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Dedicated to professor Mikołaj Busłowicz on the occasion of his 60th birthday

Electrical circuits with state-feedbacks and desired dynamical properties

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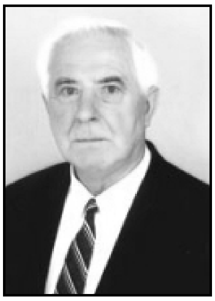

Tadeusz Kaczorek, born 1932 in Poland, currently full professor at Bialystok University of Technology. He did duties for many most important scientific institutions in Poland and over the world. He is full member of Polish Academy of Sciences (PAN), and honorary member of the Hungarian Academy of Sciences. Awarded by seven universitys by the title honoris causa doctor. His research interests cover the theory of systems and the automatic control systems theory.

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Abstract

Linear electrical circuits composed of resistors, inductances, capacitances and voltage (current) sources with state-feedbacks are addressed. It is shown that for large class of nonpositive electrical circuits it is possible to choose gain matrices of the state-feedbacks so that the closed-loop systems are positive and have some desired dynamical properties. Sufficient conditions for nonnegativity of *B* matrices of linear electrical circuits are established. Considerations are illustrated by three examples of linear electrical circuits.

Keywords: positive, linear, electrical, circuit, state-feedback, gain matrix, dynamical property.

Obwody elektryczne ze sprzężeniami zwrotnymi od wektora stanu o pożądanych właściwościach dynamicznych

Streszczenie

W pracy są rozpatrywane liniowe obwody elektryczne złożone z rezystancji, pojemności, indukcyjności i źródeł napięcia (prądu). Wykazano, że dla szerokiej klasy niedodatnich obwodów elektrycznych można dobrać macierz wzmocnień statycznych sprzężeń zwrotnych od wektora stanu tak, aby układ zamknięty był dodatni i miał pożądane właściwości dynamiczne. Podano warunki wystarczające nieujemności elementów macierzy *B* liniowych obwodów elektrycznych. Rozważania ogólne zostały zilustrowane przykładami obwodów elektrycznych.

Słowa kluczowe: dodatni, liniowy, elektryczny, obwód, sprzężenie zwrotne od stanu, macierz wzmocnień.

1. Introduction

It is well-known [1, 15, 16, 23, 24] that linear electrical circuits composed of resistances, inductances and voltage (current) sources or of resistances, capacitances and voltage (current) sources are examples of linear positive systems. A dynamical system is called positive if its trajectory starting from any nonnegative initial states remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive systems theory is given in the monographs [17, 20]. Electrical circuits composed of resistances, capacitances, inductances and voltage (current) sources are not in general case positive systems but by suitable choice of gain matrices of state-feedbacks the closed-loop circuits can be positive with desired dynamical properties [22].

Stability of linear continuous-time fractional systems with delays of retarded type has been investigated in [13] and positive linear discrete-time systems with delays in [14]. Different problems of analysis of electrical circuits have been investigated by Busłowicz in [2-12].

In this paper it will be shown that for large class of nonpositive electrical circuits it is possible to choose the gain matrices of state-feedbacks so that the closed-loop circuits are positive and have some desired dynamical properties.

The paper is organized as follows. Some preliminaries concerning linear positive systems and nilpotent matrices are recalled in section 2. Problem formulation is given in section 3. Problem solution and electrical circuits with desired dynamical properties are presented in section 4. Concluding remarks are given in section 5.Some lemmas concerning nilpotent matrices are given in appendix A. Sufficient conditions for nonnegativity of matrices *B* of linear electrical circuits are established in appendix B.

2. Preliminaries

Let $\Re^{n\times m}$ be the set of $n\times m$ real matrices. The set $n\times m$ matrices with nonnegative entries will be denoted by $\Re_+^{n\times m}$ and $\Re_+^n = \Re_+^{n\times 1}$. The set of nonnegative integers will be denoted by Z_+.

Consider the linear continuous-time system

$$\dot{x} = Ax + Bu \tag{1}$$

where $x = x(t) \in \Re^n$ is the state vector, $u = u(t) \in \Re^m$ is input vector and $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$.

Definition 1. The system (1) is called positive if and only if $x(t) \in \Re_+^n,\ t \ge 0$ for any initial conditions $x_0 = x(0) \in \Re_+^n,\ t \ge 0$.

Theorem 1. [20, 17] The system (1) is positive if and only if

$$A \in M_n \text{ and } B \in \Re_+^{n\times m} \tag{2}$$

where M_n is the set of $n\times n$ Metzler matrices, i.e. real matrices with nonnegative off diagonal entries.

Definition 2. The positive system (1) is called asymptotically stable if and only if

$$\lim_{t\to\infty} x(t) = 0 \text{ for all } x_0 \in \Re_+^n \tag{3}$$

Theorem 2. [20, 17] The positive system (1) is asymptotically stable if and only if its characteristic polynomial

$$\det[I_n s - A] = s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0 \tag{4}$$

has all positive coefficients, i.e. $a_k > 0,\ k = 0,1,...,n-1$.

Theorem 3. [20, 17] The positive system (1) is unstable if at least one diagonal entry of the matrix $A = [a_{ij}]$ is positive, i.e. $a_{ii} > 0$ for some $i \in (1,2...,n)$.

Definition 3. A real matrix $A \in \Re^{n\times n}$ is called nilpotent if there exist a natural number $v \le n$ such that $A^{v-1} \ne 0$ and $A^v = 0$. The natural number *v* is called the nilpotency index of the matrix *A*.

It is well-known [19] that the strictly upper triangular matrices and the strictly lower triangular matrices $A \in \Re^{n\times n}$ are nilpotent matrices with nilpotency indices $v \le n$. These matrices have the characteristic polynomials of the form

$$\det[I_n\lambda - A] = \lambda^n \tag{5}$$

and all their eigenvalues are equal to zero.

In appendix A some lemmas concerning nilpotent matrices and the nilpotency indices are presented.

3. Problem formulation

It is well-known [18, 21, 23] that any linear electrical circuits consisting of resistors, capacitors, coils and voltage (current) sources can be described by the state equation

$$\dot{x} = Ax + Be \tag{6}$$

where $x \in \Re^n$ is the state vector, $e \in \Re^m$ is the input vector, $A \in \Re^{n\times n}$ and $B \in \Re^{n\times m}$.

As state variables $x_1, x_2, \ldots, x_n$ (the components of x) usually the currents in the coils and voltages across the capacitors are chosen. The components of the input vector e are the source voltages or source currents.

Consider the electrical circuit (6) with the state-feedback

$$e = Kx \tag{7}$$

where $K \in \Re^{m\times n}$ is gain matrix.

Remark 1. Note that electrical circuits with state-feedbacks (7) are equivalent to linear circuits with controlled voltage (current) sources.

Substitution of (7) into (6) yields

$$\dot{x} = A_c x \tag{8}$$

where

$$A_c = A + BK \tag{9}$$

We are looking for a gain matrix such that the closed-loop system (8) has desired dynamics, for example

1. the matrix A_c has prescribed eigenvalues in the left half of the complex plane
2. the matrix A_c has desired nilpotency index $v = 2$

4. Electrical circuits with desired dynamical properties

It is assumed that the matrix B in (9) has nonnegative entries and the matrix A may not be a Metzler matrix.

Let A_c be a Metzler matrix with desired dynamical properties. If for given A_c the following condition is met

$$rank\,B = rank[B,\ A_c - A] \tag{10}$$

then the equation

$$BK = A_c - A \tag{11}$$

has a solution K.

In this case we have the following theorem.

Theorem 4. If the condition (10) is satisfied then there exists a gain matrix K such that the closed-loop system is positive with desired dynamical properties.

Proof. If for a chosen Metzler matrix A_c with desired dynamical properties the condition (10) is satisfied then by Kronecker-Cappely theorem the equation (11) has a solution K such that the closed-loop matrix is equal to A_c.■

Theorem 5. Let the condition (10) for a matrix A_c with nilpotency index $v = 2$ be satisfied. Then there exists a gain matrix K such that the state variables of the closed-loop circuit are linear functions of time for any given initial conditions $x(0) = x_0$.

Proof. If the condition (10) is satisfied then the equation (11) has a solution K for given matrices A, A_c and B. If the matrix A_c has nilpotency index $v = 2$ then

$$A_c^k = 0 \ \text{ for } \ k = 2,3,.. \tag{12}$$

and this implies

$$x(t) = e^{A_c t}x_0 = \sum_{k=0}^{\infty}\frac{(A_c t)^k}{k!}x_0 = (I_n + A_c t)x_0 \tag{13}$$

In this case state variables of the closed-loop system are linear functions of time for any initial conditions x_0.■

Example 1. Consider the electrical circuit shown on Fig. 1 with given resistances R_1, R_2 capacity C, inductances L_1, L_2 and voltage sources e_1, e_2

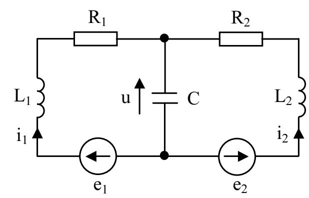

Fig. 1. Electrical circuit no1
Rys. 1. Obwód elektryczny nr1

Using the Kirchhoff's laws we may write the equations

$$e_1 = R_1 i_1 + L_1\frac{di_1}{dt} + u \tag{14a}$$

$$e_2 = R_2 i_2 + L_2\frac{di_2}{dt} + u \tag{14b}$$

$$C\frac{du}{dt} = i_1 + i_2 \tag{14c}$$

The equations (14) can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ u \end{bmatrix} = A\begin{bmatrix} i_1 \\ i_2 \\ u \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{15}$$

where

$$A = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & \frac{-R_2}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix},\quad B = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \tag{15a}$$

The following two cases will be considered.
In **case 1** we choose the matrix A_c of the form

$$A_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix} \tag{16}$$

It is easy to check that the condition (10) is satisfied and the equation (11) takes the form

$$\begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix} K = \begin{bmatrix} \frac{R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & \frac{R_2}{L_2} & \frac{1}{L_2} \\ 0 & 0 & 0 \end{bmatrix} \tag{17}$$

and its solution is

$$K = \begin{bmatrix} R_1 & 0 & 1 \\ 0 & R_2 & 1 \end{bmatrix} \tag{18}$$

From (13) we have

$$\begin{bmatrix} i_1(t) \\ i_2(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{C}t & \frac{1}{C}t & 1 \end{bmatrix} \begin{bmatrix} i_{10} \\ i_{20} \\ u_0 \end{bmatrix} = \begin{bmatrix} i_{10} \\ i_{20} \\ u_0 + \frac{i_{10}}{C}t + \frac{i_{20}}{C}t \end{bmatrix} \tag{19}$$

where $i_{10} = i_1(0),\ i_{20} = i_2(0),\ u_0 = u(0)$.

In **case 2** we choose the matrix A_c of the form

$$\begin{bmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & 0 \\ \frac{1}{C} & \frac{1}{C} & -\lambda_3 \end{bmatrix} \quad (\lambda_k > 0,\ k = 1,2,3) \tag{20}$$

It is easy to see that in this case the condition (10) is not satisfied and we are not able assign the eigenvalues of A_c in desired positions by suitable choice of the gain matrix K since the third row of the matrix B is zero row.

Example 2. Consider the electrical circuit shown on Fig. 2 with given resistances R_1, R_2 capacitances C_1, C_2, inductance L and voltage sources e_1, e_2, e_3.

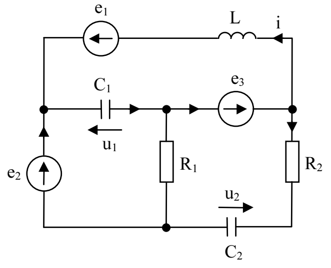

Fig. 2. Electrical circuit no 2
Rys. 2. Obwód elektryczny nr 2

Using the Kirchhoff's laws we may write the equations

$$\begin{aligned} e_1 + e_3 &= L\frac{di}{dt} + u_1 \\ e_2 &= R_1\left(-i + C_1\frac{du_1}{dt} - C_2\frac{du_2}{dt}\right) + u_1 \\ e_3 &= R_2C_2\frac{du_2}{dt} + u_2 + R_1\left(i - C_1\frac{du_1}{dt} + C_2\frac{du_2}{dt}\right) \end{aligned} \tag{21}$$

The equations (21) can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} = A\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{22a}$$

where

$$A = \begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{1}{C_1} & -\frac{R_1+R_2}{C_1R_1R_2} & -\frac{1}{C_1R_2} \\ 0 & -\frac{1}{C_2R_2} & -\frac{1}{C_2R_2} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L} & 0 & \frac{1}{L} \\ 0 & \frac{R_1+R_2}{C_1R_1R_2} & \frac{1}{C_1R_2} \\ 0 & \frac{1}{C_2R_2} & \frac{1}{C_2R_2} \end{bmatrix} \tag{22b}$$

Note that the matrix A is not a Metzler one since it has some negative off-diagonal entries and the matrix B has nonnegative entries. Therefore, the electrical circuit is not a positive system.
Let

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = K\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} \tag{23}$$

where $K \in \Re^{3\times 3}$ is a gain matrix.
Substitution of (23) into (22a) yields,

$$\frac{d}{dt}\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} = A_c\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} \tag{24}$$

where

$$A_c = A + BK \tag{25}$$

We are looking for a gain matrix K such that the closed-loop matrix A_c is a Metzler matrix with nilpotency index $v = 2$ of the form [11]

$$A_c = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & 0 \\ 0 & a_2 & 0 \end{bmatrix} \tag{26}$$

where a_1, a_2 are some positive real parameters.
In this case the matrix B is nonsingular and from (25) we obtain

$$K = B^{-1}\left(A_c - A\right) \tag{27}$$

In particular case for (26) we have

$$K=\begin{bmatrix}\frac{1}{L} & 0 & \frac{1}{L}\\ 0 & \frac{R_1+R_2}{C_1R_1R_2} & \frac{1}{C_1R_2}\\ 0 & \frac{1}{C_2R_2} & \frac{1}{C_2R_2}\end{bmatrix}^{-1}\begin{bmatrix}0 & a_1+\frac{1}{L} & 0\\ -\frac{1}{C_1} & \frac{R_1+R_2}{C_1R_1R_2} & \frac{1}{C_1R_2}\\ 0 & a_2+\frac{1}{C_2R_2} & \frac{1}{C_2R_2}\end{bmatrix}= \tag{28}$$
$$=\begin{bmatrix}-R_1 & a_1L+1-a_2R_1C_2-a_2C_2R_2 & -1\\ -R_1 & 1-a_2R_1C_2 & 0\\ R_1 & (R_1+R_2)C_2a_2 & 1\end{bmatrix}$$

and

$$\begin{bmatrix}i(t)\\ u_1(t)\\ u_2(t)\end{bmatrix}=\begin{bmatrix}1 & a_1t & 0\\ 0 & 1 & 0\\ 0 & a_2t & 1\end{bmatrix}\begin{bmatrix}i_0\\ u_{10}\\ u_{20}\end{bmatrix}=\begin{bmatrix}i_0+a_1u_{10}t\\ u_{10}\\ u_{20}+a_2u_{10}t\end{bmatrix} \tag{29}$$

where $i_0=i(0),\ u_{10}=u_{10}(0),\ u_{20}=u_{20}(0).$

5. Concluding remarks

Linear electrical circuits composed of resistances, inductances, capacitances and voltage (current) sources with state-feedbacks have been addressed. It has been shown that for nonpositive electrical circuits with nonnegative *B* matrices if the condition (10) is met then it is possible to find gain matrices of the state-feedback such that the closed-loop circuits have some desired dynamical properties (Theorem 4). If for a matrix A_c with nilpotency index $v=2$ the condition (10) is satisfied then there exists a gain matrix *K* such that the state-variables of the closed-loop circuit are linear functions of time for any given initial conditions (Theorem 5). In Appendix A some lemmas concerning nilpotent matrices have been given and in appendix B sufficient conditions for nonnegativity of *B* matrices of linear electrical circuits have been established. The main result of the paper have been illustrated by linear electrical circuits.

Appendix A
Nilpotent matrices

Lemma A1. Matrices of the form

$$A=\begin{bmatrix}0 & a_{12} & \dots & a_{1,n-1} & 0\\ 0 & 0 & \dots & 0 & 0\\ \vdots & \vdots & \dots & \vdots & \vdots\\ 0 & 0 & \dots & 0 & 0\\ 0 & a_{n,2} & \dots & a_{n,n-1} & 0\end{bmatrix}\in\Re^{n\times n} \tag{A1}$$

have the nilpotency index $v=2$ for any values of the entries $a_{12},\dots a_{1,n-1}$, $a_{n,2},\dots a_{n,n-1}$ and the characteristic polynomials of the form

$$\det[I_n\lambda-A]=\lambda^n \tag{A2}$$

Proof. Using (A1) it is easy to check that $A^2=0$ and

$$\det[I_n\lambda-A]=\det\begin{bmatrix}\lambda & -a_{12} & \dots & -a_{1,n-1} & 0\\ 0 & \lambda & \dots & 0 & 0\\ \vdots & \vdots & \dots & \vdots & \vdots\\ 0 & 0 & \dots & \lambda & 0\\ 0 & -a_{n,2} & \dots & -a_{n,n-1} & \lambda\end{bmatrix}=\lambda^n.\ \blacksquare \tag{A3}$$

Lemma A2. Matrices of the form

$$A=\begin{bmatrix}0 & A_{12}\\ 0 & 0\end{bmatrix}\in\Re^{2n\times 2n} \tag{A4}$$

have the nilpotency index $v=2$ and the characteristic polynomial of the form

$$\det[I_{2n}\lambda-A]=\lambda^{2n} \tag{A5}$$

for any submatrices $A_{12}\in\Re^{n\times n}$.

Proof. Using (A4) it is easy to verify that $A^2=0$ and

$$\det[I_{2n}\lambda-A]=\det\begin{bmatrix}I_n\lambda & -A_{12}\\ 0 & I_n\lambda\end{bmatrix}=\lambda^{2n}.\ \blacksquare \tag{A6}$$

From the well-known property of the transposition (denoted by upper index *T*) of the matrix *A*, $(A^k)^T=(A^T)^k$ for $k=1,2,...$ we have the following remark.

Remark A1. The transpose matrix A^T has a nilpotency index *v* if and only if the matrix *A* has the same nilpotency index *v*.

Lemma A3. A diagonal matrix *A* with at least one nonzero entry is not the nilpotent matrix.

Proof. This follows immediately from the relation

$$A^k=(diag[a_1,...,a_n])^k=diag[a_1^k,...,a_n^k]\neq 0 \tag{A7}$$

for $k=1,2,...$ if at least one from the entries $a_1,...,a_n$ are nonzero.■

Lemma A4. Nonnegative matrix $A\in\Re_+^{n\times n}$ with at least one nonzero diagonal entry is not nilpotent matrix.

Proof. Let decompose the matrix *A* as the sum of the diagonal matrix *D* and the nonnegative matrix *B* with zero diagonal entries. Then

$$A^k=(D+B)^k=D^k+BD^{k-1}+...+B^k\ \text{ for } k=1,2,... \tag{A8}$$

If the matrix *A* has at least one nonzero diagonal entry then $D\neq 0$ and by Lemma A3 $D^k\neq 0$ for $k=1,2,...$. From (A8) we have $A^k\neq 0$ for $k=1,2,...$ since $D^k\neq 0$ and the remaining matrices are nonnegative.■

Appendix B
Electrical circuits with nonnegative *B* matrices

Theorem B1. Entries of *B* matrices of electrical circuits composed of resistances, capacitances and voltage sources are positive if directions of currents caused separately by each voltage source are consistent in all capacitors.

Proof. Substituting $u_i=0$ for $i=1,...,n$ and $e_j=1$, $e_i=0$ for $i\neq j$, $j=1,...,m$ to the equation

$$\frac{du_k}{dt}=\sum_{i=1}^{n}a_{ki}u_i+\sum_{j=1}^{m}b_{kj}e_j,\quad k=1,...,r \tag{B1}$$

we obtain

$$\frac{du_k}{dt} = b_{kj}, \quad j = 1,...,m; \quad k = 1,...,n \tag{B2}$$

Note that at $t = 0+$ the capacitors are short circuits and the current $i_{C_k}(0+)$ caused by voltage source $e_j = 1$ is equal to

$$C_k \left.\frac{du_k}{dt}\right|_{t=0+} = i_{C_k}(0+) \quad k = 1,...,n \tag{B3}$$

From (B2) and (B3) we have

$$b_{kj} = \frac{1}{C_k} i_{C_k}(0+), \quad k = 1,...,n; j = 1,...,m \tag{B4}$$

Therefore, to find the coefficient b_{kj} we have to compute the current in the short circuit *k*th capacitor caused by the *j*th voltage source $e_j = 1, \quad j = 1,...,m.$ If the directions of currents caused separately by each voltage source are consistent then the entries of the matrix *B* are positive. ■

Example B1. Consider the electrical circuit shown on Fig. B1 with given resistances R_1, R_2, R_3, capacitances C_1, C_2 and voltage sources e_1, e_2.

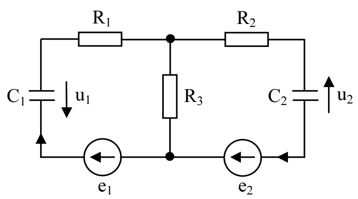

Fig. B1. Electrical circuit no3
Rys. B1. Obwód elektryczny nr 3

Using the Kirchhoff's laws we may write the equations

$$\begin{aligned} e_1 &= u_1 + (R_1 + R_3)C_1 \frac{du_1}{dt} - R_3 C_2 \frac{du_2}{dt} \\ e_2 &= u_2 + (R_2 + R_3)C_2 \frac{du_2}{dt} - R_3 C_1 \frac{du_1}{dt} \end{aligned} \tag{B5}$$

which can be written in the form

$$\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + B \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{B6}$$

where

$$\begin{aligned} B = -A &= \begin{bmatrix} (R_1 + R_3)C_1 & -R_3 C_2 \\ -R_3 C_1 & (R_2 + R_3)C_2 \end{bmatrix}^{-1} = \\ &= \frac{1}{\Delta}\begin{bmatrix} (R_2 + R_3)C_2 & R_3 C_2 \\ R_3 C_1 & (R_1 + R_3)C_1 \end{bmatrix}, \\ \Delta &= [R_1(R_2 + R_3) + R_2 R_3]C_1 C_2 \end{aligned} \tag{B7}$$

From (B7) it follows that the matrix *B* has positive entries and the matrix *A* has negative entries. Note that in the electrical circuit shown on Fig. B1 the directions of currents in the short circuit capacitors caused separately by the voltage sources $e_1 = 1$ and $e_2 = 1$ are consistent. Therefore, the entries of the matrix *B* given by (B7) are positive.

Theorem B1 can be extended for electrical circuits composed of resistances, capacitances, inductances and voltage (current) sources as follows.

Theorem B2. Entries of *B* matrices of electrical circuits composed of resistances, capacitances and voltage (current) sources are nonnegative if the directions of currents and the directions of voltages on gaps caused separately by each voltage source are consistent in all capacitors and on all coils.

Proof. The idea of the proof is similar. In this case instead of the equation (B1) we have

$$\frac{di_k}{dt} = \sum_{j=1}^{r} a_{kj} i_j + \sum_{i=r+1}^{n} a_{ki} u_i + \sum_{l=1}^{m} b_{kl} e_l, \quad k = 1,...,r \tag{B8a}$$

and

$$\frac{du_k}{dt} = \sum_{j=1}^{r} a_{kj} i_j + \sum_{i=r+1}^{n} a_{ki} u_i + \sum_{l=1}^{m} b_{kl} e_l, \quad k = r+1,...,n \tag{B8b}$$

Substituting $i_j = 0, \ j = 1,...,r, \ \ u_i = 0, \ i = r+1,...,n$ and $e_l = 1$ for $l = p$ and $e_l = 0$ for $l \neq p,$ from (B8a) we obtain

$$\frac{di_k}{dt} = b_{kp}, \quad k = 1,...,r; \ p = 1,...,m \tag{B9}$$

Note that at $t = 0+$ the capacitors are short circuit and the coils are gaps. The voltage across the *k*th gap caused by the voltage source $e_p = 1$ is equal to

$$L_k \left.\frac{di_k}{dt}\right|_{t=0+} = u_k(0+), \quad k = 1,...,r \tag{B10}$$

From (B9) and (B10) we have

$$b_{kp} = \frac{1}{L_k} u_k(0+), \quad k = 1,...,r; \ p = 1,...,m \tag{B11}$$

Therefore, to find the coefficient b_{kp} we have to compute the voltage on the *k*th gap caused by the voltage source $e_p = 1.$ If the directions of voltages on the gaps caused separately by each voltage source are consisted then the entries of the matrix *B* are nonnegative. The remaining part of the proof is similar to the proof of Theorem B1.■

The paper was supported by Ministry of Science and Higher Education in Poland under work S/WE/1/06.

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otrzymano / received: 01.02.2010
przyjęto do druku / accepted: 12.04.2010 *artykuł recenzowany*