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# Contour compression scheme combining spectral and spatial domain methods

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### Abstract

A two-stage contour data compression scheme, based on transform coding technique (the first stage) combined with a selected spatial method (the second stage), is presented in the paper. The goal of the transform coding is to achieve the compression ratio as high as possible. The tangent method, a selected spatial domain contour approximation algorithm, is used to compensate the quantization error introduced at the first stage. Advantages of the proposed scheme were examined according to the compression ratio as well as to the Mean Square Error and the Signal to Noise Ratio. Compensation abilities of the tangent method were proved with respect to a new measure, referred to as the Area Error.

**Keywords:** contour compression and approximation, transform coding, piecewise linear transform, the Lloyd-Max quantizer.

## Schemat kompresji konturów łączący metody operujące w dziedzinie widmowej i przestrzennej

### Streszczenie

W artykule zaprezentowano dwustopniowy schemat kompresji konturów, który łączy w sobie metodę kodowania transformatowego z wybraną metodą przestrzennej aproksymacji konturów. Celem zastosowania kodowania transformatowego w pierwszym, kodującym stopniu prezentowanego schematu jest uzyskanie jak najwyższego stopnia kompresji. Wraz ze wzrostem stopnia kompresji narasta jednak tzw. błąd kwantyzacji, a wraz z nim rośnie również zniekształcenie wprowadzane do przetwarzanych danych. Zadaniem metody tangensów - metody przestrzennej aproksymacji konturów zastosowanej w drugim, dekodującym stopniu proponowanego schematu, jest znacząca redukcja ww. zniekształcenia przy zachowaniu wysokiej wartości stopnia kompresji. Własności zaimplementowanego algorytmu kodowania transformatowego zostały przebadane zarówno pod względem stopnia kompresji jak i jakości rekonstrukcji. Własności te silnie zależą od typu zastosowanej transformaty. W pracy prezentowane są rezultaty uzyskane przy zastosowaniu transformaty DCT oraz dwóch, wybranych transformat odcinkowo-liniowych: PHL [10] i PWL [9]. Zdolność metody tangensów do kompensacji błędu kwantyzacji została wykazana na podstawie badania stosunku powierzchni wyznaczanych przez oryginalny kontur wejściowy i kontur zrekonstruowany na wyjściu schematu. Na potrzeby ww. badania zdefiniowano osobny wskaźnik AE (Area Error). Efektywność kompletnego, proponowanego dwustopniowego schematu kompresji została zaprezentowana przy wykorzystaniu wybranych konturów testowych.

**Słowa kluczowe:** kompresja i aproksymacja konturów, kodowanie transformatowe, transformaty odcinkowo-liniowe, kwantyzator Lloyda-Maxa.

## 1. Introduction

Contours define edges which can be understood as boundaries between objects and the background of analyzed images. This

feature is used to evaluate such basic object properties as perimeter, area, shape, direction, etc. Properties like those are essential for object identification and classification - the sub-domains of a computer vision. Different methods of contour approximation and compression are also very useful for advanced image processing, especially of such a kind which refers to remote-controlled operating.

The fundamental goal of the digital signal compression is to reduce the bit rate for transmission and storage without introduction of significant distortion. There are two main approaches solving this problem. The first one is represented by methods operating in a spectral domain. One of the most efficient techniques of this class is the transform coding, referred also to as the block quantization method [6]. The second one is the spatial domain approach. There are many methods for contour compression and approximation operating in a spatial domain [2], [7]. The Ramer algorithm [8], based on the maximum distance of the curve from the approximating polygon, is one of the most appreciated contour approximation methods.

A two-stage contour compression scheme combining the transform coding and the selected spatial contour approximation method is presented in the paper. The implemented algorithm of the transform coding, applied at the first stage of this scheme, makes the compression ratio high. However, the contour reconstructed after the transform coding is greatly out of shape, because of new elements and irregularities introduced during the quantization step. The tangent method [1], applied at the second stage of the proposed scheme, shows very good smoothing abilities which significantly reduce the quantization error of the first stage. The fit criterion of the tangent method is the tangent of an angle between two straight lines.

## 2. Block quantization

Quantization [3, 4] is the step subsequent to sampling in signal digitization. During the quantization process a continuous variable  $u$  is mapped into a discrete variable  $u^*$ , which takes values from the finite set. The scalar quantization defines two sets of the increasing values: the set of decision levels  $p = \{t_k, k = 1, \dots, L+1\}$ , with  $t_1$  and  $t_{L+1}$  as the minimum and maximum values of  $u$ , respectively, and the set of reconstruction levels  $u^* = \{r_1, \dots, r_L\}$ , defined with accordance to the set  $p$ . Continuous values of  $u$  from the range  $[t_k, t_{k+1})$  are approximated by a single number  $r_k$ . A quantizer introduces distortion  $Q_E = u^* - u$ , which any reasonable design method must attempt to minimise. Distortion  $Q_E$  can be measured by the Mean Squared Quantization Error, defined as follows:

$$\varepsilon_q = \sum_{i=1}^L \int_{t_i}^{t_{i+1}} (u - r_i) \cdot p(u) du, \quad (1)$$

The quantizer which best minimises the introduced distortion for a given number of decision levels  $L$  and for a given probability density function  $p(u)$  is the optimal Lloyd-Max Quantizer [5]. Two commonly used probability density functions for quantization of image-related data are the Gaussian and Laplacian functions, which are described by the following formulas [6]:

$$\text{Gaussian: } p_L(u) = \frac{1}{\sqrt{2\sigma^2}} \exp\left(-\frac{\sqrt{2}|u|}{\sigma}\right), \quad (2)$$

$$\text{Laplacian: } p_G(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(u-\mu)^2}{2\sigma^2}\right),$$

where:  $\sigma^2$  and  $\mu$  - variance and the mean value of  $u$ , respectively.

### 3. The algorithm

The implemented algorithm of the transform coding begins by normalizing the input, one-dimensional signal. The input signal (e.g. a vector representing the input contour in the Cartesian coordinate system) is normalized using the following procedure:

$$x_{norm} = \sum_{i=0}^{D-1} (x_i - x_{ave}), \quad (3)$$

where:  $x_{ave}$  - average value of the input signal  $x$ ;  $D$  - length of the input signal.

Vector  $x_{norm}$  is divided into sub-vectors of the length  $M$ . Each of these sub-vectors is then processed using selected transform, which produces  $D/M \times M$  array of spectrum coefficients. In the next step, the array of spectrum coefficients is transposed, making the coefficients grouped according to their position. Variances which are calculated next, for each row of the grouped coefficients array, are put together into a vector required for the normalization process. The normalization process lets the same quantizer be used for data series of different variances. The vector of variances is also required to create a sequence of bit allocation. This sequence is created by the Integer Bit Allocation procedure in respect to the assumption that more bits should be allocated for coefficients with higher variance.

The implemented Integer Bit Allocation procedure is as follows:

step 1. Initialization:  $b_k = 0$ ,  $d_k = \sigma_k^2$ ,  $z = 0$ , for  $k = 1, 2, \dots, M$ ;

step 2. while ( $z < M \cdot a$ ):

- finding position  $k^*$  at which  $d_k$  is maximal;

- allocating one bit to the  $k^*$ -th position:

$$b_{k^*} = b_{k^*} + 1, \quad d_{k^*} = d_{k^*} / 2;$$

- incrementing  $z$ ;

step 3. correction of  $b_k$  in respect to the variability of  $v_k$ :

- finding  $p_k$ ;

- finding  $bb_k$ :  $bb_k = \log_2(p_k) / \log_2 2$ ;

- if  $b_k > bb_k$  than  $b_k = bb_k$ .

where:  $M \times n$  - size of the array of grouped coefficients ( $n = D/M$ );  $v_k$  -  $k$ -th row of the array of grouped spectrum coefficients,  $k = 1, 2, \dots, M$ ;  $\sigma_k^2$  - variance for a given  $v_k$  vector;  $a$  - mean number of bits allocated for the quantized vector;  $b_k$  - number of bits allocated to  $v_k$ ;  $z$  - total number of allocated bits;  $p_k$  - number of different values in  $v_k$ ;  $bb_k$  - maximum number of bits that can be allocated to  $v_k$ .

The vector of bit allocation determines quantization of  $v_k$  vectors. Because Gaussian or Laplacian probability density functions had been expected, the Lloyd-Max Quantizer was applied. The approach described above, referred to as a zonal

sampling, is especially efficient for signals with uniformly distributed values - e.g. contours in the Cartesian representation.

The quantized values are, at the final step of the implemented transform coding method, coded and stored (or transmitted). Coded information contains:

- value  $x_{ave}$  (in fixed-point representation),
- value  $\log_2 D/M$ , where  $D/M$  indicates number of rows of the array of spectrum coefficients;
- vector  $pv$  with binary encoded information of positions of transmitted (1) or no (0)  $v_k$  vectors;
- variances of these  $v_k$  vectors for which  $pv_k = 1$  (in floating-point representation);
- vectors with differentially encoded (using the Adaptive Huffman coding) of quantized values and their indices.

Fractional parts of quantized values are rounded to five digits. According to the expected probability density functions, 99,73% of all values of  $v_k$  vectors lie inside the range  $[v_{ave} - 3\sigma, v_{ave} + 3\sigma]$ . In other words, almost 100% of coded values are greater than -3,96875 and less than 3,96875, which limits the size of coded alphabet to 255 letters.

### 4. The tangent method

The fit criterion of the tangent method (TM) algorithm [1] is the tangent of an angle between two straight lines, called opening and closing lines. These two lines determine the width of the analyzed segment - a set of input sequence points which is going to be replaced by an edge of the approximating polygon.

The fit criterion of the TM algorithm is illustrated in Fig. 1.

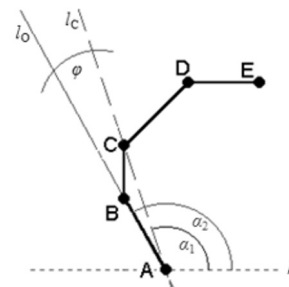


Fig. 1. Fit criterion for the tangent method

Rys. 1. Kryterium dopasowania dla metody tangensów

The opening line is passing through the first two points of a new segment under the investigation. The first point of each segment is the starting point SP. The SP point and the third point - the ending point EP - of the investigated segment determine the closing line. If the tangent of the  $\phi$  angle is less than a given threshold  $p$ , then EP point is shifted to the next point and the new closing line is drawn. Otherwise, the SP and EP points determine vertices of an edge of the approximating polygon. Then the EP is marked as the SP point of a the new segment.

TM method can be applied to contours in polar, Cartesian or generalized representation described by  $(\theta, l)$  co-ordinates.

The fit criterion in Cartesian representation can be obtained from the following equation

$$\text{tg } \phi = \frac{m_2 - m_1}{1 + m_2 \cdot m_1}, \quad (4)$$

where:  $m_1$  - slope of the opening line;  $m_2$  - slope of the closing line.

### 5. Applied measures

Analysis of compression abilities of the implemented transform coding algorithm were performed using the following measures:

– compression ratio

$$\eta = \frac{(b-c) \cdot 100\%}{b}, \quad (5)$$

where:  $b$  – number of the input signal bits before the compression;  
 $c$  – number of bits of coded information.

– Mean Square Error (MSE)

$$\varepsilon = \frac{1}{N_x} \cdot \sum_{i=1}^{N_x} (x_i - \tilde{x}_i)^2, \quad (6)$$

where:  $x$ ,  $\tilde{x}$  – the input and the output (reconstructed) signals,  
 respectively;

– Signal-to-Noise Ratio (SNR)

$$SNR(\text{dB}) = 10 \cdot \log \frac{\sigma_x^2}{\varepsilon}, \quad (7)$$

where:  $\sigma_x^2 = \frac{1}{N_x} \cdot \sum_{i=1}^{N_x} (x_i - x_{\text{ave}})^2$  – variance of the input signal;

$x_{\text{ave}} = \frac{1}{N_x} \cdot \sum_{i=1}^{N_x} x_i$  – average value of the input signal.

Effectiveness of the presented transform coding scheme depends mainly on two parameters:

$M$  – the length of sub-vectors into which the normalized input signal is divided;

$a$  – average number of bits allocated for one quantized vector.

The threshold parameter  $a$  is inversely proportional both to the Mean Square Error and the compression ratio, which is illustrated in Fig. 2.

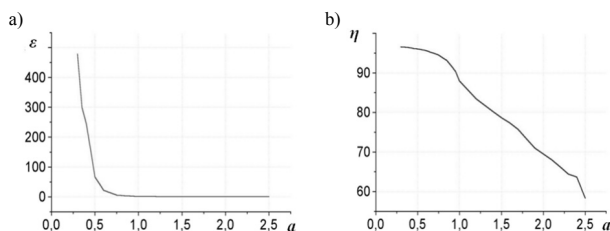


Fig. 2. MSE (a) and compression ratio (b) versus  $a$  parameter ( $M = 8$ )

Rys. 2. Zależności błędu średniokwadratowego (a) i współczynnika kompresji od parametru  $a$

The length  $M$  is also very significant for the presented compression scheme. Selection of the most adequate value of  $M$  parameter was performed according to the results obtained for different types of the applied transforms and a wide range of the test contours. A selected subset of these results is presented in Table 1.

The results presented in Table 1 are divided into two main categories:

- the first one, listed in grey-colored columns, includes only these cases where SNR is equal or greater than 30 dB (this level of SNR guaranties the acceptable quality of reconstruction);
- the second one, represented by white columns, shows the results which were obtained for the same average number of bits.

The presented results show that the length of the  $v_k$  sub-vectors is inversely proportional to the compression ratio and directly proportional to the SNR, and that the best results can be obtained for  $M = 4$ .

Tab. 1. Selected results of the transform coding scheme for different types of the applied transforms

Tab. 1. Wybrane rezultaty metody kodowania transformatowego dla różnych rodzajów zastosowanych transformat

	$M$	$a$		SNR [dB]		$\eta$ [%]	
DCT	4	1,50	0,45	34,50	6,50	92,21	95,10
	8	0,90	0,45	34,71	17,89	91,76	96,32
	16	0,45	0,45	33,85	33,85	92,23	92,23
	32	0,25	0,45	34,18	37,87	92,32	87,25
	64	0,22	0,45	34,77	38,86	90,59	82,06
PHL	4	1,80	0,80	35,24	17,95	89,43	94,48
	8	1,20	0,80	35,00	29,93	84,34	94,01
	16	0,80	0,80	33,93	33,93	84,52	84,52
	32	0,59	0,80	34,19	37,57	83,30	78,36
	64	0,43	0,80	34,33	39,46	81,95	72,22
PWL	4	1,50	0,90	34,50	17,96	92,21	94,55
	8	0,90	0,90	34,71	34,71	91,76	91,76
	16	0,90	0,90	34,19	34,19	79,13	79,13
	32	1,38	0,90	34,32	32,36	58,58	72,38
	64	1,71	0,90	34,38	30,29	25,31	59,47

Examples of selected test contour and the contour obtained as a result of applying the presented transform coding scheme to it (with use of the PWL transform [9]) are depicted in Fig. 3.

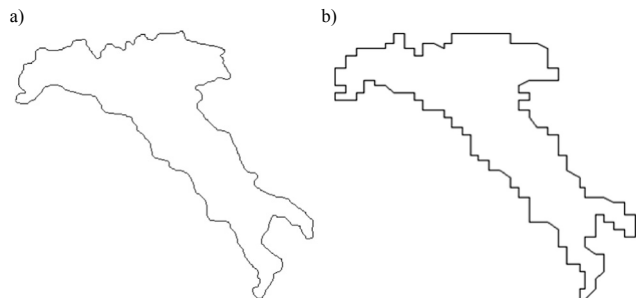


Fig. 3. Original input contour (a) and contour obtained at the end of the proposed transform coding scheme (b):  $a = 1,48$ ;  $M = 4$ ; SNR = 29,66;  $\eta = 93,45\%$

Rys. 3. Oryginalny kontur wejściowy (a) i kontur zrekonstruowany na wyjściu proponowanej metody kodowania transformatowego (b):  $a = 1,48$ ;  $M = 4$ ; SNR = 29,66;  $\eta = 93,45\%$

## 6. Two-stage contour compression scheme

Fig. 3b shows a typical staircase-like distortion introduced into the processed signal as a result of the quantization error. To improve the quality of reconstructed contours in the proposed compression scheme, the tangent approximation method [1] was used. The tangent method was selected because of its smoothing abilities. Moreover, the tangent method algorithm is simple (easy to implement) and fast. The smoothing procedure of the tangent method was used as a complement of the final, decoding part of the proposed compression scheme.

The following Area Error (AE) measure was defined to evaluate the strength of the smoothing effect:

$$AE = \frac{|A_O - A_R|}{A_O} \cdot 100\%, \quad (8)$$

where:  $A_O$  – area of the original input contour;  $A_R$  – area of the reconstructed contour.

The Area Error determines a percentage of the area being the difference between  $A_O$  and  $A_R$ . The smaller value of the Area Error, the greater smoothing effect and better effectiveness of the proposed two-stage compression scheme.

The complete decoding part of the proposed contour compression scheme is illustrated in Fig. 4.

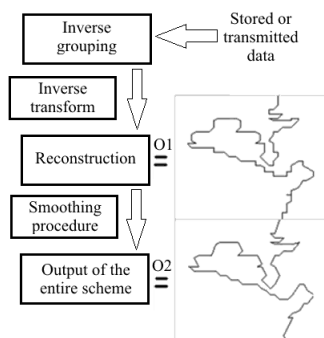


Fig. 4. Organization chart of the decoding part of the proposed two-stage contour compression scheme

Rys. 4. Schemat organizacyjny części dekodującej proponowanej, dwustopniowej metody kompresji konturów

Selected original test contours and the related results (including the Area Error) obtained at outputs O1 and O2 of the presented two-stage contour compression scheme are depicted in Fig. 5.

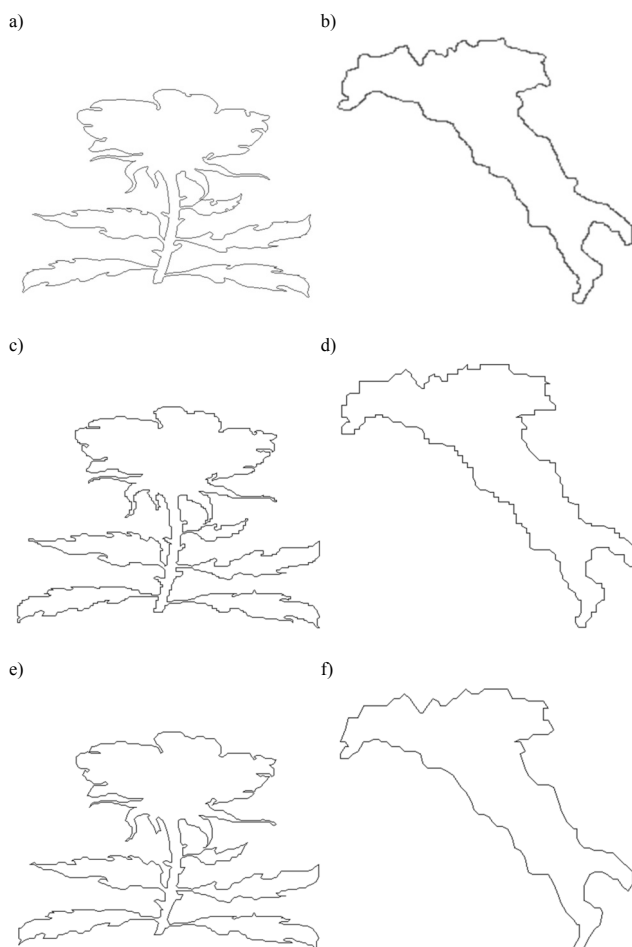


Fig. 5. Selected test contours (a, b) and related results obtained at output O1: c)  $a = 1,75$ ,  $\epsilon = 2,46$ ,  $SNR = 37,42$ ,  $\eta = 92,79\%$ ,  $AE_{(O1)} = 0,44\%$ , d)  $a = 1,74$ ,  $\epsilon = 2,64$ ,  $SNR = 34,50$ ,  $\eta = 92,21\%$ ,  $AE_{(O1)} = 2,29\%$ , and at output O2: e)  $AE_{(O2)} = 0,05\%$ , and f)  $AE_{(O2)} = 0,44\%$ , of the presented two-stage contour compression scheme

Rys. 5. Wybrane kontury testowe (a, b) i uzyskane dla nich rezultaty na wyjściu O1: c)  $a = 1,75$ ,  $\epsilon = 2,46$ ,  $SNR = 37,42$ ,  $\eta = 92,79\%$ ,  $AE_{(O1)} = 0,44\%$ , d)  $a = 1,74$ ,  $\epsilon = 2,64$ ,  $SNR = 34,50$ ,  $\eta = 92,21\%$ ,  $AE_{(O1)} = 2,29\%$ , i na wyjściu O2: e)  $AE_{(O2)} = 0,05\%$ , i f)  $AE_{(O2)} = 0,44\%$ , prezentowanego dwustopniowego schematu kompresji konturów

## 7. Conclusions

A new two-stage contour data compression scheme, combining the transform coding technique and the tangent contour spatial approximation method, is presented. The implemented transform coding algorithm, used at the first stage of the proposed scheme, gives possibility to achieve high compression ratio. However, the block quantization procedure of this technique introduces quite big distortion associated with the low quality of reconstruction. The introduced quantization error is visible (Figs. 5c, 5d) as staircase-like irregularities that make the reconstructed contours greatly out of shape. Therefore, the proposed compression scheme was complemented, at the second decoding stage, by the tangent method algorithm – contour approximation method which shows very good smoothing abilities. These smoothing abilities reduce the quantization error of the first stage to the acceptable level (Figs. 5e, 5f).

The results presented in Fig. 5 show that the proposed two-stage contour compression scheme is very effective. As it is depicted in Figs. 5c and 5d, the implemented modified algorithm of the transform coding guarantees that the compression ratio is greater than 92%, while the SNR is much greater than 30 dB. Simultaneously, smoothing abilities of the tangent method allow rejecting most of the irregularities introduced during the first stage, which is indicated by the Area Error measure - values of the AE calculated for reconstructions illustrated in Figs. 5e and 5f are more than 5 times smaller than in the cases presented in Figs. 5c and 5d, respectively.

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## 8. References

- [1] Baran R., Dziech A.: Tangent Method and the other Efficient Methods of Contour Compression, WSEAS Transactions on Computers, Issue 7, Vol. 4, pp. 805-813, 2005.
- [2] Dziech A., Ukasha A., Baran R.: Fast Method for Contour Approximation and Compression, WSEAS Transactions on Communications, Issue 1, Vol. 5, pp. 49-56, 2006.
- [3] Baran R., Dziech W.: Kompresja konturów z wykorzystaniem kwantyzacji skalarnej, Materiały Krajowej Konferencji Radiokomunikacji, Radiofonii i Telewizji, Gdańsk, s. 283-286, 2002.
- [4] Sayood K.: Introduction to Data Compression, Second Edition, Morgan Kaufmann Publishers, 2000.
- [5] Lloyd S. P.: Least Squares Quantization in PCM, IEEE Trans. on Information Theory, Vol. 28, pp. 127-135, 1982.
- [6] Jain A. K.: Fundamentals of Digital Image Processing, Prentice-Hall, Englewood Cliffs, New Jersey, 1989.
- [7] Montanari U.: A Note on Minimal Length Polygonal Approximation to a Digitized Contour, Comm. ACM, Vol. 13, pp. 41-47, 1970.
- [8] Ramer U.: An Iterative Procedure for the Polygonal Approximation of Plane Curves, Computer Graphics and Image Processing, Academic Press, 1, pp. 244-256, 1972.
- [9] Belgassem F., Dziech A., Aboudeana M., Dziech W.: Improved Fast Algorithm for PWL Transform, Proc. of the Int. Workshop on Systems, Signals and Image Processing IWSSIP, Zagreb, Croatia, 1998.
- [10] Dziech A., Belgassem F., Aboukres S., Nabout A.: Periodic Haar Piecewise Linear Series and Transforms, Proc. of the CESA'96, IEEE Multiconference, Lille, France, 1996.

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