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Software for approximation of sensor transfer function and its sensitivity function

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Abstract

In this paper, a specialized software package named "Program package for continuous and smooth uniform spline approximation of high-accuracy cryosensors' transfer functions" is presented. The software is worked out for solving problems of optimization of analytical formulae describing transfer functions of cryogenic (or of wider transfer range) temperature sensors. The problem of reconstruction of a sensor transfer function and its sensitivity function can be reduced to constructing a continuous and smooth spline approximation that meets the Chebyshev criterion on all subintervals of the approximation interval.

Keywords: Chebyshev approximation, sensor transfer function, sensor sensitivity.

Oprogramowanie do aproksymowania funkcji przetwarzania sensora i funkcji jego czułości

Streszczenie

Przedstawiono właściwości specjalizowanego pakietu oprogramowania ApproCryo przeznaczonego do wyznaczania ciągłej i gładkiej funkcji sklejanej (splajnu) aproksymującej funkcje przetwarzania kriogenicznych (lub o szerszym zakresie przetwarzania) sensorów temperatury. Program opracowano do zagadnień optymalizacji wyrażeń analitycznych opisujących charakterystyki sensorów temperatury w zakresie kriogenicznym (lub szerszym). W celu uzyskania najlepszej dokładności celowe jest stosowanie aproksymacji minimaksowej splajnem, dla wszystkich ogniw, dla których spełnione jest kryterium Czebyszewa. Program ApproCryo pozwala dla danej postaci wyrażenia aproksymującego o określonym zbiorze parametrów wyznaczyć funkcję aproksymującą o najmniejszej możliwej wartości błędu aproksymacji. Efektywność pakietu ApproCryo wykazano na przykładzie diodowego sensora niskotemperaturowego; dokładność aproksymacji funkcji przetwarzania sensora i jego czułości okazała się zadowalająca. Oprogramowanie jest przystosowane do systemu operacyjnego Windows 98/2000/XP/... (wymagane jest zainstalowanie platformy Visual Basic), a menu użytkownika jest intuicyjnie zrozumiałe i przystępne w obsłudze.

Słowa kluczowe: aproksymacja Czebyszewa, funkcja przetwarzania sensora, czułość sensora.

1. Introduction

The ApproCryo software package is written for constructing continuous and smooth minimax spline-approximations based on experimental data. The package is orientated towards solving optimization problems by means of the Chebyshev approximation method, under condition that the tabulated high-accuracy measurement points of the characteristic of the low temperature sensor are given. The ApproCryo software has been created on the Visual Basic platform, and can be implemented on any PC standard configuration based on the operating system Windows 98/2000/XP/… with installed the Visual Basic platform. The peculiarities of ApproCryo package have been described in detail in [1].

2. Functional features of the ApproCryo package

The package allows for creating a continuous and smooth minimax spline approximation

$$S(x)=F_m(a^{(j)}; x), \tag{1}$$

where $t_j \le x \le t_{j+1}$, j=1,…, q; $t_1 = x_1$ and $t_{q+1} = x_n$. The spline is created over the interval $[\alpha, \beta]$ according to the continuously differentiable function $f(x)$, $(f(x)\in(C^1[\alpha,\beta]))$, the values of which are tabulated for a set of points

$$X=\{\alpha \le x_1 < x_2 < ... < x_n \le \beta\}. \tag{2}$$

The points t_j (j=1,…, q+1) of the spline (1) are the knots at which the spline pieces join continuously together. The intervals $[t_j , t_{j+1}]$ (j=1,…, q) are the spline segments over which the spline values are determined by a given expression $F_m(a^{(j)}; x)$. This expression depends on m ($m \ge 4$) real parameters a ($a\in A$, $A\subseteq R_m$). Each expression $F_m(a^{(j)}; x)$, (j=1,…, q) is a Chebyshev approximation – over the interval $[t_j , t_{j+1}]$ – of the function $f(x)$ that satisfies the conditions

$$F_m(a^{(j-1)}; t_j)= F_m(a^{(j)}; t_j)=f(t_j) \tag{3}$$

$$F'_m(a^{(j-1)}; t_j)= F'_m(a^{(j)}; t_j)=f'(t_j) \tag{4}$$

at the knots t_j (j=2,…, q).

Let G_j denotes the value of the approximation error for the j-th spline piece; then the total approximation error G of the spline approximation is equal to

$$G= \max_{1\le j\le q} G_j \tag{5}$$

In order to yield a good continuous and smooth minimax spline approximation, subjected to the constraint of a given value of the maximum approximation error, we should to find the Chebyshev approximation with such expressions $F_m(a^{(j)}; x)$, that the approximation error - for each of spline pieces - does not exceed the given value G_{max}. The continuity and smoothness of the spline is obtained by satisfying the conditions (3) and (4) imposed on the values of the approximated function and its derivatives in the spline knots. In those knot points, the values of approximating functions and their derivatives are equal to the tabulated values of experimental data. The algorithm how to obtain such spline approximation was described in [2]. According to this algorithm, the maximum length of each of spline pieces is fitted so that for the given value of approximation error, the minimum number of spline pieces is found.

The ApproCryo software allows the user to obtain the continuous and smooth spline approximation for a given value of either absolute or relative error, with the following analytical expressions:

– polynomial

$$P_m(a;x) = \sum_{i=0}^{m} a_i x^i \tag{6}$$

– the sum of polynomial and exponential function

$$Q_m(a;x) = \sum_{i=0}^{m} a_i x^i + Ae^{qx} \tag{7}$$

– the sum of polynomial and logarithmic function

$$L_m(a;x) = \sum_{i=0}^{m} a_i x^i + A\ln(x+p) \tag{8}$$

where a_i (i=0,…, m) and A are unknown parameters; q and p are fixed parameters. The package allows also obtaining the continuous and smoothed spline approximation for a given value of absolute error using the following nonlinear expressions:

– the sum of polynomial and exponential function

$$E_m(a;x) = \sum_{i=0}^{m} a_i x^i + Ae^{px} \tag{9}$$

– the sum of polynomial and power function

$$E_m(a;x) = \sum_{i=0}^{m} a_i x^i + Ae^{px} \tag{10}$$

where the value of the power p is to be found by the approximation procedure.

The fundamental pieces of the ApproCryo package are the modules oriented on calculating of Chebyshev approximation with the expressions (6)-(10) for given interpolation conditions. The package contains a set of fifty-two approximation modules which are intended to calculate the parameters of Chebyshev approximation keeping the exact values of the interpolated function and its derivative at the knot points and endpoints of the approximation interval; moreover, the minimum value of the absolute or relative error is obtained. The ApproCryo package contains also a set of seventeen interpolation modules that are intended to interpolate functions using the formulae (6)-(10). Fifteen of the modules allow for calculating the Hermite interpolations with the formulae (6)-(10) keeping the exact values of the derivative of interpolated function at the knot points and endpoints of the approximation interval. The methods of performing the Chebyshev approximation and interpolation using the formulae (6)-(10) are described in detail in [1-5].

3. Input and output data of the ApproCryo package

The information required for starting the operation of the program is: the tabulated values of the argument, the approximated function and its derivative, the degree of the polynomial in approximating expression, and the value of the acceptable level of approximation error (absolute or relative). The input data can be entered into the input data window displayed on the screen (cf. Fig. 1). The values of the function to be approximated and of its derivative can also be read from a text file saved in the computer's memory. It is not obligatory to input the values of the derived function. If these values are not inputted into the computer, they are calculated automatically using well-known difference equations.

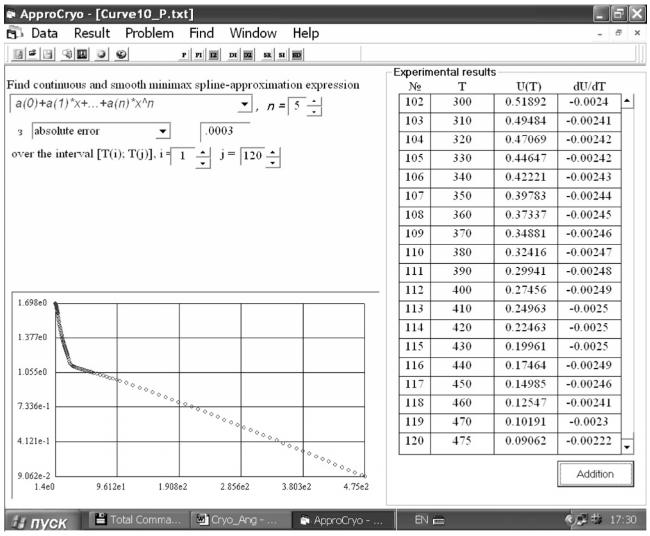

Fig. 1. The view of the program working area showing given input data
Rys. 1. Widok okna roboczego programu z oknem wprowadzania danych wejściowych

For selecting one of the functions (6)-(10) and a given type of the approximation error (absolute or relative), the boxes containing pop-up lists of options are available from the working area.

As the result of the program execution, a continuous and smooth spline is obtained. For a given value of the approximation error, the number of segments of the approximating spline is minimal. The ApproCryo package allows calculating a continuous and smooth minimax spline approximation as well as a continuous (but not smooth) or discontinuous one.

The view of the working area of the program presenting the results of its execution is shown in Figure 2. In this window, the results of calculating a continuous and smooth minimax spline that closely matches the transfer function of a diode-type cryogenic sensor DT-471 [6] are shown. The experimental characteristic is given as a set of 120 data points over the temperature range from 1.4 K to 475 K; the absolute approximation error does not exceed 0.0003 K. The high accuracy is obtained using only six spline segments; each segment is a fifth-degree polynomial. The value of the error for the sensor's sensitivity function approximated by the derivative of the minimax spline is no higher than 0.283 mV/K.

For each segment of the calculated minimax spline approximation, a set of output data is displayed: the ordinal number of the segment, the endpoints of the segment, and the coefficients of the approximation. The approximation error of the given function which met the Chebyshev's criterion under certain conditions imposed on the endpoints of a segment, as well as the reconstruction error of the derived function approximated with the spline derivative can be presented on the screen.

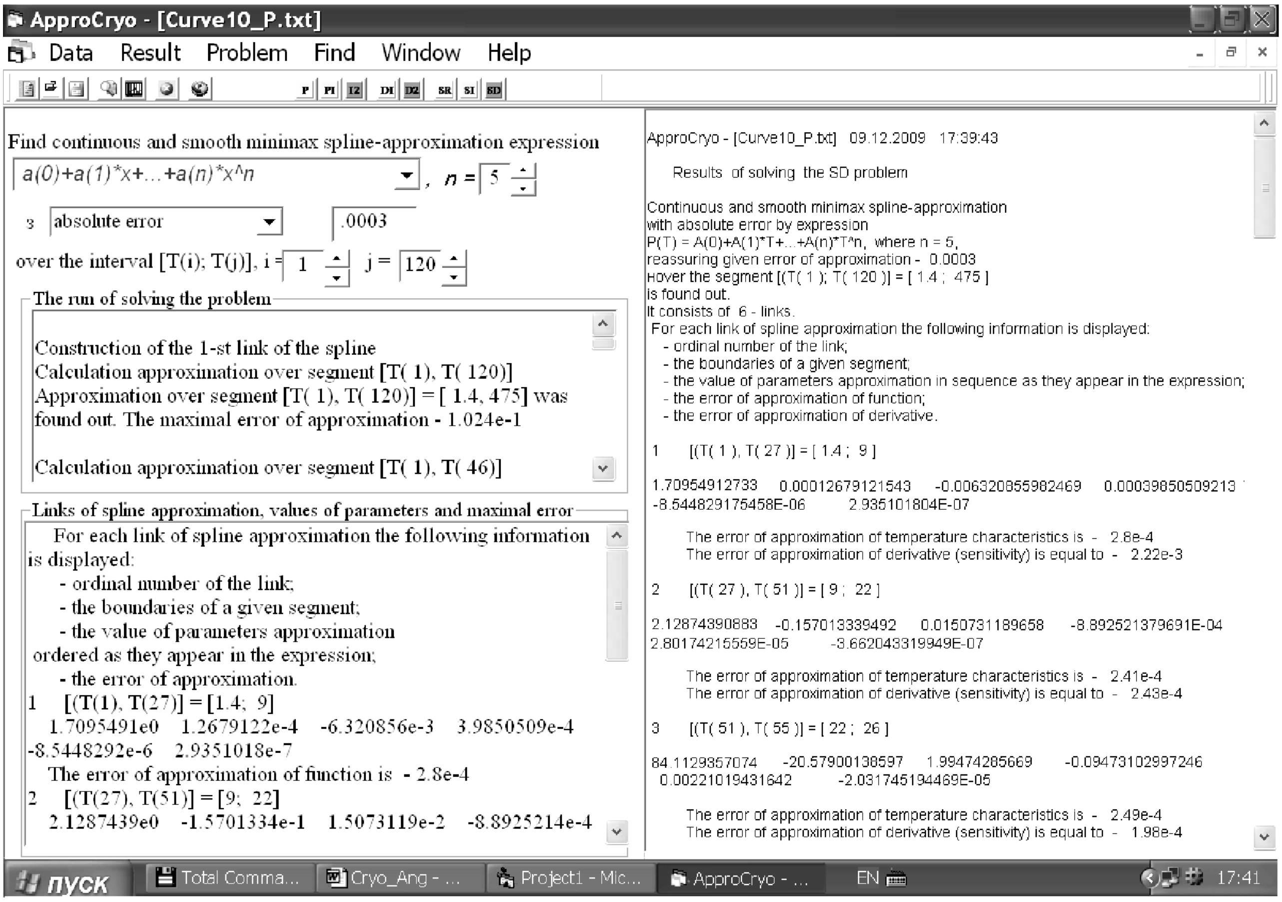

Fig. 2. The view of the program working area showing the results of problem solving
Rys. 2. Widok okna roboczego programu z wyprowadzaniem wyników rozwiązania zadania

The program lets the user to create a graph of the calculated spline showing the data points of the target function; the same can be done for the spline derivative function. Moreover, the program provides the possibility to show the train of the problem solving in detail. The user can look through a table containing both a given value of the function to be approximated and the value calculated from the spline approximation for each data point. The approximation error of the function, the value of derived function and its approximation, and the approximation error of the derived function are also presented in the table. The results can be run over in the panel "Experimental results" in the working area (Fig. 1).

In order to detect the input data which exhibit large random errors, the program allows the user to analyze the set of experimental data points. The analysis is based on the condition of keeping the sensor's transfer function monotonic and on the detection of the values which are spread more than two standard deviation (2σ) from the root-mean-square model.

The package gives the opportunity to trace the train of problem solving. In the panel "The run of solving the problem" of the working area (Fig. 2) the information about the search of optimal length for each spline segment and the values of approximation error are given. According to that information, the process of calculating the spline approximation can be traced back.

Except the main task – to construct a continuous and smooth minimax spline approximation – the package allows solving subsidiary problems which help the user to impose the endpoints of segments, as well as the interpolating conditions, for examining the approximation error in some intervals.

The program offers multitask operation mode. The possibility of solving several problems simultaneously allows the user to compare different versions of admissible results in on-line mode.

4. Conclusions

The software package ApproCryo is provided to calculate optimal continuous and smooth minimax spline approximations in terms of both absolute-error criterion and relative-error one. The transfer function can be approximated with polynomials, exponential and logarithmic expression as well as a sum of a polynomial and an exponential function. The package works under the Windows operating system and its interface is user-friendly and intuitively easy to understand.

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