hypothesis based on relative events. In navigation one tries to fix position based on imprecise indications of various aids. The Theory exploits events that can be expressed by fuzzy sets. In the presented application membership grades are degree of location of each element of the search space within selected ranges. Fuzzy sets contain allocation of points within an area of possible ship position. It will be shown how to use the scheme of Dempster-Shafer combination in order to fix position of the ship based on distances and/or bearings taken in terrestrial navigation. The method for adjustment of search space point location is discussed.

Keywords: belief structures, scheme of combination, fuzzy reasoning, terrestrial navigation.

Algorytmy wnioskowania rozmytego dla określania pozycji

Streszczenie

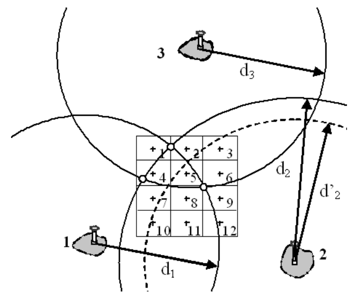
Artykuł poświęcony jest algorytmom określania pozycji na podstawie obserwacji obiektów stałych. W poprzednim swoim opracowaniu autor przedstawił koncepcję wykorzystania Matematycznej Teorii Ewidencji do wyznaczania pozycji statku. Teoria wykorzystuje miary przekonania i domniemania i operuje na strukturach przekonań. Struktury te pozwalają zakodować wiedzę nawigatora dokonującego pomiarów, jak też wyrazić jego niepewność. Składanie rozmytych struktur przekonań prowadzi do selekcji punktu będącego obserwowaną pozycją statku. Zaproponowano prosty algorytm rozwiązania postawionego problemu, był on jednak wrażliwy na niewłaściwe, początkowe rozmieszczenie punktów przestrzeni poszukiwań. Prezentowany algorytm likwiduje to ograniczenie i poprawia jakość otrzymanej pozycji. Siatka określająca zbiór punktów przestrzeni poszukiwań jest wielokrotnie losowo przemieszczana. Dla każdego położenia rejestrowana jest wartość najlepszego rozwiązania, odpowiednia lokalizacja jest następnie wykorzystywana do dalszych obliczeń.

Słowa kluczowe: nawigacja, struktury przekonań, Matematyczna Teoria Ewidencji.

1. Introduction

In terrestrial navigation determination of the ship position is based on fixed objects. Terrestrial landmarks such as lighthouses, buoys, shore contours and other are used. Distances and bearings enable position fixing. Graphic methods are widely used to achieve that goal. Three or more distances or bearings result in numerous intersection points. Based on their allocation, the final reasoning on the true position is to be carried out. Experience and knowledge of a navigator is helpful whenever the ambiguity occurs. Intersection points create a kind of a frame of discernment. As shown in figure 1, the frame of discernment can be seen in the shape of a grid covering the area of possible ship position. Centres of numbered grid cells are points that belong to certain ranges created by measured distances.

Table 1 presents binary vectors which express membership of each centre within the three ranges. The meaning of their consecutive elements refers to particular points as presented in figure 1. Value 1 assigned to the *ij-th* element means that the *j-th* point is embraced by the *i-th* range, 0 is assigned otherwise. The data gathered in Table 1 express appropriate binary values.



Rys. 1. Określenie pozycji z trzech odległości
Fig. 1. Position fixing with three distances

Apart from the vectors containing membership of the search space points within each range, Table 1 also presents their combinations. Row $\mu_1 \wedge \mu_2$ includes the results of selection of a smaller value from vectors μ_1 and μ_2. It is seen that this combination selects points located within the intersection of the two ranges. Row $\mu_1 \wedge \mu_2 \wedge \mu_3$ shows the result set of combination of all three vectors. The only unity elements in this set are at the forth and fifth position, which indicates that these points are located within the common area. A combination of the location vectors results in selecting the points situated within an intersection of the ranges. This is important observation for the proposed approach to position fixing.

Tab. 1. Binarne wektory położeń i ich składanie
Tab. 1. Binary location vectors and their combination

	1	2	3	4	5	6	7	8	9	10	12	*m*(..)
μ_1	{1	0	0	1	1	0	1	1	0	1	0}	0,6
μ_2	{0	1	1	1	1	1	1	1	1	1	1}	0,5
$\mu_1 \wedge \mu_2$	{0	0	0	1	1	0	1	1	0	1	0}	0,3
μ_3	{1	1	1	1	1	1	0	0	0	0	0}	0,7
$\mu_1 \wedge \mu_2 \wedge \mu_3$	{0	0	0	1	1	0	0	0	0	0	0}	0,21
pl(..)	0,20	0,16	0,16	0,41	0,41	0,16	0,13	0,19	0,05	0,19	0,05	
bel(..)	0	0	0	0	0	0	0	0	0	0	0	

Note that data referring to point are missed due to column with limitation

The last column in Table 1 contains masses assigned to the consecutive location vectors. They express credibility, a kind of confidence related to a particular distance. Rows *pl*(..) and *bel*(..) contain the result masses calculated for each element from the frame of discernment. Assuming the additive property of non-negative attributes, the fourth and fifth points in the considered example receive the highest values of 0.41. The plausibility values are calculated from the formula:

$$pl(x_l) = \sum_k \mu_k(x_i) * \frac{m(\mu_k(x_i))}{\sum_i \mu_k(x_i)} \tag{1}$$

and the belief figures from the formula:

$$bel(x_l) = \sum_{k=1}^{n} m(\mu_k(x_i)) \min_{x_i \in \Omega;\, i \neq l} (\neg \mu_k(x_i)) \quad (2)$$

Note that all belief values are null. For this reason belief cannot be considered as a primary factor in the considered position fixing problem [2].

Let us reconsider the situation shown in figure 1 with a modified layout of measured distances. Distance 2 was reduced due to embracing the single point number 5 within a common intersection area. Table 2 presents new locations vectors and the same as before initial masses. Two last rows present new plausibility and belief values. It can be seen that the rank of number 5 is the highest one. Note that only row marked with $\mu_1 \wedge \mu_2 \wedge \mu_3$ receives the value of $\min_{x_i \in \Omega;\, i \neq l} (\neg \mu_k(x_i))$ equal to one. It is also worth noticing that belief can be greater than zero for points which are alone within some areas. An obvious conclusion is that the point number 5 receives the highest plausibility and this point is the unique solution to the problem.

Tab. 2. Binarne wektory położeń i ich składanie dla zmienionych odległości
Tab. 2. Binary location vectors and their combination for modified distances

	1	2	3	4	5	6	7	8	9	10	12	m(..)
μ_1	{1	0	0	1	1	0	1	1	0	1	0}	0,6
μ_2	{0	0	1	0	1	1	0	1	1	1	1}	0,5
$\mu_1 \wedge \mu_2$	{0	0	0	0	1	0	0	1	0	1	0}	0,3
μ_3	{1	1	1	1	1	1	0	0	0	0	0}	0,7
$\mu_1 \wedge \mu_2 \wedge \mu_3$	{0	0	0	0	1	0	0	0	0	0	0}	0,21
pl(..)	0,20	0,12	0,18	0,20	0,55	0,18	0,09	0,22	0,06	0,22	0,06	
bel(..)	0	0	0	0	0,21	0	0	0	0	0	0	

Note that data referring to point are missed due to column with limitation

The presented examples deliver the following useful conclusions important for the proposed approach:
1. association of the location vectors results in selection points situated within the intersection areas,
2. equal plausibility values are assigned to selected points within the single common region,
3. null belief values are assigned to selected points within the single common region,
4. not null belief value is assigned to the selected single point within the common region.

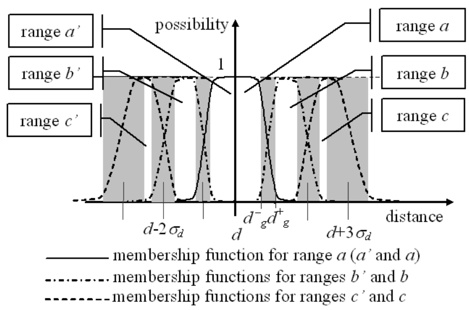
In order to fix a position one has to define the search space, create the location vectors and assign masses to each of them. In the next step the vectors are combined and a search space element of the highest value selected. It follows Dempster-Shafer scheme of reasoning. The scheme uses belief structures that are assignments of masses to events. The method proved to be universal and its application to position fixing appeared to be successful [2, 3].

2. Measuring distances and belief structures

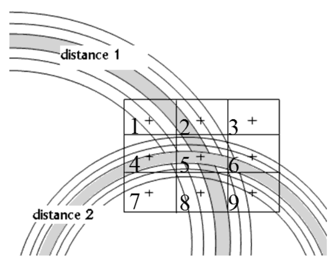
The example presented above includes crisp location values of the search space elements within wide ranges limited by measured distances. The ranges are to be narrowed to make the concept practical. In order to reduce the considered ranges, one has to use his knowledge regarding distribution of the distances measured, with particular navigational aid, around the true value. It is widely assumed that the measured distance is a random value and the true distance is somewhere in its vicinity [4]. The Gaussian function is usually used to determine an error distribution. Standard deviation σ_d is a parameter that uniquely defines the normal distribution. The parameter is estimated based on experiments. The standard deviation somewhat varies depending on an adopted testing method as well as current conditions. It is assumed that the deviation is an imprecise interval value $[\sigma^-_d, \sigma^+_d]$ (see figure 2). Therefore, at each side of the measured distance there are three ranges with imprecise borders. For each of them probabilities of containing the true distance are estimated as: 0.34, 0.14 and 0.02.

Imprecise limits create transition zones between the selected ranges. Any point within these zones belongs partially to any range. In particular, the point in the centre of a zone is located with the same possibility of 0.5 within left and right range.

To calculate locations within the selected ranges, a family of sigmoid membership functions [5] whose diagrams are shown in figure 2 was adopted Within any transition zone there are: descending slope of a membership function for the left range and ascending part of the same function for the right range. Locations of the grid cell centres within the selected ranges are to be calculated. The result sets express fuzzy membership of each of the centres inside the selected ranges with respect to a single distance. The values can be obtained with use of the membership functions defined above. The initial masses of evidence assigned to each vector are related to probabilities associated with the selected ranges.



Rys. 2. Wyróżnione zakresy i sigmoidalne funkcje przynależności
Fig. 2. Selected ranges and sigmoid membership functions



Rys. 3. Dwie nieprecyzyjne odległości i siatka poszukiwań na ich przecięciu
Fig. 3. Two imprecise distances and search space grid located at their intersection

Figure 3 presents two imprecise distances and the search space grid located at their intersection. Each grid center must be located within the defined ranges. The references are separately made to two distances.

Tab. 3. Zbiór faktów dotyczących przykładowego zestawu punktów
Tab. 3. Collection of facts regarding the presented set of search points

point	locations related to distance 1	locations related to distance 2
1	within range c'	outside the ranges
2	partly in b and c	outside the ranges
3	outside the ranges	outside the ranges
4	outside the ranges	partly in a and b)
5	within range a	within range a
6	outside the ranges	partly in b and c
7	outside the ranges	outside the ranges
8	partly in b' and c'	outside the ranges
9	outside the ranges	within range c'

Table 3 contains the collection of facts regarding the presented set of search points. The collection contains description of each point location within the defined ranges. The references are made to two distances as shown in figure 3.

The collected data enable defining the major parts of two belief structures. To complete the structures one has to introduce the credibility regarding each of the measured distances.

Tab. 4. Dwie subnormalne struktury przekonań
Tab. 4. Two subnormal belief structures

		1	2	3	4	5	6	7	8	9	$m(..)$
distance 1	μ_{1a}	{0	0	0	0	1	0	0	0	0}	0.41
	μ_{1b}	{0	0.8	0	0	0	0	0	0.6	0}	0.16
	μ_{1c}	{1	0.2	0	0	0	0	0	0.4	0}	0.03
	μ_{1u}	{1	1	1	1	1	1	1	1	1}	0.40
distance 2	μ_{2a}	{0	0	0	0.9	1	0	0	0	0}	0.55
	μ_{2b}	{0	0	0	0.1	0	0.2	0	0	0}	0.22
	μ_{2c}	{0	0	0	0	0	0.8	0	0	1}	0.03
	μ_{2u}	{1	1	1	1	1	1	1	1	1}	0.20

It is assumed that the confidence attributed to distance 1 is 0.6. The level of credibility to distance 2 is higher and equal to 0.8. The complements of these values are assigned to all vectors that express the uncertainty. The credibility levels affect probabilities assigned to each range whose products are adopted as ultimate masses of evidence. The exemplary set of figures is shown in the last column of Table 4. The table contains the sets of data of the aggregated locations within ranges *b, b'* and *c*, *c'*. Aggregation reduced the total number of the presented figures.

The belief structure is a mapping or assignment of masses to normal location sets. The sum of all masses should be one in order to call the whole assignment a correct belief structure. The mapping is a pseudo belief structure otherwise [6]. For both structures the sums of masses are equal to one. It should be noted that Table 4 presents two rows with subnormal fuzzy sets. Their highest grades are less than one. Pseudo belief structures are usually created during combination. The phenomenon occurs when a non-zero mass is assigned to an empty or subnormal set. The occurrence is also called as inconsistency. As one can reason from figure 1, all null location vectors result from combining the ranges with empty intersection. In a particular application it results from the lack of support for a given set of positions of the search space elements within the other combination of locations. Pseudo structures have some undesirable properties [6]. For this reason the pseudo belief structures are to be normalized. Two different approaches to normalization proposed by Yager and Dempster are used. Despite a few drawbacks, the method proposed by Yager can be used for nautical applications [2]. In the approach all grades of subnormal sets are increased by a complement of the highest one. At the same time the masses assigned to the null sets increase uncertainty. In Dempster method all grades are divided by the highest one. The masses assigned to not null sets are modified during normalization.

The combination table can be used to carry out association of belief structures with fuzzy events [7]. In case of more than two belief structures, the next association steps are necessary. The normalized result of the first step is to be combined with the third structure, then with next ones if available. The final belief structure consists of a family of fuzzy locations sets $\{\mu_k(x_i)\}$ and collection of masses assigned to each set $\{m(\mu_k(x_i))\}$. Given these data sets, support for the hypothesis represented by a set of $\mu_A(x_i)$ is sought. It expresses the basic ability of MTE to reason on a certain hypothesis based on relative ones. A navigator reasons on the true distance, given measured ones and knowledge on used aids and observed objects.

Plausibility and belief of the proposition represented by $\mu_A(x_i)$ included in the collection of sets $\{\mu_k(x_i)\}$ can be calculated using the formulas presented in [1]. Specificity of the reference set enable calculating plausibility in a simpler way from the formula:

$$pl(x_l) = \sum_{k=1}^{n} m(\mu_k(x_i)) * \mu_k(x_l) \qquad (3)$$

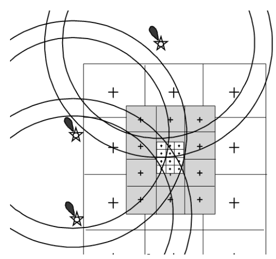
3. Calculations

The presented rational enable selecting a search space element with the highest plausibility regarding true distances to given locations. In case when the search space is small enough, the selected point can be estimated as a position of the ship. Therefore the iterative algorithm is proposed to fix the position based on taken measurements for selected objects. The search space area is reduced in each of the iterations. The center of a new area is placed at the location with the highest metric. Figure 4 presents the idea of the proposed computation concept; the general scheme describes algorithm I. In the previous paper by the author [2] the details of the non-iterative version of this algorithm were presented.

Algorithm I

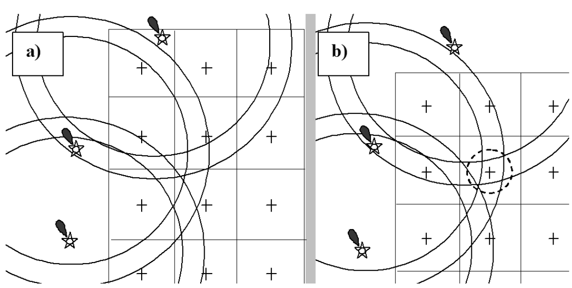
1. adjust position of the initial search space area
2. calculate fuzzy elements of the location vectors; normalize vectors and assign their masses of confidence in order to define the belief structures
3. combine all structures, select a point with the maximum value of plausibility
4. if the accuracy is satisfying, then quit; if not, reduce the search space area and place it around the point of pl_{max}. and go to 1.

The presented algorithm is prone to misbehavior due to misplacement of the search grid. Figure 5 shows the misplaced and properly located search grids. Part a) of the figure presents the grid location for which all search space elements receive plausibility values close to zero. The solution is vague; there is no single point with supreme metric. Part b) contains the circled point for which plausibility is significantly greater than for other points.



Rys. 4. Określanie pozycji jako proces iteracyjny z ograniczaną przestrzenią poszukiwań
Fig. 4. Position fixing as an iterative process with decreasing search space

At the beginning of computation, misplacement can make the approximate location of intersection of all observations impossible.



Rys. 5. Niepoprawnie a) i poprawnie b) położona siatka poszukiwań
Fig. 5. Misplaced a) and well located b) search grid

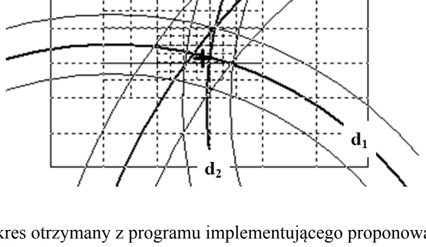
The search grid position adjustment algorithm is to be used to avoid misplacement. The grid initial location is randomly shifted within the area of a single cell and the value of maximum plausibility recorded. The position with the highest value is returned. It is worth emphasising that the iterating process produces a non-decreasing series of the calculated plausibility values. The middle of the search grid is repeatedly located at the point with the highest metric. Nonetheless, the adjustment algorithm should be also invoked in order to discover local extremes.

Algorithm II (search space grid position adjustment algorithm)

1. accept the top left and bottom right coordinates of search space area *(x1, y1, x2, y2)* and initial plausibility pl_{init};
   assign $max= pl_{init}$; *xmax=x1; ymax=y1; x3:= x1; y3= y1; x4= x2; y4= y2*
2. for the given search space define the belief structures regarding each measurement
3. combine structures and select the search space element with pl_{max}
4. if max < pl_{max} then: $max= pl_{max}$; *xmax=x3; ymax=y3*
5. if stop condition then quit, else:
   shift_x = random(width);
   shift_y= random(height);
   x3=x1 + shift_x;
   y3=y1+ shift_y;
   x4=x2 + shift_x;
   y4=y2+ shift_y;
   go to 2.

The search space grid position adjustment algorithm accepts: the initial plausibility value, the top left and bottom right grid corner coordinates. At the first step of computation the initial plausibility is supposed to be zero, and iterations count determined to break the loop. The previous step plausibility should be used as an invocation parameter during consecutive iterations. In this case the stop condition can be related to achieving any greater plausibility or exhausted loop count. The algorithm, presented below, adjusts position of the search space grid spanned over the potential position area. Random adjustment is limited by dimensions of a single cell. Function *random(width); random(height);* returns the integer value less than the width and height (in meters) of a single cell. For each location of the grid the maximum plausibility value is calculated and coordinates of the grid recorded.

Figure 6 presents the output generated by the software implementing the proposed approach. There were three distances available, each of them characterized by the interval valued estimates of the standard deviation. Distances were treated as random values with normal distributions. Each of them is shown with three rings of radiuses equal to: $d^-_i = d_i - 3\sigma_{di}$, d_i, $d^+_i = d_i + 3\sigma_{di}$, respectively. Measuring conditions were subjectively assessed and appropriate structures created. The search space grids are presented in the figure; the grids cover decreasing areas during consecutive iterations. Therefore three iterations were made in the presented example. The grid cell with the highest calculated plausibility was marked with a cross. The point is assumed to be the ship position.



Rys. 6. Wykres otrzymany z programu implementującego proponowane rozwiązanie
Fig. 6. Output generated by software implementing the proposed approach

4. Summary and conclusions

A new method for position fixing in terrestrial navigation is proposed. Measurements are treated as a random variable of known distribution, theoretical or empirical functions can be accepted for calculations. The true observation is somewhere in the vicinity of the measured one. To define probability of the true distance, six ranges of distances were introduced. The probability levels assigned to each range were calculated based on the features of a particular distribution. The length of a taken distance, the radar echo signature and the height of an object should be taken into account while evaluating measurement conditions. All these factors contribute to the value of a mass assigned to the uncertainty. Although it is assumed that the mass is a crisp value, subjective linguistic terms like good, medium or poor can be used instead. The mass of uncertainty affects credibility assigned to the location vectors.

Imprecise interval valued limits of ranges are assumed. Consequently, the sigmoid membership functions were introduced and used in the presented numerical examples. Thanks to the membership functions, the elements of fuzzy locations vectors are calculated. The number of the location vectors within a single structure is the same as the number of the distinguished ranges. One part of the belief structure consists of the location vectors supplemented with the set expressing the uncertainty. Another part embraces masses of the initial credibility assigned to each of the fuzzy sets. The complete belief structure is related to each measurement.

The belief structures are a subject of combination. During association process the search space points within the common intersection region are selected. The result of association is further explored in order to reason on the final position. There are two formulas enabling calculations of belief and plausibility of the proposition, represented by a certain fuzzy set, and included in the result belief structure. The formulas were simplified due to the unique property of the reference set.

In order to achieve the required accuracy, the iterative algorithm for position fixing is proposed. The search area with the constant number of cells is decreased during iterations. At the beginning the rough proximity of the fix is to be obtained. At this step the search grid covers a large potential area of the ship position. Unfortunately, scarce distribution of the search points can cause that the proximity cannot be obtained. The wrong coverage of the search area can result in an inconclusive output. To avoid misplacement, the grid can be randomly shifted within a single cell in order to discover the local maxima. Iterations end when the grid cells are small enough compared to the minimum standard deviation of the used aids.

5. References

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