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Wireless sensor convergecast based on random operations procedure

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Abstract

The widespread availability of inexpensive equipment for creating the wireless sensor networks (WSN) [1, 2], has enabled their manifold and innovative applications in numerous fields. PASTA system (Poisson Arrivals See Time Averages) [3, 4] for modeling “many-to-one” type WSN network has been used in the paper. In the proposed solution, single sensor-sender units remain inactive all the time except for the moments when they transmit information about the measured value randomly at Poisson epochs [5] to a central base station. Probability of collision occurrence has been determined for the fixed operating conditions of the network. The concept of a collective radio-operated measuring network with one-way transmission system (simplex) based on random access of particular sensor-sender units, constitute the most innovative aspect of the paper. The proposed solution enables the sensors with a simple hardware part to be used and allows a very short transmission protocol and the complete independence of each particular sensor-sender. Consequently, it results in saving the energy from the power source of sensors-senders.

Keywords: wireless sensor network, probability of collision, PASTA system, Poisson process.

Bezprzewodowa zbiorcza sieć pomiarowa z losową procedurą działania

Streszczenie

Powszechna dostępność niedrogiego sprzętu do budowy bezprzewodowych sieci czujników (Wireless Sensor Network WSN) umożliwia ich wielorakie i nowatorskie wykorzystania w wielu dziedzinach. W pracy wykorzystujemy system PASTA (Poisson Arrivals See Time Averages) [3, 4] do modelowania sieci zbiorczej WSN [3, 4]. W proponowanym rozwiązaniu pojedyncze czujniki-nadajniki pozostają nieaktywne przez cały czas, poza momentami, kiedy wysyłają informację o wielkości mierzonej w momentach losowych poissonowskich [5] do stacji bazowej. Wyznaczono prawdopodobieństwo występowania kolizji w transmisji dla ustalonych warunków pracy sieci. Innowacją tej pracy jest koncepcja zbiorczej radiowej sieci pomiarowej z transmisją jednokierunkową (Simplex) w oparciu o losowy sposób dostępu poszczególnych czujników-nadajników. Proponowane rozwiązanie jest niezwykle proste sprzętowo. Pozwala zastosować bardzo krótki protokół transmisyjny i daje pełną niezależność czujników-nadajników (brak synchronizacji i kanału powrotnego). W konsekwencji skutkuje to dużą oszczędnością energii ze źródeł zasilających czujniki-nadajniki, łatwością realizacji i rekonfiguracji sieci.

Słowa kluczowe: bezprzewodowe sieci czujników, prawdopodobieństwo kolizji, proces Poissona, system PASTA.

1. Introduction

The advent of Micro Electro Mechanical Systems (MEMS) technology has made it technologically feasible and economically viable to develop large scale networks composed of low power devices that integrate general-purpose computing with multi-

purpose sensing and wireless communications into “wireless sensor networks” (WSN). A wide range of applications is being conceptualized, designed and developed. The most promising applications are (see [1, 2]) in industrial control and monitoring, home automation and consumer electronics, security, and health monitoring. With wireless sensor networks it becomes possible to inexpensively and reliably monitor temperature, vibration, and other critical state variables of machine components. This in turn enables the optimization of maintenance schedules with minimal impact on throughput. Retrofitting of cables is expensive, particularly in harsh environments or in situations where moving parts could damage the cabling. In all of the above WSN application examples, multiple sensor nodes in a common neighborhood sense an event. These sensor nodes subsequently transmit sensed information to a remote processing unit or base station. Base stations are responsible for collecting and processing data from the physical layer (sensor nodes).

We apply Poisson Arrivals See Time Averages (PASTA) to modeling a wireless sensor network type many-to-one. PASTA (see [3, 4]) is a well known property applicable to many stochastic systems.

Section 2 presents assumptions of our wireless sensor network. A major properties of Poisson process are described in Section 3. In Section 4 present multi-sensor measurement model based on Poisson's process is described (see [5]). Finally, we summarize results and draw conclusions.

2. Network Model and Assumption

Realization of wireless sensor Network used for measurement results obtained at different points to be transmitted to one receiving point, is a reverse issue to that of broadcast networks. This issue is much more difficult as a certain fixed access procedure between particular sensor-sender units and receiver is required, with one single industrial channel assumed to be at disposal. This issue can be solved in a number of ways but each time the access procedure makes data transmission system more complicated. This complication appears in respect of both hardware and software. On hardware side, it is necessary to ensure synchronization and the reception of return information, i.e. to equip the senders with a backward channel receiver allowing the access procedure to function in a correct way. This makes the design of measurement sensors be much more complicated as, apart from radio transmitter, they must be additionally equipped with receivers and control units. Moreover, it is connected with an increased energy consumption of particular measurement points, which is very disadvantageous. Wireless sensors are used in order to avoid the application of signal leads and, especially, the power cables. Thus, the sensors are battery-operated which means a limited capacity and time of use.

In the case of some special applications of wireless measurement networks, a completely new approach to this issue can be proposed. Let us define the range of application for the proposed solution and the assumptions for such a network design. This work has been aimed at analyzing the operation correctness model for the proposed network.

Let us assume a measurement network to be performed operating in one radio channel (at one frequency) comprising an n -number of sensors which are capable of emitting the information about the value of a physical quantity measured at the points where the sensors are placed and set to the selected radio frequency with a preset power level. Changes in the measured parameters are assumed to take effect very slowly in relation to the time of measurement protocol transmission. One-way transmission is assumed (from the sensor to information exit, i.e. for example, to the computer equipped with data transmission radio receiver and the appropriate communication software destined for information processing, especially for data base archiving and processing according to the necessary algorithms).

There is quite significant demand for such type of convergecast (many-to one) network in numerous engineering areas. For example, it can be the monitoring of temperature or other physical parameters of the environment (humidity, sun exposure, rainfall level, atmospheric pressure, etc.), on a certain selected area within the network radio range.

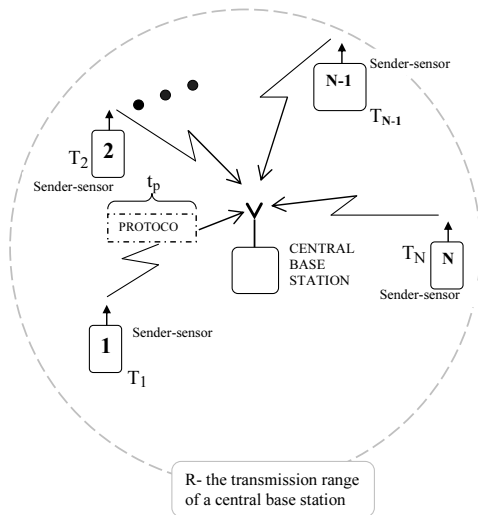


Fig. 1. In many-to one network topology each sender-sensor transmit its information in the protocol at t_p duration time, to a central base station randomly at Poisson times T_1, T_2, \dots .

Rys. 1. W sieci o topologii wiele-do-jednego każdy czujnik nadawczy transmituje swoją informację w formie protokołu w czasie t_p do centralnej stacji bazowej, w przypadkowych chwilach czasu T_1, T_2, \dots , o rozkładzie Poissona

Characteristics of the proposed network:

1. Quite a large number of sensor-sender units (N) (Fig.1).
2. Sensor-sender units remain completely independent and switching them on or off is of no influence on network operation.
3. All sensor-sender units, or a part of them, may be mobile provided that they are located within the radio range of the receiving station.
4. The slowly changing physical parameters are subjected to measurements what means there is no need to transmit the data very frequently (e.g. every several minutes or several dozens of minutes).

5. The transmission is of one-way type, i.e. from the sensor-sender unit up to the receiving point at T average time intervals. Information is transmitted in the protocol at t_p duration time.
6. Any selected sensor starts transmitting randomly at Poisson times. PASTA will be used to justify the sending of probes at Poisson epochs.
7. All sensor-sender units remain randomly independent and they will transmit the information at a randomly selected moment of time of t_p duration and of T average time of repetition.
8. If one or more sensors start transmitting while the protocol of t_p duration is being transmitted from another sensor, such a situation is called the collision. Collision makes it impossible for central base station to receive the information in a correct way.

3. Stochastic model

Here we give a formal definition for a Poisson process [5]. This process we used to modeling our wireless network. A Poisson process is the stochastic process in which events occur continuously and independently of one another. Often the arrival process of customers can be described by a Poisson process. In teletraffic theory the “customers” may be calls or packets. Mathematically the process is described by so called counter process N_t or $N(t)$. The counter tells the number of arrivals (events) that have occurred in the interval $(0, t]$ ($t \geq 0$).

Definition 3.1.

Let $\lambda > 0$. By definition, a Poisson process with rate λ is a random process $N(t)$, ($t \geq 0$) such that

- N1. $N(t)$ is a counting process,
- N2. $N(t)$ has independent increment (the number of occurrences counted in disjoint intervals are independent from each other),
- N3. $N(t) - N(s)$ has the Poisson ($\lambda(t-s)$) distribution for $t \geq s \geq 0$,

$$P\{N(t) - N(s) = k\} = e^{-\lambda(t-s)} \frac{[\lambda(t-s)]^k}{k!}, (k = 0, 1, 2, \dots)$$

The rate parameter λ is the expected number of events that occur per unit time. If T_k denotes the time of the k -th arrival (event) for $k \geq 1$, then $N(t)$ can be described by the sequence (T_k) . Or, if $U_1 = T_1$ and $U_k = T_k - T_{k-1}$ for $k \geq 2$, then $N(t)$ can be described by the sequence (U_k) . The random variables T_1, T_2, \dots are called the arrival-times and the random variables U_1, U_2, \dots are called the inter-arrival times. It is the following well-known characterization of a Poisson process (see [5]).

Proposition 3.2.

Let $N(t)$ be a counting process, and let $\lambda > 0$. The following are equivalent:

- (a) $N(t)$ is a Poisson process with rate λ .
- (b) The inter-arrival-times U_1, U_2, \dots are mutually independent, $\text{Exp}(\lambda)$ random variables (mean $1/\lambda$).

By Proposition 3.2 the inter-arrival times are exponentially distributed with parameter λ , and the arrival times $T_n = U_1 + U_2 + \dots + U_n$ ($n=1,2,\dots$) are gamma-distributed with a shape parameter n , and an inverse scale parameter λ . In the next proposition we show, that for each $t > 0$, $n=1,2,\dots$ and assuming $N_t = n$, the vectors (U_1, \dots, U_n) and (T_1, \dots, T_n) are uniformly distributed (on the sets Ω_t^* and Ω_t , respectively).

Proposition 3.3.

Let $\lambda > 0$, $t > 0$, $n=1,2,\dots$. Then assuming $N_t = n$

- (a) the arrival times of n arrival during $(0, t]$ are the same as n -independent uniformly distributed on the interval $[0, t]$ random variables, reordered to be non-decreasing. That is (T_1, \dots, T_n) is uniformly distributed on the set $\Omega_t = \{(t_1, \dots, t_n) : t_1, \dots, t_n \geq 0, 0 \leq t_1 \leq t_2 \leq \dots \leq t_n \leq t\}$ with the conditional density $f(t_1, \dots, t_n | N_t = n) = n! / t^n$, for $(t_1, \dots, t_n) \in \Omega_t$, and 0 else.
- (b) the random vector (U_1, \dots, U_n) is uniformly distributed on the set $\Omega_t^* = \{(u_1, \dots, u_n) : u_1, \dots, u_n \geq 0, u_1 + \dots + u_n \leq t\}$, with the conditional density $f(u_1, \dots, u_n | N_t = n) = n! / t^n$, for $(u_1, \dots, u_n) \in \Omega_t^*$, and 0 else.

Proof. By Proposition 3.2 the inter-arrival times U_1, U_2, \dots are mutually independent, $\text{Exp}(\lambda)$ random variables for some $\lambda > 0$. Thus for $n \geq 1$, the first n inter-arrival times (U_1, \dots, U_n) have the joint probability density function $f_{(U_1, \dots, U_n)}(u_1, \dots, u_n) = \lambda^n e^{-\lambda(u_1 + \dots + u_n)}$, for $u_1 > 0, \dots, u_n > 0$; 0 else. The vector (U_1, \dots, U_n) is the image of (T_1, \dots, T_n) under the mapping $(t_1, \dots, t_n) \rightarrow (u_1, \dots, u_n)$ defined by $u_1 = t_1$, $u_k = t_k + t_{k-1}$, for $k \geq 2$. The mapping is invertible, because $t_k = u_1 + \dots + u_k$ for $1 \leq k \leq n$. The Jacobian $\frac{\partial u}{\partial t}$ has unit determinant, and the Jacobian $\frac{\partial t}{\partial u}$ has unit determinant. Thus the probability distribution of the first n arrivals T_1, \dots, T_n is given by $f_{(T_1, \dots, T_n)}(t_1, \dots, t_n) = \lambda^n e^{-\lambda t_n}$ if $0 < t_1 < \dots < t_n$; 0 else. Fix $t > 0$ and an integer $n \geq 1$. The event $\{N_t = n\}$ is equivalent to the event $(T_1, \dots, T_{n+1}) \in B_{n,t}$, where

$$B_{n,t} = \{(t_1, \dots, t_{n+1}) : 0 < t_1 < \dots < t_n < t < t_{n+1}\}.$$

The conditional probability density function of (T_1, \dots, T_{n+1}) , given that $\{N_t = n\}$, is obtained by starting with the joint probability density function of (T_1, \dots, T_{n+1}) , namely $\lambda^{n+1} e^{-\lambda(t_{n+1})}$ on $\{(t_1, \dots, t_{n+1}) : 0 < t_1 < \dots < t_{n+1}\}$. Setting it equal to zero off of the set $B_{n,t}$, and scaling it up by the factor $1/P\{N_t = n\}$ on $B_{n,t}$: $f(t_1, \dots, t_{n+1} / N_t = n) = \lambda^{n+1} e^{-\lambda t_{n+1}} / P\{N_t = n\}$ for $0 < t_1 < \dots < t_n < t < t_{n+1}$; 0 else. The joint density of (T_1, \dots, T_n) given that $\{N_t = n\}$, is obtained for each (t_1, \dots, t_n) by integrating the density $f(t_1, \dots, t_n / N_t = n)$ with respect to t_{n+1} over R . If $0 < t_1 < \dots < t_n \leq t$, then this density is non-zero for $t_{n+1} \in (t, \infty)$. This integrating with respect to t_{n+1} over (t, ∞) yields

$f(t_1, \dots, t_n / N_t = n) = \lambda^n e^{-\lambda t} / P(N_t = n)$ for $0 < t_1 < \dots < t_n < t$; 0 else. Thus the conditional density is constant over the set $\Omega_t = \{(t_1, \dots, t_n) : 0 < t_1 < t_2 < \dots < t_n \leq t\}$ and this constant is $n! / t^n$. Thus a) is proved. Taking into account that the Jacobian $\frac{\partial u}{\partial t}$ has the unit determinant we obtain that the conditional density $f(u_1, \dots, u_n | N_t = n) = n! / t^n$ for $(u_1, \dots, u_n) \in \Omega_t^*$. The theorem is proved.

4. Node Model. Probability of collision

Before going into the details of node transmission modeling, let us state our main assumptions.

There are N identical sensors observing a dynamical system and reporting to a central location over the wireless sensor network with one radio channel. For simplicity, we assume our sensor network to be a single hop network with star topology. We also assume every node (sender-sensor, shortly sensor) always has packet ready for transmission. We assume that sensors send probe packets at Poissonian times. The average time between sending (the wake-up- times) of a sensor is T (the epoch period), and the duration of the on-time t_p (the awake interval). Assume that the wake-up- times corresponding to sensors are independent each of other. Let $N(t)$ be the Poisson process representing the time counter of sending sensors. Let T_1, T_2, \dots be the sending times (the wake-up- times) of sensors, U_1, U_2, \dots the inter-sending times. Then the average time between sending of sensors is T/N , the average number of sending sensors in the time interval of length T equals N . We say that a collision occurs in the time interval of length t_p , if at least two sensors start sending within this interval. We say that a collision occurs in the time interval of length s , if there exist at least two sensors which start sending within this interval with the difference between the beginning of their sending time not exceeding the value of t_p .

By Definition 3.1

$$P(N_t = k) = e^{-\lambda t} \frac{[\lambda t]^k}{k!} \quad (k = 0, 1, \dots),$$

where $\lambda = \frac{N}{T}$. Let p_k ($k = 0, 1, 2, \dots$) be the probability that the number of sensor transmissions that have occurred in the interval $[0, t_p]$ equals k . Then, for $k = 0, 1, 2, \dots$

$$p_k = e^{-\lambda t_p} \frac{[\lambda t_p]^k}{k!} = e^{-N \frac{t_p}{T}} \frac{[N \frac{t_p}{T}]^k}{k!}. \quad (4.1)$$

In other words, for $k = 2, 3, \dots$ p_k is the probability of exactly k collisions in the interval $[0, t_p]$, and consequently, by stationarity of Poisson process on every interval $[t, t + t_p]$ ($t > 0$).

Let A_s , A'_s be the events that denote the collisions occur and the lack of collisions, respectively, on the interval $[0, s]$ ($s > 0$). Let $P(A_{t_p})$ be the probability of collisions on the interval $[0, t_p]$.

Then

$$P(A_{t_p}) = 1 - p_0 - p_1 = 1 - e^{-N \frac{t_p}{T}} - N \frac{t_p}{T} e^{-N \frac{t_p}{T}}.$$

Thus we have

$$P(A_{t_p}) = 1 - [1 + \alpha]e^{-\alpha}, \quad (4.2)$$

where $\alpha = \frac{Nt_p}{T}$.

5. Examples

Let $t_p = 3,2 \cdot 10^{-5} s$,

- $T = 10s$, $N = 20$, $\alpha = 6,4 \cdot 10^{-5}$, $P(A_{t_p}) = 2,05 \cdot 10^{-9}$,
- $T = 10s$, $N = 100$, $\alpha = 3,2 \cdot 10^{-4}$, $P(A_{t_p}) = 5,12 \cdot 10^{-8}$,
- $T = 180s$, $N = 20$, $\alpha = 3,56 \cdot 10^{-6}$, $P(A_{t_p}) = 6,32 \cdot 10^{-12}$,
- $T = 180s$, $N = 100$, $\alpha = 1,78 \cdot 10^{-5}$, $P(A_{t_p}) = 1,58 \cdot 10^{-10}$.

Consider the interval $[0, s]$, where $s > t_p$. Assume $N(s) = n$, i.e. the number of sensor transmissions on the interval $[0, s]$ equals n ($n \geq 1$). By Proposition 3.3 the random vector (U_1, \dots, U_n) of inter-transmission-times is uniformly distributed on the set $\Omega_t^* = \{(u_1, \dots, u_n) : u_1 + \dots + u_n \leq s\}$ with the conditional density $f(u_1, \dots, u_n | N(s) = n) = n! / s^n$ for $(u_1, \dots, u_n) \in \Omega_t^*$, and 0 else. Then the conditional probability of the lack of collisions on the interval $[0, s]$, assuming $N(s) = n$ is

$$P(A_s' / N(s) = n) = P(U_1 > t_p, \dots, U_n > t_p) = \left(1 - \frac{nt_p}{s}\right)^n.$$

Thus we have

$$P(A_s' / N(s) = n) = \left(1 - \frac{nt_p}{s}\right)^n. \quad (4.3)$$

Note that the expected number of sensor transmissions on the interval $[0, T]$ equals N . By (4.3) with $s = T$ and $n = N$, we obtain that the conditional probability of the lack of collisions on the interval $[0, T]$, assuming $N(T) = N$, is

$$P(A_T' / N(T) = N) = \left(1 - \frac{Nt_p}{T}\right)^N, \quad (4.4)$$

and the conditional probability of collisions on the interval $[0, T]$, assuming $N(T) = N$, is

$$P(A_T / N(T) = N) = 1 - \left(1 - \frac{Nt_p}{T}\right)^N. \quad (4.5)$$

6. Summary

The probability of collision is quite low when memorylessness is assumed between subsequent sensor-sender unit's transmission moments sender-sensor property PASTA, when t_p protocol duration is very short in relation to T average repetition time and the number of sensor-sender units is not large. As the transmission

is effected randomly, the network realization is simplified significantly. No synchronization, no backward channel (no communication between the receiving station and sensors) simplify the communication procedure and reduce the complexity of sensor-sender design which results in t_p time decrease, in the lowering of collision probability and also in the decreased energy demand of particular sensors (they are usually battery-operated devices which imposes strong limitations).

7. Index

- A_s - the event that denotes the collisions occurring on the interval $[0, s]$ ($s > 0$),
- A_s' - the event that denotes the lack of collisions on the interval $[0, s]$ ($s > 0$),
- n - the number of sensor transmissions on an interval,
- N - the number of sensor-sender units,
- $N(t)$, N_t - the Poisson process representing the time counter of sending sensors.
- $P(A_{t_p})$ - the probability of collisions on the interval $[0, t_p]$,
- p_k - the probability that the number of sensor transmissions that have occurred in the interval $[0, t_p]$ equals k ($k = 0, 1, 2, \dots$),
- t , s - the moments of time,
- T - the average time between sending (the wake-up- times) of a sensor (the epoch period),
- t_p - the duration time of the protocol (the awake interval),
- T_1, T_2, \dots - the sending times (the wake-up- times) of sensors,
- U_1, U_2, \dots - the inter-sending times of sensors,
- λ - the rate of a Poisson process $N(t)$ (the expected number of events that occur per unit time),
- MEMS - Micro Electro Mechanical System,
- PASTA - Poisson Arrivals See Time Averages,
- WSN - Wireless Sensor Network,
- R - the transmission range of a central base station.

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