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Elimination of beat effects in structures by added lumped mass

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Abstract

A formulation for eliminating beat effects by using additional lumped masses or/and changing damping coefficients is presented in the paper. The Rayleigh's model of viscous damping, being a linear combination of the system mass and stiffness parameters, is adopted in the equations of motion. As an example, a truss dome system, treated as a lattice shell, is considered. Input data of geometrical and material properties, as well as of an assumed impulse excitation, are given. Numerical results are verified alternatively by the mode superposition method and direct step-by-step integration one. Analytical and computational aspects of the problem at hand are discussed and compared through obtained illustrative results. It is turned out that, for such a class of structures, the application of lumped masses should be much more efficient rather than the change of damping properties.

Keywords: dynamics, damping, beat, finite element.

Eliminacja efektów dudnienia w konstrukcjach za pomocą dodatkowych mas

Streszczenie

W pracy przedstawiono sformułowanie eliminacji efektów dudnienia poprzez dodatkowe masy skupione i zmiany tłumienia w dynamice konstrukcji. Zagadnienie dyskretyzowano w ramach modelu przemieszczeniowego metody elementów skończonych. Otrzymano układ równań ruchu o wielu stopniach swobody. Z uwagi na złożony układ konstrukcyjny w analizie wykorzystano macierz tłumienia Rayleigh'a, będącą liniową kombinacją sztywności i masy. Dla przykładowej kopuły kratowej opisano materiałowe i geometryczne dane wejściowe oraz przyjęto obciążenie dynamiczne typu nagłego uderzenia. W celu zweryfikowania poprawności wyników sprawdzono układ alternatywnie dwiema metodami - całkowania bezpośredniego i superpozycji modalnej. Aspekty konstrukcyjne i komputerowe rozpatrywanego układu przedyskutowano szczegółowo poprzez ilustrujące wyniki liczbowe. Wykazano, że zastosowanie dodatkowych mas znacznie efektywniej redukuje dudnienie niż tłumienie.

Słowa kluczowe: dynamika, tłumienie, dudnienie, element skończony.

1. Introduction

A tendency to create structures with amazing shapes, which cross the barriers of height, span and slender can be seen in civil engineering presently. These modern and slim forms strike with an amazing impression but slenderness makes them much more sensitive with respect to their parameters. It is essential to consider these dynamic loads at the designing stage, because increasing cross-sectional area undoubtedly beneficial from the point of view of load the capacity of structure can cause magnifying displacement under dynamic load.

The goal of this work is supposed to be dynamic analysis of the dome and during the research of dynamic loads the beat phenomenon that means periodic changes of amplitude due to an interference between two vibrations with slightly different frequencies is observed. The beat phenomenon is eliminated in two ways - by using dampers and by added lumped mass. It turns out that the latter is significantly effective in this case.

2. Beat and damping phenomena

It is commonly known that the beat is observed in many vibration kinds. Lets consider two harmonic vibrations with the same amplitude A and almost equal frequencies ω_1 and ω_2. Their equations of motion can be written respectively as

$$q_1(t) = A\sin\omega_1 t;\;\; q_2(t) = A\sin\omega_2 t \tag{1}$$

so that

$$q(t) = q_1(t) + q_2(t) = 2A\cos\frac{\omega_1 - \omega_2}{2}t\sin\frac{\omega_1 + \omega_2}{2}t \tag{2}$$

The arising vibration has the frequency which is an arithmetic average from ω_1 and ω_2 but the amplitude changes with time periodically, creating such an envelope.

On the other hand, damping in a structure is very difficult to describe and there is a problem to take it into consideration not only in numerical modelling and computations. The main reason of this situation is that many sources of damping have influence on structural response. In dynamics a few types of energy loss are distinguished by the characteristics of structural materials, geometry discontinuities and/or surrounding media related to aerodynamics, hydrodynamics, electrodynamics, acoustics and radiation damping, for instance. Also, magnetic and thermal effects or atomic reconstruction have influence of dissipating energy too. As damping is one of the most sophisticated topics in engineering, for years scientists have tried to describe this phenomenon and make models which can accurately describe reality. Three damping models may be classified [4]:

1. Viscous Damping – damping force F_d is directly proportional to the velocity, i.e.

$$F_d = c\dot{q} \tag{3}$$

where c is viscous damping constant with displacement $\dot{q}$. The equation of motion for a single degree of freedom takes form

$$m\ddot{q} + c\dot{q} + kq = 0 \tag{4}$$

Energy loss in a cycle of the vibration can be adopted as

$$\Delta E = \pi c\omega A^2 \tag{5}$$

with ω being the natural frequency and A the displacement amplitude.

2. Hysteretic Damping – F_d is independent of the frequency and proportional to the displacement when the velocity is different from zero. Taking h as hysteretic damping constant, damping force under steady state excitation reads [4]

$$F_d = \text{sign}(\ddot{q})h\text{abs}(q) \tag{6}$$

Energy loss in the cycle of the vibration can be then presented as

$$\Delta E = \pi h A^2 \tag{7}$$

3. Coulomb Damping provides a constant force

$$F_d = \text{sign}(\dot{q})d \tag{8}$$

with d being constant, and equation of motion takes the form

$$m\ddot{q} + d\text{sign}(\dot{q}) + kq = 0 \tag{9}$$

This method is characterized by energy loss proportional to the amplitude and independent of frequencies. This model is simple in mathematical definition that's why it is often used for presentation damping effect.

Almost all modern computational programs are recently based on the finite element method (FEM). When considering complex structural systems with many degrees of freedom, it is very difficult or even impossible to describe the problem of damping using the above models. For that reason some simplifications in damping models are applied in the text.

Equations of motion in the global coordinate systems can be written as [1, 5]

$$\tilde{M}\ddot{\bar{q}} + \tilde{C}\dot{\bar{q}} + \tilde{K}\bar{q} = \bar{Q}(t) \tag{10}$$

where $\tilde{M}$, $\tilde{C}$, and $\tilde{K}$ are mass, damping and stiffness matrices, respectively. In this paper for the sake of simplicity the damping terms are assumed as a linear combination of the mass and stiffness terms, so that the Rayleigh's damping matrix can be written as

$$\tilde{C} = \alpha\tilde{M} + \beta\tilde{K}, \tag{11}$$

α and β are the coefficients, obtained in an experimental way. Solving the differential equation by mode superposition we obtain set of uncoupled equations. In numerical analysis is input damping factor λ, which can be expresses as [5]

$$\lambda = \frac{1}{2}\left(\frac{\alpha}{\omega} + \beta\omega\right) \tag{12}$$

3. Structure description – FEM model

Research object is dome presented in Figs. 1. and 2. with its diameter of 10 m and height of 5 m. The set of bars is designed as a steel pipe with cross-sectional area 20 cm^2 and Young's modulus 205 GPa. The whole structure is modeled by 3D truss element in number – 81. The total number of system degrees of freedom is 171. Static analysis is consider for only one vertical force put in the top of the dome, at node 31 (Fig. 2). Hinged supports are defined at nodes 1, 3, 5, 7, 9.

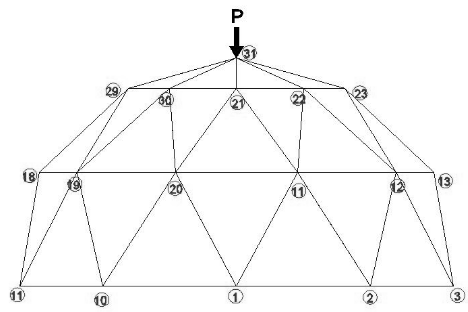


Fig. 1. Structure in front view
Rys. 1. Widok z przodu konstrukcji

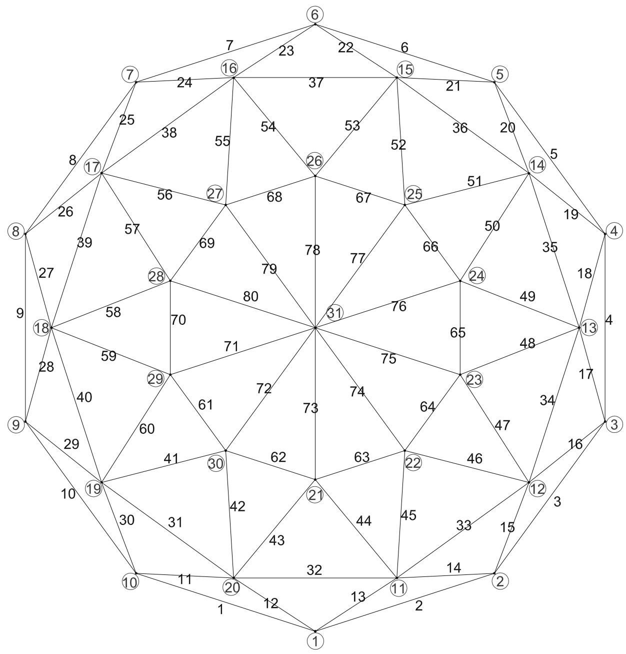


Fig. 2. Structure in bird's eye view – finite element setting
Rys. 2. Widok konstrukcji z lotu ptaka – siatka elementów skończonych

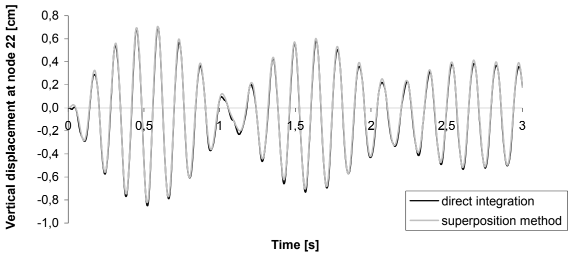
4. Numerical results

Static analysis is supposed to verify structure's correct work and choice of load which can be put in the top of the dome with the dead-weight ignored. Main subject of the study are displacements nodes under dynamic excitation. In order to confirm results the system is computed independently with the aid of superposition method and direct integration. Since the outcomes turned out to be well-converged (see Fig. 3.), further analysis is conducted by the means of the superposition method.

In dynamic analysis we consider the case of a sudden hit of a significant amount on to the top of the dome. The system is integrated over 6000 time steps Δt with 0,005 s each. The eigenproblem is solved for the first 12 eigenpairs and converged after 14 iterations. We apply the constant vertical impulse of 10^3 kN in the top of the dome during 41 seconds.

The progress of vibration at chosen nodes (Fig. 3.) revealed the existence of beat phenomenon. It is undoubtedly harmful effect. Because of the symmetry, slightly different circular frequencies interfere so the amplitude of the vibration changes periodically. The main aim of this paper became an attempt of beat phenomena elimination by two methods: using damping and adding inertial masses.

a)



b)

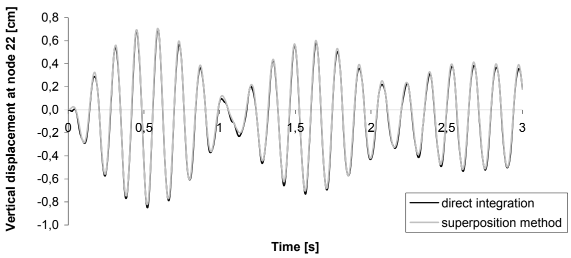
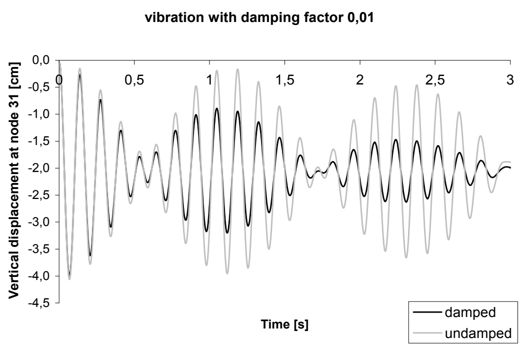


Fig. 3. a) Vertical displacement at node 22, b) vertical displacement at node 31

Rys. 3. a) Pionowe przemieszczenie węzła 22, b) pionowe przemieszczenie węzła 31

The first way did not give a satisfying effect, because with damping considered vibration decreased, however an obvious elimination of beat did not occur. The assumed damping factor is λ=0,01 and λ=0,05. Comparison of damped and undamped system for chosen nodes is presented in Fig. 4.

a)



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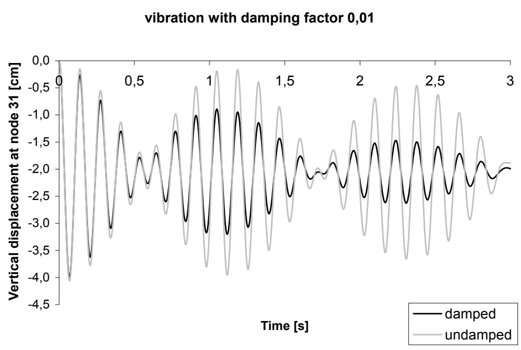

Fig. 4. a) Vertical displacement at node 31 with damping factor 0,01, b) vertical displacement at node 31 with damping factor 0,05

Rys. 4. a) Pionowe przemieszczenie węzła 31 ze współczynnikiem tłumienia 0,01, b) Pionowe przemieszczenie węzła 31 ze współczynnikiem tłumienia 0,05

The second method turns out more successful. In the beginning it seemed that unsymmetrical placement of masses would give the expected effect yet the computational results showed that such masses do not interfere in a satisfying way the beat in the structure. Following an existing engineering solution of a damper in Taipei 101 building, inertial mass was put centrically in the top point of the dome. Presentation of time dependent displacement for system with and without mass is in Fig. 5.

The values of circular frequencies for the undamped system, with damping and lumped mass are shown in Tab. 1.

Tab. 1. Natural frequencies of the system

Tab. 1. Częstości własne układu

Natural frequency, 1/s	
Undamped system without added mass	System with added mass
28,06	13,25
28,07	16,92
42,24	16,92
46,2	44,31
46,21	45,59
47,5	45,6
47,51	47,5

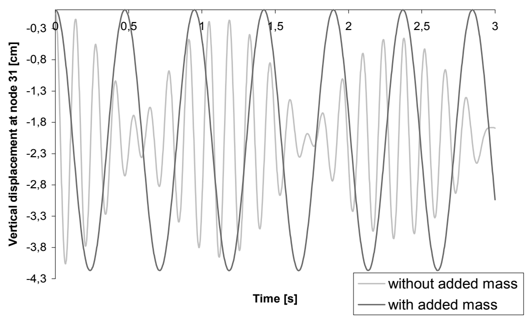

Fig. 5. Vertical displacement at the top (node 31) with and without added mass

Rys. 5. Pionowe przemieszczenie wierzchołka (węzeł 31), z i bez dodatkowych mas

5. Concluding remarks

Beat effects, so interesting and useful in many engineering branches such as acoustic, music etc. may stand for an inherent disadvantages in many structures, these with regular, repeating – segment geometry in particular. Trial to elimination this phenomena by using more damping and adding lumped mass give surprising result. It tuned out that the use of lumped mass effectively reduces beat effects and it may be used with success in civil engineering.

6. References

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