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Spatial characteristics of geometric deviations of free-form surfaces determined in coordinate measurements

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Abstract

Geometric deviations of free-form surfaces are determined as normal deviations of measurement points from the nominal surface. Different sources of errors in the manufacturing process result in deviations of different character, deterministic and random. Geometric deviations of 3D free-form surfaces may be treated as a spatial process, and spatial data analysis methods can be applied in order to conduct research on the relationships among them. The paper presents theoretic bases for testing spatial dependence of measurement data, as well as the results of tests on geometric deviations involving testing their spatial autocorrelation with the use of the Moran's *I* statistics.

Keywords: geometric deviations, free-form surface, coordinate measurements, spatial autocorrelation.

Charakterystyka przestrzenna odchyłek geometrycznych powierzchni swobodnych wyznaczanych w pomiarach współrzędnościowych

Streszczenie

Pomiary współrzędnościowe są źródłem cyfrowych danych w postaci współrzędnych punktów pomiarowych o dyskretnym rozkładzie na mierzonej powierzchni. Odchyłki geometryczne powierzchni swobodnych wyznacza się w każdym punkcie jako odchyłki normalne tych punktów od powierzchni nominalnej (modelu CAD). Różne źródła błędów w procesie wytwarzania powodują powstawanie odchyłek o odmiennym charakterze, deterministycznych i losowych (rozdz. 2). Udział zjawisk losowych na powierzchni zależy od rodzaju obróbki. W artykule zaproponowano stosowanie metod analizy danych przestrzennych do badań losowości lokalnych odchyłek geometrycznych powierzchni swobodnych, polegających na testowaniu ich przestrzennych powiązań. W badaniach przestrzennej autokorelacji odchyłek powierzchni frezowanej wykorzystano statystykę *I* Morana (rozdz. 3). Wykrycie dodatniej korelacji przestrzennej dowodzi istnienia systematycznych błędów obróbki (rozdz. 4), charakter błędów pozwala na określenie ich przestrzennego modelu, a następnie eliminację przez usunięcie źródeł błędów czy korekcję programu obróbkowego.

Słowa kluczowe: odchyłki geometryczne, powierzchnia swobodna, pomiary współrzędnościowe, autokorelacja przestrzenna.

1. Introduction

The coordinate measurement technique consists in determining the coordinate values of measurement points located on the surface of an object. As a result of the measurement, a set of discrete data is obtained in the form of coordinates of the measurement points. From the point of view of CAD/CAM techniques, the most important feature of the coordinate measurement is providing data concerning the object's geometry in the digital form.

Machine parts composed of free-form 3D surfaces are more often designed. Such parts are shaped by surfaces, which cannot be described by the use of simple mathematical equations. In designing, producing and measuring of free-form surfaces, CAD/CAM techniques are applied. The accuracy inspection consists in digitalizing the workpiece under research (coordinate measurement with the scanning method), followed by comparing the obtained coordinates of the measurement points with the CAD design (model) [1, 2]. The values of local geometric deviations of the free-form surface, or normal deviations of measurement points from the nominal surface, may be calculated by previously determining the deviation components in the *X*, *Y*, *Z* directions [3, 4]. Software of coordinate measurement machines automatically performs such calculation for each measurement point in the UV scanning option. The processing accuracy inspection results may be presented in the form of a three-dimensional plot or a map [5].

Geometric deviations of surfaces are attributed to many phenomena that occur during the machining, both deterministic and random in character. These phenomena with their consequent machining errors can be described in space domain. In machining workpieces including free-form surfaces a multi-axis machining is applied. Different combinations of machining parameters may produce variations in the final product surface quality. In coordinate measurements of free-form surfaces, spatial data are obtained which provide information on the processing and on geometric deviations in the spatial aspect. Deterministic deviations are spatially correlated however, a lack of spatial correlation indicates their spatial randomness. Calculating the values of local geometric deviations solely does not provide much information, neither with regard to the surface properties nor to the course of the machining process. Deviations of random values may be spatially correlated which is reflected in their determined distribution on a surface and is indicative of the existence of a systematic source in the course of processing.

This paper suggests applying methods of analysing spatial data to research on them since spatial analysis of data makes it possible to determine the similarities and differences between the specified areas and also to quantify spatial dependencies. The literature states that the Moran's *I* statistic is used in the majority of cases in order to test the existence of spatial autocorrelation. Cliff and Ord [6] justify that choice. Identifying spatial correlation of measurement data proves the existence of a systematic, repetitive processing error. In such a case, the spatial modelling methods suggested by Cliff and Ord [6], as well as by Kopczewska [7], may be applied to fitting a surface regression model describing the deterministic deviations. The first step in model diagnosing is to examine the model residuals for the existence of spatial autocorrelation. The Moran's *I* test is also used for this purpose.

2. Characteristics of discrete geometric deviations

Geometric deviations may be decomposed into three components: shape deviations, waviness, and roughness [8]. The components connected with the form deviations and waviness are surface irregularities superimposed on the nominal surface, resulting in a smooth surface and most often deterministic in character. The component connected with random phenomena, including the surface roughness, is irregularity of high frequency. The actual surface is the effect of superimposition of the shape deviations, waviness and roughness on the nominal surface [8]. The contribution of random phenomena on the surface depends on the type of processing. The literature data indicate that after the finished milling process, values of random geometric deviations of the surface are greater than those of the deterministic deviations.

Shape deviations are caused, among others, by deviations of machine tool ways, deviations of the machine tool parts, and improper fixing. The surface waviness results from, among others, geometric deviations or tool movement deviations, and vibrations of the machine tool or the processing tool. Roughness is a result of the shape of the tool blade and the tool's longitudinal feed or in-feed as well as of vibrations at the workpiece-tool contact.

In coordinate measurements, the coordinates of a finite number of points on the surface of the workpiece are determined. The aim is to determine a smooth surface superimposed on the nominal surface. However, in the measurement process, the random component and the deterministic component overlap each other. In consequence, the spatial coordinates assigned at each measurement point include two separate components. The component connected with the deterministic deviations represents the smooth surface trend and is spatially correlated. The random component, on the other hand, is weakly correlated and is considered to be of a spatially random character. A surface constructed on measurement points is therefore more complex than a nominal surface.

3. Measuring spatial autocorrelation

Autocorrelation is a characteristic of data obtained from a process that is articulated in one or more directions and describes the error structure of the data. Spatial autocorrelation refers to systematic spatial changes. In general, positive autocorrelation means that the observed feature values in a selected area are more similar to the features of the contiguous areas than it would result from the random distribution of these values. In the case of negative spatial autocorrelation, the values in the contiguous areas are more different than it would result from their random distribution. A lack of spatial autocorrelation means spatial randomness. The values observed in one area do not depend on the values observed in the contiguous areas, and the observed spatial pattern is as much probable as any other spatial pattern.

In order to test the existence of spatial dependence, global and local Moran's and Geary's statistics for a given variable are applied.

The literature data state that the Moran's I statistic is used in the majority of cases; it can be applied to analysing spatial data of both normal and unknown (randomisation) probability distribution [6, 7].

In adapting methods of spatial statistics, concerning research on spatial autocorrelation, to research on geometric deviations, the following needs to be determined:

ε_i – geometric deviation at each measurement point,

$\bar{\varepsilon}$ – arithmetic mean of geometric deviation at n – measurement points,

w_{ij} – weighting coefficients, elements of weighting matrices reflecting spatial relations between ε_i and ε_j.

A spatial weighting matrix defines the structure of the spatial neighbourhood. The matrix measures spatial connections and is constructed in order to specify spatial dependence. One of the possible dependence structures is assumed, e.g. neighbourhood along a common border, neighbourhood within the adopted radius or within the inverse of distance. In research on geometric deviations, it is most suitable to make the spatial interrelations dependent on the distance between the measurement points, in particular on the inverse of the minimum straight-line distance.

As a result of scanning, the coordinates (as well as geometric deviations) of the points distributed on the surface along a regular grid are obtained. The distance between the i-th and j-th point, according to the Euclidean metric, is as follows:

$$d_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{\frac{1}{2}}, \tag{1}$$

where:

x_i, y_i – i-th point coordinates,

x_j, y_j – j-th point coordinates,

d_{ij} – distance between the i-th and j-th measurement point.

If it is assumed that the dependence between the data values at the i and j points decreases when the distance increases, this relation can be described in the following way:

$$w_{ij} = d_{ij}^{-k}, \tag{2}$$

where:

$w_{ij} = 0$ for $i = j$,

k – constant ($k \geq 1$).

The spatial autocorrelation coefficient has the following form covariance/variance ratio with a weighting scheme based on spatially symmetric interactions:

$$I = \frac{n}{S_0} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(\varepsilon_i - \bar{\varepsilon})(\varepsilon_j - \bar{\varepsilon})}{\sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon})^2}, \tag{3}$$

where:

$$S_0 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}, \quad (i \neq j). \tag{4}$$

The Moran's I statistic has an asymptotically normal distribution (for $n \to \infty$). In order for assessing the Moran's statistics to be correct, the analysed variable must have a constant variance.

The Moran's I statistic indicates whether there is a spatial effect of agglomeration or not. Positive and significant values of the statistics imply the existence of positive spatial autocorrelation, i.e. a similarity of observation in the specified d distance. Negative statistics values mean negative autocorrelation, i.e. diversification of the tested observations and are indicative of the fact that the neighbouring areas are more different than it would result from their random distribution.

After having determined the I coefficient, the null hypothesis of no spatial autocorrelation at the assumed significance level needs to be verified, examples showed Upton and Fingleton in [9]. The distribution moments of Moran's I statistic can be determined both at the assumption that the data (deviations) comes from the normal distribution population and at the assumption that it comes from the population of an unknown probability distribution. When the number of localities is large it is reasonable to use the normal approximation. In the case of assuming the normal distribution, the expected value and the variance depend exclusively on spatial weights. If randomisation is assumed, these moments depend also on the value of the variable under research.

Assuming a normal probability distribution for geometric deviations, the E(I) expected value and the var(I) variance are calculated using the formulae [6, 9]:

$$\mathrm{E}(I) = \frac{-1}{n-1}, \tag{5}$$

$$\mathrm{var}(I) = \frac{\left(n^2 S_1 - n S_2 + 3 S_0^2\right)}{(n-1)(n+1) S_0^2}, \tag{6}$$

where:

$$S_1 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_{ij} + w_{ji}\right)^2 , \quad \left(i \neq j\right), \tag{7}$$

$$S_2 = \sum_{i=1}^{n}\left(w_{i(.)} + w_{(.)j}\right)^2 , \tag{8}$$

$$w_{i(.)} = \sum_{j} w_{ij} , \tag{9}$$

$$w_{(.)j} = \sum_{i} w_{ji} . \tag{10}$$

The expected value (4) of the Moran I statistic approaches 0, which might be interpreted as randomness [6, 7, 9].

Verifying the hypothesis of no spatial autocorrelation (or randomness) in the data set under research follows the plan listed below:

1. Formulating the H_0 null hypothesis: data is not spatially correlated. The alternative H_1 hypothesis: data is spatially correlated.
2. Assuming the significance level or the probability of rejecting the null hypothesis when it is true.
3. Calculating the test statistics (standard normal deviate) $z = I_p - E(I)/\sqrt{\mathrm{var}(I)}$, I_p – the coefficient evaluated from experimental sample (3), distribution moments calculated using the formulae (5) and (6).
4. Determining the limit value of the z test statistics; for the adopted significance level $z(\alpha) = z_\alpha$, which means that if the $z < z_\alpha$ there is no reason for rejecting the null hypothesis, and in that case the null hypothesis is accepted, otherwise the alternative hypothesis is accepted.

In tests on geometric deviations, accepting the null hypothesis means that the tested deviations are spatially random.

4. Experimental investigations

The experiments were performed on a free-form surface of a workpiece made of aluminium alloy with the base measuring 100x100 mm (Fig. 1), obtained in the milling process using ball-end mill 6 mm in diameter, rotational speed equal to 7500 rev/min, working feed 300 mm/min and zig-zag cutting path in the XY plane. The measurements were carried out on a Mistral Standard 070705 Brown&Sharpe CMM (PC-DMIS software, MPE_E = 2,5 + L/250), equipped with a Renishaw TP200 touch trigger probe (3D form measurement deviation = ±1μm), a 20 mm stylus with a ball tip of 2 mm in diameter.

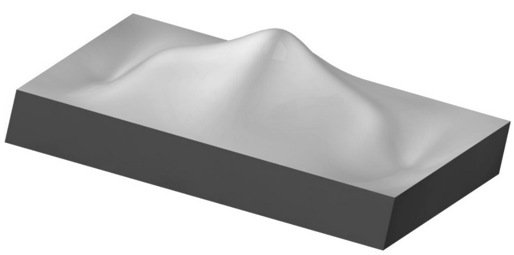


Fig. 1. Model CAD of the surface
Rys. 1. Model CAD powierzchni

4.1. Characteristics of the measured surface

The surface was scanned (not applying radius compensation) with the UV scanning option (the option built in PC-DMIS software), 2500 uniformly distributed measurement points were scanned from the surface (50 rows and 50 columns), and the process of fitting the data to the nominal surface was then carried out in which the least square method was applied and all the measurement points were used [10]. In this way the position deviation was minimized. Measurement process was subsequently repeated, geometric deviations ε were computed.

The obtained measurement data are presented in a graphical form. Fig. 2 shows a spatial plot of the ε deviations with reference to the x and y nominal coordinates. The deviation distribution indicates that the measurement points contain both the deterministic and the random component (Fig. 2, Fig. 3).

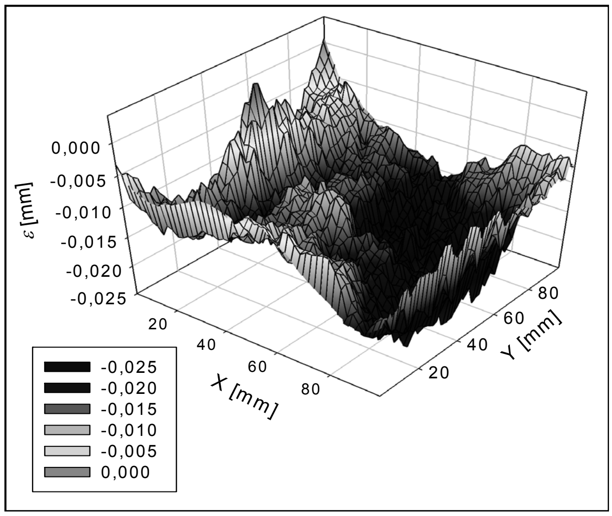


Fig. 2. Spatial plot of geometric deviations versus XY plane
Rys. 2. Przestrzenny wykres odchyłek geometrycznych względem płaszczyzny XY

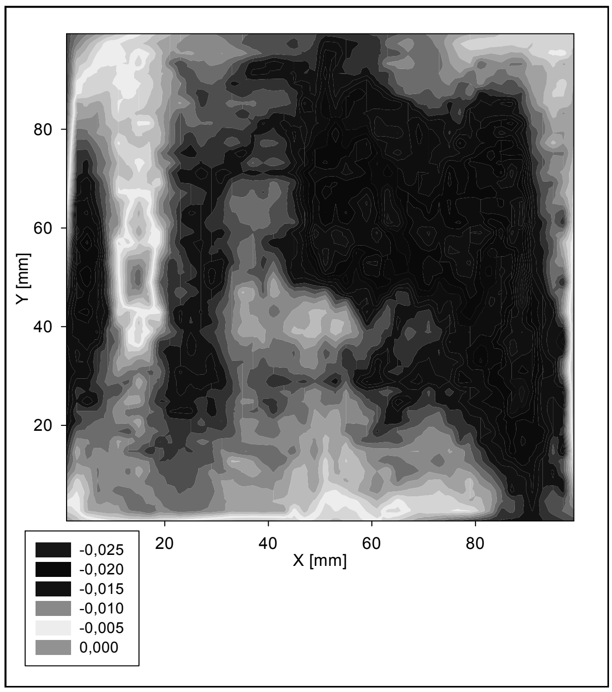


Fig. 3. Map of geometric deviations
Rys. 3. Mapa odchyłek geometrycznych

In a test on randomness of the values of the obtained data it turned out that the values of geometric deviations undergo a normal probability distribution, which is illustrated in Fig. 4. In the Kolmogorov-Smirnov test on normality, if calculated ratio d is greater then the limit value d_α, then reject the null hypothesis of normality at the chosen significance level α. In this case the null hypothesis was adopted ($\alpha = 0{,}05$; $d = 0{,}0266$; $d_\alpha = 0{,}0272$; $d < d_\alpha$).

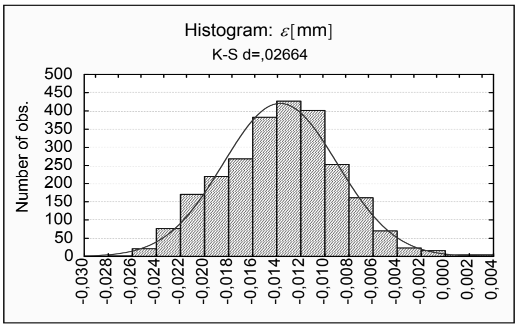
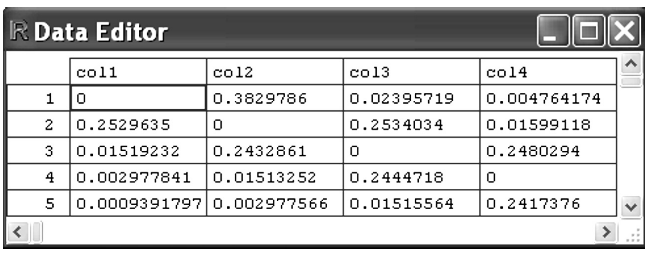


Fig. 4. Probability distribution of geometric deviations
Rys. 4. Rozkład prawdopodobieństwa odchyłek geometrycznych

4.2. Tests on spatial autocorrelation of deviations

Tests on spatial autocorrelation of geometric deviations were subsequently carried out. The relationships between the deviations were made dependent on the reciprocal distances determined from the (1) formula. The elements of weight matrices, defining the dependencies between deviations at points *i* and *j* were calculated from the (2) formula, assuming the value of the constant as $k = 4$. A fragment of the weight matrix is shown in Fig. 5. Moving successively from the dependence (3) to (6), and later according to the items of the plan described in p. 3, the spatial autocorrelation coefficient was determined, and the null hypothesis of the lack of geometric deviations autocorrelation *I* was verified, assuming a normal plot approximation, at the significance level $\alpha = 0{,}05$ (the upper point of a standard normal distribution $z_\alpha = 1{,}645$). The computations were performed in the *R-Gui* programme. Fig. 6 presents the print screen image with the computation results.


Data Editor

	col1	col2	col3	col4
1	0	0.3829786	0.02395719	0.004764174
2	0.2529635	0	0.2534034	0.01599118
3	0.01519232	0.2432861	0	0.2480294
4	0.002977841	0.01513252	0.2444718	0
5	0.0009391797	0.002977566	0.01515564	0.2417376

Fig. 5. The top left corner of the *W* matrix
Rys. 5. Górny lewy narożnik macierzy wag *W*

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        Moran's I test under normality

data:  odchylki
weights: wagi

Moran I statistic standard deviate = 87.9371, p-value < 2.2e-16
alternative hypothesis: greater
sample estimates:
Moran I statistic       Expectation          Variance
     0.8861198923     -0.0004001601      0.0001016326
```

Fig. 6. Print screen image of *R-Gui* programme with computation results
Rys. 6. Zrzut z ekranu z wynikami obliczeń w programie *R-Gui*

The null hypothesis of the lack of spatial autocorrelation was rejected ($I = 0{,}886$, $z = 87{,}937$, z_α=1,645, $z > z_\alpha$). The computation results show a clear positive autocorrelation of geometric deviations. In this case it is possible to predict in approx. 80% the values in the neighbouring points on the basis of the deviation value at any point. Considering that, increasing the number of measurement points (as opposed to the case with spatially independent values) does not provide much additional information because the deviations values can be predicted on the basis of the deviations at the neighbouring points.

5. Conclusions

Methods of spatial data analysis are most suitable in research on geometric deviations of free-form surfaces, because they allow for obtaining information on spatial dependence between the deviations values at individual measurement points. This is significant information concerning the accuracy of surfaces, both with regard to the surface properties and to the course of the machining process. These methods may be applied both to analysing raw data or data obtained directly from measurements, and also to researching residuals from surface regression models in tests of models' adequacy. Detecting a positive spatial autocorrelation is a proof that a systematic processing error has appeared, while the character of the error makes it possible to determine its value (spatial model) and later to minimize the error by removing its source and/or by correcting the processing programme.

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