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Pointwise completeness and pointwise degeneracy of standard and positive Roesser models

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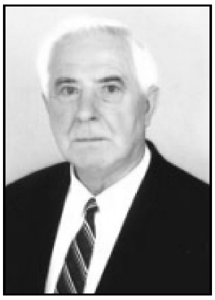

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Abstract

The pointwise completeness and pointwise degeneracy of standard and positive Roesser models are addressed. Necessary and sufficient conditions for pointwise completeness and pointwise degeneracy of standard and positive Roesser models are established. The considerations are illustrated by numerical examples.

Keywords: pointwise completeness, pointwise degeneracy, positive Roesser model.

Punktowa zupełność i punktowa degeneracja standardowych i dodatnich modeli Roessera

Streszczenie

W systemach dodatnich wartości sygnałów wejściowych, wyjściowych i zmiennych stanu przyjmuj ą jedynie wartości dodatnie. Przykładami takich systemów są m.in. procesy przemysłowe w reaktorach chemicznych, wymiennikach ciepła, kolumnach destylacyjnych, zbiornikach, a także modele zanieczyszczeń wody i atmosfery. Systemy liniowe dodatnie są definiowane na przestrzeniach stożkowych, dlatego też ich teoria jest bardziej skomplikowana i mniej rozwinięta. Najpopularniejsze modele liniowe dwuwymiarowe Roessera, Fornasiniego-Marchesiniego oraz Kurka są rozszerzone także na zastosowania w systemach dodatnich. W pracy została przedstawiona punktowa zupełność i punktowa degeneracja standardowych i dodatnich modeli Roessera. Rozważania oparto na niezbędnych formalizmach matematycznych. Podane zostały warunki konieczne i wystarczające punktowej zupełności i punktowej degeneracji takich standardowych modeli Roessera. Rozważania zilustrowano przykładami numerycznymi. W pracy znajduje się wiele odniesień do innych prac źródłowych rozszerzających obszar zagadnienia.

Słowa kluczowe: punktowa zupełność, punktowa degeneracja, dodatni model Roessera.

1. Introduction

In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of state of the art in positive systems theory is given in the monographs [4, 8].

The most popular models of two-dimensional (2D) linear systems are the discrete models introduced by Roesser [14], Fornasini-Marchesini [5, 6] and Kurek [11]. The models have been extended for positive systems. An overview of positive 2D system theory has been given in monographs [4, 8].

A dynamical system described by homogenous equation is called pointwise complete if every given final state of the system can be reached by suitable choice of its initial state. A system which is not pointwise complete, is called pointwise degenerated.

The pointwise completeness and pointwise degeneracy of linear continuous-time system with delays have been investigated in [3, 12, 13] and of discrete-time and continuous-time systems of fractional order in [1, 10] and with delays in [2]. The pointwise completeness of linear discrete-time cone-systems with delays has been analyzed in [15]. The pointwise completeness and pointwise degeneracy of standard and positive linear systems with state-feedbacks have been investigated in [9].

In this paper the pointwise completeness and pointwise degeneracy of standard and positive 2D Roesser models will be addressed.

Structure of the paper is the following. In section 2 preliminaries are given and in section 3 necessary and sufficient conditions are established for the pointwise completeness of standard Roesser model. The pointwise degeneracy of standard Roesser model is investigated in section 4 and the pointwise degeneracy in section 5. Concluding remarks are given in section 6.

2. Preliminaries

Let $R^{n\times m}$ be the set of $n \times m$ real matrices. The set $n \times m$ matrices with nonnegative entries will be denoted by $R_+^{n\times m}$ and $R_+^n = R_+^{n\times 1}$. The set of nonnegative integers will be denoted by Z_+ and the $n \times n$ identity matrix will be denoted by I_n.

Consider the autonomous 2D Roesser model [7, 8, 14]

$$\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix}, \ i, j \in Z_+ \tag{1}$$

where $x_{ij}^h \in R^{n_1}$ and $x_{ij}^v \in R^{n_2}$ are the horizontal and vertical state vectors at the point (i, j) and $A_{kl} \in R^{n_k \times n_l}$; $k, l = 1,2$.

Boundary conditions for (1) have the form

$$x_{0j}^h \in R_+^{n_1}, \ j \in Z_+ \text{ and } x_{i0}^v \in R_+^{n_2}, \ i \in Z_+ . \tag{2}$$

The model (1) is called (internally) positive Roesser model if $x_{ij}^h \in R_+^{n_1}$, $x_{ij}^v \in R_+^{n_2}$, $i, j \in Z_+$ for any boundary conditions (2).

Theorem 1. [8] The 2D Roesser model is positive if and only if

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \in R_+^{n\times n} . \tag{3}$$

Theorem 2. [7, 14] The solution of the autonomous Roesser model (1) with boundary conditions (2) is given by

$$\begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} = \sum_{k=0}^{i} T_{i-k,j} \begin{bmatrix} 0 \\ x_{k,0}^v \end{bmatrix} + \sum_{l=0}^{j} T_{i,j-l} \begin{bmatrix} x_{0,l}^h \\ 0 \end{bmatrix} \tag{4}$$

where the transition matrix T_{ij} is defined by

$$T_{ij} = \begin{cases} I_n & \text{for} \quad i=j=0 \\ T_{10}T_{i-1,j} + T_{01}T_{i,j-1} = T_{i-1,j}T_{10} + T_{i,j-1}T_{01} & \text{for} \, i,j \geq 0 \, (i+j>0) \\ 0 & \text{for} \quad i<0 \, \text{or} \, j<0 \end{cases} \quad (5)$$

$$T_{10} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}, \quad T_{01} = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}$$

3. Pointwise completeness of standard Roesser model

Definition 1. The standard Roesser model (1) is called pointwise complete at the point (p,q) if for every final state $x_f \in R$ there exist boundary conditions

$$x_0 = \begin{bmatrix} x_{00}^h \\ x_{00}^v \end{bmatrix}, \; x_{0j}^h = 0, \; j = 1,2,... \text{ and } x_{i0}^v = 0, \; i = 1,2,... \quad (6)$$

such that $x_{pq} = x_f$.

Theorem 3. The standard Roesser model (1) is pointwise complete if and only if

$$\text{rank} T_{pq} = n. \quad (7)$$

Proof. From Definition 1, (4) and (6) for $i = p, j = q$ we have

$$x_f = \begin{bmatrix} x_{pq}^h \\ x_{pq}^v \end{bmatrix} = T_{pq} x_0. \quad (8)$$

By Kronecker-Capelly theorem the equation (8) has a solution x_0 for any given x_f if and only if the condition (7) is satisfied. □

Theorem 4. The standard Roesser model (1) is not pointwise complete if

$$\text{rank} \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & A_{21} & A_{22} \end{bmatrix} < n \quad (9a)$$

or

$$\text{rank} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = 0. \quad (9b)$$

Proof. From (5) for $i = p, j = q$ we have

$$T_{pq} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} T_{p-1,q} \\ T_{p,q-1} \end{bmatrix} = \begin{bmatrix} T_{p-1,q} & T_{p,q-1} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \\ 0 & 0 \\ A_{21} & A_{22} \end{bmatrix} \quad (10)$$

and the condition (7) is not satisfied if (9a) holds. From (10) it follows also that if (9b) holds then rankT_{pq} < n, since rank$AB \leq$ min(rank$A$, rank$B$). □

From Theorem 4 we have the following corollary.

Corollary 1. The standard Roesser model (1) is not pointwise complete if the matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (11)$$

has at least one zero row.

Example 1. Check the pointwise completeness at the point (2,1) of the standard Roesser model (1) witch the matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}. \quad (12)$$

Using (5) we compute

$$\begin{aligned} T_{20} &= T_{10}^2 = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \\ T_{11} &= T_{10}T_{01} + T_{01}T_{10} = \begin{bmatrix} -6 & -4 \\ 3 & -6 \end{bmatrix} \\ T_{21} &= T_{10}T_{11} + T_{01}T_{20} = \begin{bmatrix} -12 & 8 \\ 3 & -6 \end{bmatrix}. \end{aligned} \quad (13)$$

In this case the condition (7) is met since detT_{21} = 48, Therefore, by Theorem 3 the standard Roesser model with (12) is pointwise complete at the point (2,1).

4. Pointwise degeneracy of standard Roesser model

Definition 2. The standard Roesser model (1) is called pointwise degenerated at the point (p,q) in the direction v if there exist a non-zero vector $v \in R^n$ such that for all boundary conditions (6) the solution (4) for $i = p, j = q$ satisfies the condition $v^T x_{pq} = 0$, where T denotes the transpose.

Theorem 5. The standard Roesser model (1) is pointwise degenerated at the point (p,q) in the direction v if and only if

$$\text{rank} T_{pq} < n. \quad (14)$$

Proof. From (8) we have

$$v^T x_{pq} = v^T T_{pq} x_0 = 0. \quad (15)$$

Note that (15) holds for all boundary conditions (6) if and only if

$$v^T T_{pq} = 0 \quad (16)$$

and this is satisfied for $v \neq 0$ if and only if the condition (14) is met. □

Theorem 6. The standard Roesser model (1) is pointwise degenerated if

$$\text{rank} \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & A_{21} & A_{22} \end{bmatrix} < n. \quad (17)$$

Proof. From (10) we have

$$v^T T_{pq} = v^T \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} T_{p-1,q} \\ T_{p,q-1} \end{bmatrix} \quad (18a)$$

and if condition (17) is met then (16) holds for $v \neq 0$. The vector v can be found from the equation

$$v^T \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}. \; □ \quad (18b)$$

From Theorem 6 we have the following corollary.

Corollary 2. The standard Roesser model (1) is pointwise degenerated if the matrix (11) has at least one zero row.

5. Pointwise completeness and pointwise degeneracy of positive Roesser model

Definition 3. The positive Roesser model (1) is called pointwise complete at the point (*p*,*q*) if for every final state $x_f \in R_+^n$ there exist boundary conditions

$$x_0 = \begin{bmatrix} x_{00}^h \\ x_{00}^v \end{bmatrix} \in R_+^n,\ x_{0j}^h = 0,\ j = 1,2,... \text{ and } x_{i0}^v = 0,\ i = 1,2,... \quad (19)$$

such that $x_{pq} = x_f$.

A matrix $A \in R_+^{n \times n}$ is called monomial if and only if every its row and every its column has only one positive entry and the remaining entries are zero.

Theorem 7. The positive Roesser model (1) is pointwise complete at the point (*p*,*q*) if and only if the matrix T_{pq} is monomial.

Proof. From (8) it follows that for $x_f \in R_+^n$ there exist boundary conditions (19) if and only if the matrix T_{pq} is monomial since $T_{pq}^{-1} \in R_+^{n \times n}$ if and only if T_{pq} is monomial. □

Example 2. Find values of the parameter $a \geq 0$ for which the positive Roesser model (1) with matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix} \quad (20)$$

is pointwise complete at the point (1, 2).

Using (5) we compute

$$\begin{aligned} T_{02} &= T_{02}{}^2 = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 \\ 2 & 4 \end{bmatrix} \\ T_{11} &= T_{10}T_{01} + T_{01}T_{10} = \begin{bmatrix} a & 2a \\ 1 & a \end{bmatrix} \\ T_{12} &= T_{10}T_{02} + T_{01}T_{11} = \begin{bmatrix} 2a & 4a \\ 2+a & 4a \end{bmatrix} \end{aligned} . \quad (21)$$

From (21) it follows that for all values of the parameter $a \geq 0$ the matrix T_{12} is not monomial. Therefore, the positive Roesser model (1) with (20) is not pointwise complete at the point (1,2).

Definition 4. The positive Roesser model (1) is called pointwise degenerated at the point (*p*,*q*) in the direction *v* if there exists a nonzero vector $v \in R^n$ such that for all boundary conditions (19) the solution (4) for $i = p, j = q$ satisfies the condition $v^T x_{pq} = 0$.

Theorem 8. The positive Roesser model (1) is pointwise degenerated at the point (*p*,*q*) in the direction *v* if the condition (14) is met. The vector *v* can be found from the equation (16).
The proof is similar to the proof of Theorem 5.

Theorem 9. The positive Roesser model (1) is pointwise degenerated at the point (*p*,*q*) in the direction *v* if the condition (17) is met.

The proof is similar to the proof of Theorem 6. From Theorem 9 we have the following corollary.

Corollary 3. The positive Roesser model (1) is pointwise degenerated if the matrix (11) has at least one zero row.

Those considerations can be extended to the standard and positive Roesser model (1) with nonzero boundary conditions

$$x_{0j}^h,\ j = 1,2,...,p\ ;\ x_{i0}^v,\ i = 1,2,...,q \quad (22a)$$

and

$$x_{0j}^h = 0,\ j = p+1, p+2,...;\ x_{i0}^v = 0,\ i = q+1, q+2,.... \quad (22b)$$

6. Concluding remarks

Necessary and sufficient conditions for the pointwise completeness and pointwise degeneracy of standard and positive Roesser models have been established. The considerations have been illustrated by numerical examples. Those considerations can be extended to the 2D general models with and without delays. Extensions of these considerations for standard, positive and fractional 2D continuous-time linear systems are open problems.

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