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Stabilisation of inertial processes with time delay using a fractional order PI controller

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Abstract

The paper presents the stability problem of control systems composed of a fractional-order PI controller and an inertial plant of a fractional order with time delay. A simple and efficient computational method for determining stability regions in the controller and plant parameters space for specified gain and phase margins requirements is given. If these regions are known tuning process of the fractional-order PI controller can be made. The method proposed is based on the classical D-partition method.

Keywords: PID controllers, fractional system, stability, delay, D-partition method.

Stabilizacja układów inercyjnych z opóźnieniem za pomocą regulatora PI ułamkowego rzędu

Streszczenie

W pracy rozpatrzono problem stabilności układów regulacji automatycznej złożonych z regulatora PI ułamkowego rzędu oraz obiektu inercyjnego ułamkowego rzędu z opóźnieniem. Rozpatrywany układ regulacji automatycznej jest stabilny, gdy jego quasi-wielomian charakterystyczny ułamkowego stopnia (3) jest stabilny. tzn. wszystkie jego zera mają ujemne części rzeczywiste. Wykorzystując klasyczną metodę podziału D podano prostą analityczno-komputerową metodę wyznaczania obszarów stabilności na płaszczyźnie parametrów modelu obiektu regulacji (1) i regulatora (2). Wyznaczono analityczne zależności określające granice obszarów stabilności w przestrzeni parametrów (X, Y), gdzie $X = Kk_p$, $Y = Kk_ih^{\lambda}$. Obszar stabilności leży pomiędzy granicą zer rzeczywistych $Y = 0$ i granicą zer zespolonych o opisie parametrycznym (10), (11). Otrzymane opisy granic stabilności umożliwiają także wyznaczenie obszarów stabilności dla zadanego zapasu modułu A i fazy ϕ. Przy wyznaczaniu obszarów stabilności dla określonego zapasu modułu A należy przyjąć $\phi = 0$, natomiast dla określonego zapasu fazy ϕ należy przyjąć $A = 1$. Na podstawie znajomości tych obszarów można w prosty sposób określić nastawy regulatora, dla których rozpatrywany układ regulacji charakteryzuje się określonymi zapasami stabilności. Przedstawiony przykład potwierdza rezultat otrzymany na podstawie metody podziału D, że punkt z wyznaczonego obszaru stabilności (rys. 3) zapewnia określone wartości zapasu fazy.

Słowa kluczowe: regulator PID, układ ułamkowego rzędu, stabilność, opóźnienie, metoda podziału D.

1. Introduction

Since they have a simple structure, Proportional - Integral - Derivative (PID) controllers are widely applied. PID-control has been the subject of many publications (see, e.g. [1-5]). Many methods of tuning PID controllers for satisfactory behaviour have been proposed in the literature [3]. These methods are based on the mathematical description of the process. The first order-plant with time delay is the most frequently used model for tuning PID controller [1, 3].

In recent years, considerable attention has been paid to control systems whose processes and/or controllers are of a fractional order (see, e.g. [6-8]). The fractional PID controllers, namely PI$^{\lambda}$D$^{\mu}$ controllers, including an integrator of a λ order and a differentiator of a μ order were proposed in [8]. Several design methods of tuning the PI$^{\lambda}$D$^{\mu}$ controllers for systems without time delay have been presented [9-11]. It has been shown that the PI$^{\lambda}$D$^{\mu}$ controller which has five degrees of freedom enhances the system control performance when used for control systems with integer order plants and fractional order plants. A computation method of stabilizing fractional-order PI$^{\lambda}$D$^{\mu}$ controllers for fractional-order time delay systems is presented in [12].

In the paper [13] a simple method for determining the stability region in the parameter space of an inertial plant of a fractional order with time delay and a fractional-order PI controller is given.

This work extends the means of obtaining stability regions for specified gain and phase margin requirements. Using this region, a very fast and simple way of calculating the stabilising values of PI$^{\lambda}$ controllers is obtained.

2. Problem formulation

Consider the feedback control system shown in Fig. 1 in which the process to be controlled is described by an inertial plant with time delay

$$G(s) = \frac{Ke^{-sh}}{1+s^{\alpha}T}, \tag{1}$$

where K, T, h are positive real numbers and the order α can be integer ($\alpha = 1$) or fractional with $0 < \alpha < 1$.

Let the controller $C(s)$ be the fractional PI controller described by the transfer function [8]

$$C(s) = k_p + \frac{k_i}{s^{\lambda}}, \tag{2}$$

where k_p and k_i denote the proportional and integral gains of the controller and λ is the fractional order of the integrator (the order may assume positive real noninteger values). Clearly, on selecting $\lambda = 1$, a classical PI controller can be obtained.

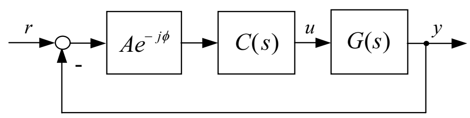

Fig. 1. Feedback control system structure
Rys. 1. Rozpatrywana struktura układu regulacji automatycznej

The main path of control includes the gain-phase margin tester $A\exp(-j\phi)$, where A and ϕ are gain margin and phase margin, respectively (Fig.1). This tester does not exist in the real control system, it is only used for tuning the controller. A system may be designed to have specified gain and phase margins. In typical control systems the phase margin is from 30° to 60° whereas the gain margin is from 5dB to 10dB. Gain and phase margins are measures of relative stability for a feedback system, though frequently only the phase margin is used rather than both margins. The phase margin is closely related to transient response, i.e. overshoot.

The characteristic function of the closed-loop system with plant (1), controller (2) and the gain-phase margin tester is given by

$$w(s) = KAe^{-j\phi}(k_p s^\lambda + k_i)e^{-sh} + (1+Ts^\alpha)s^\lambda. \quad (3)$$

The closed-loop system in Fig. 1 is said to be bounded-input bounded-output stable if and only if all the zeros of the characteristic function (3) have negative real parts. It is noted that (3) is a quasi-polynomial which has an infinite number of zeros. This makes the problem of analysing the stability of the closed-loop system difficult. There is no general algebraic methods available in the literature for the stability test of quasi-polynomials. The next problem of synthesis of the closed-loop system is how to choose such a fractional order λ of the integrator that the closed-loop system will be stable and characterized by specified gain and phase margins.

The main aim of the paper is to give the method for determining the stability region in the parameters space for specified gain and phase margins requirements.

3. Main Result

On multiplying quasi-polynomial (3) by $\exp(sh)$ we obtain a new quasi-polynomial in the form

$$w(s) = K(k_p s^\lambda + k_i)Ae^{-j\phi} + s^\lambda(1+Ts^\alpha)e^{sh}, \quad (4)$$

which has exactly the same zeros as quasi-polynomial (3). Substituting $z = sh$ in quasi-polynomial (4) after transformations we obtain the quasi-polynomial

$$w(z) = (Xz^\lambda + Y)Ae^{-j\phi} + z^\lambda(1+pz^\alpha)e^z, \quad (5)$$

where $X = Kk_p$, $Y = Kk_i h^\lambda$, $p = T/h^\alpha$.

Using the D-partition method [2] the asymptotic stability region in the parameter plane (X, Y) may be determined and the parameters can be specified. For $A = 1$ and $\phi = 0$, the stability boundaries are determined.The plane (X, Y) is decomposed by the boundaries of D-partition into finite number regions $D(k)$. Any point in $D(k)$ corresponds to such values of X and Y that quasi-polynomial (5) has exactly k zeros with positive real parts. The region $D(0)$, if exists, is the stability region of quasi-polynomial (5). The D-partition boundaries are curves on which each point corresponds to quasi-polynomial (5) having zeros on the imaginary axis. It may be the real zero boundary or the complex zero boundary. It is easy to see that quasi-polynomial (5) has zero $z = 0$ if $Y = 0$ (the real zero boundary). The complex zero boundary corresponds to the pure imaginary zeros of (5). We obtain this boundary by solving the equation

$$w(j\omega) = [X(j\omega)^\lambda + Y]Ae^{-j\phi} + (j\omega)^\lambda[1 + p(j\omega)^\alpha]e^{j\omega} = 0, \quad (6)$$

which we get by substituting $z = j\omega$ in quasi-polynomial (5) and equating to 0. The term of j^λ which is required for equation (6) can be expressed by

$$j^\lambda = \cos\left(\frac{\pi}{2}\lambda\right) + j\sin\left(\frac{\pi}{2}\lambda\right). \quad (7)$$

Using (7) the complex equation (6) can be rewritten as a set of real equations in the form

$$\mathrm{Re}[w(j\omega)] = 0, \quad (8)$$

$$\mathrm{Im}[w(j\omega)] = 0, \quad (9)$$

where $\mathrm{Re}[w(j\omega)]$ and $\mathrm{Im}[w(j\omega)]$ denote the real and the imaginary parts of (6), respectively.

Finally, by solving the equations (8) and (9) we get

$$X = \frac{-1}{A\sin\left(\frac{\pi}{2}\lambda\right)}\left[p\omega^\alpha \sin\left(\frac{\pi}{2}(\lambda+\alpha)+\omega+\phi\right) + \sin\left(\frac{\pi}{2}\lambda+\omega+\phi\right)\right], \quad (10)$$

$$Y = \frac{1}{A\sin\left(\frac{\pi}{2}\lambda\right)}\left[p\omega^{\lambda+\alpha} \sin\left(\frac{\pi}{2}\alpha+\omega+\phi\right) + \omega^\lambda \sin(\omega+\phi)\right], \quad (11)$$

Equations (10) and (11) determine the complex zero boundary in plane (X, Y). The real zero boundary and the complex zero boundary for $\omega > 0$ decompose plane (X, Y) into regions $D(k)$. The stability region $D(0)$ is chosen by testing an arbitrary point from each region and checking the stability of the quasi-polynomial (5) using the methods proposed in [14].

In the paper [13] only the stability region in the parameter space of quasi-polynomial (5) was determined. The influence of the value of the plant parameters on the stability regions was analysed. The use of the fractional order α of a plant causes an increase in the stability regions. The increasing value of p results in larger stability regions.

To determine the complex zero boundary for a given value of gain margin A of the control system, we should set $\phi = 0$ in (10) and (11). On the other hand by setting $A = 1$ in (10) and (11), we can obtain the boundary for a given phase margin ϕ.

The stability regions of quasi-polynomial (5) for $p = 4$, $\alpha = 1$, $A = 1$, $\phi = 30°$ and different values of λ are shown in Fig. 2. The figure shows that for $\lambda < 1$ the stability regions are larger than for $\lambda = 1$. An increase in the value of λ to over one initially results in an increase in the stability region after which it begins to decrease. The value of λ at which the stability region disappears is $\lambda = 2$.

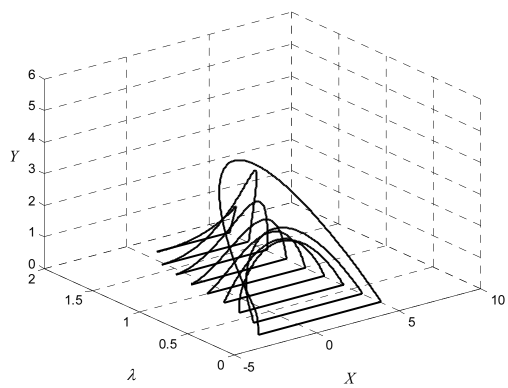

Fig. 2. Stability regions for quasi-polynomial (5) for $p = 4$, $\alpha = 1$, $A = 1$, $\phi = 30°$ and different values of λ

Rys. 2. Obszary stabilności quasi-wielomianu (5) wyznaczone dla kilku wartości λ przy $p = 4$, $\alpha = 1$, $A = 1$, $\phi = 30°$

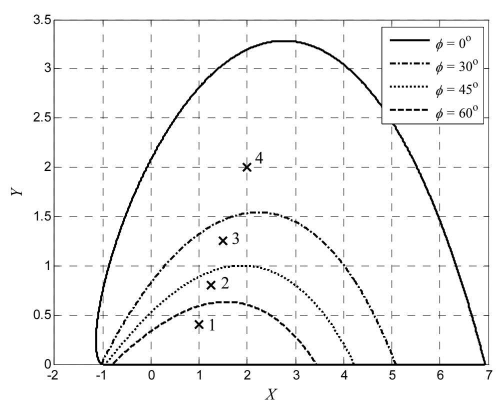

Fig. 3. Stability regions for quasi-polynomial (5) for $\lambda = 0.8$, $p = 4$, $\alpha = 1$, $A = 1$ and different values of ϕ

Rys. 3. Obszary stabilności quasi-wielomianu (5) wyznaczone dla kilku wartości ϕ przy $\lambda = 0.8$, $p = 4$, $\alpha = 1$, $A = 1$

Fig. 3 shows the stability regions of quasi-polynomial (5) for $\lambda = 0.8$, $p = 4$, $\alpha = 1$, $A = 1$ and different values of ϕ. For example, any point from the region limited by the line $Y = 0$ and the curve corresponds to $\phi = 60°$ provides the phase margin of this system greater than 60°.

4. Example

Consider the feedback control system shown in Fig. 1 in which the process to be controlled is described by transfer function (1) where $K = 1$, $T = 2$, $\alpha = 1$, $h = 0.5$.

In the example we have $p = T / h^{\alpha} = 4$. Fig. 3 shows the stability regions of quasi-polynomial (5) for $\lambda = 0.8$, $p = 4$ and different values of ϕ. Assuming the value of ϕ, e.g. $\phi = 60°$, the stability region is limited by the line $Y = 0$ and the curve corresponds to $\phi = 60°$. Choosing an arbitrary point from this region, e.g. $X = 1$, $Y = 0.4$ (point 1 in Fig. 3), we get $Kk_p = 1$, $Kk_ih^{\lambda} = 0.4$. By computation based on the above expressions the following controller parameters $k_p = 1$, $k_i = 0.7$ are obtained. Gain and phase margins for this case are illustrated in Fig. 4. The phase margin for this system is greater than 60° and equals 73.76°.

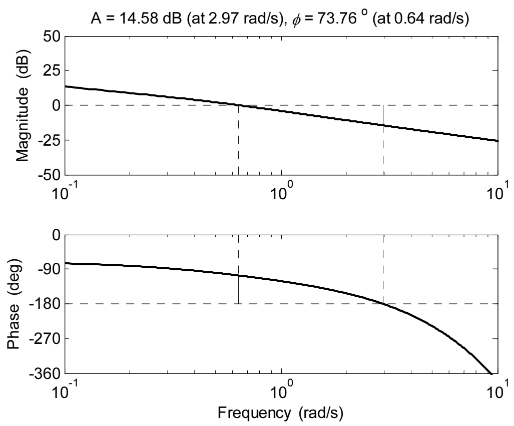

Fig. 4. Bode plot with gain and phase margins
Rys. 4. Charakterystyka Bodego z zaznaczonymi zapasami modułu i fazy

The controller gains and stability margins of the control system for all points marked in Fig. 3 are shown in Tab. 1. This confirms the results received on the basis of the D-partition method.

The step responses of the control system are presented in Fig. 5. It can be seen that the decreasing value of ϕ results in larger oscillations.

Tab. 1. Gain and phase margins
Tab. 1. Zapasy modułu i fazy

Point	Controller gains of $PI^{0.8}$	Gain margin [dB]	Phase margin [°]
1	$k_p = 1$, $k_i = 0.7$	14.58	73.76
2	$k_p = 1.25$, $k_i = 1.39$	11.01	50.98
3	$k_p = 1.5$, $k_i = 2.17$	8.00	36.08
4	$k_p = 2$, $k_i = 3.48$	4.22	19.80

5. Conclusion

In this paper, the stability problem of control systems composed of a fractional-order PI controller and an inertial plant of fractional order with time delay is examined. On the basis of the D-partition method, analytical forms expressing the D-partition boundaries of stability regions in the parameter space for the specified gain and phase margin requirements were determined. Knowledge of stability regions permits the tuning of the fractional PI type controller.

The method presented can be applied to the fractional-order time delay systems with parametric uncertainties.

The calculations and simulations were made using the Matlab/Simulink programme.

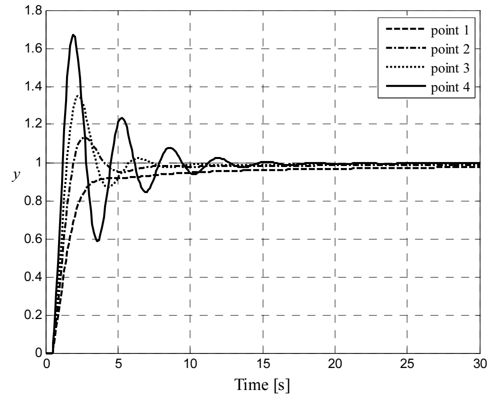

Fig. 5. Step responses of control system
Rys. 5. Odpowiedzi skokowe układu regulacji

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6. Literatura

[1] Aström K. J. and Hägglund T.: PID Controllers: Theory, Design, and Tuning, 2nd ed. Research Triangle Park, NC: Instrument Society of America, 1995.

[2] Górecki H., Fuksa S., Grabowski P. and Korytowski A.: Analysis and Synthesis of Time Delay Systems, PWN-J. Wiley, Warsaw Chichester, 1989.

[3] O'Dwyer A.: PI and PID controller tuning rules, Imperial College Press/Word Scientific, London, 2003.

[4] Silva G. J., Datta A. and Bhattacharyya S. P.: PID Controllers for Time-Delay Systems, Birkhauser, Boston, 2005.

[5] Ruszewski A.: Parametric synthesis of controllers for particular plants with uncertain parameters, PhD Dissertation, Faculty of Electrical Engineering, Białystok Technical University (in Polish), 2008.

[6] Matignon D.: Stability properties for generalized fractional differential systems, Proc. ESAIM, 145-158 (1998).

[7] Ostalczyk P.: Epitome of the fractional calculus. Theory and its applications in automatics, Publishing Department of Technical University of Łódź, Łódź, 2008, (in Polish).

[8] Podlubny I.: Fractional-order systems and $PI^{\lambda}D^{\mu}$ -controllers, IEEE Trans. on Automatic Control, 44, 208-214 (1999).

[9] Zhao C., Xue D. and Chen Y.Q.: A Fractional Order PID Tuning Algorithm for A Class of Fractional Order Plants, Proc. of the IEEE International Conference on Mechatronics & Automation, Niagara Falls, Canada, 216-221 (2005).

[10] Chen Y.Q., Dou H., Vinagre B. M. and Monje C.A.: A Robust Tuning Method for Fractional Order PI Controllers, The Second IFAC Symposium on Fractional Derivatives and Applications, Porto, Portugal (2006).

[11] Monje C.A., Vinagre B.M., Chen Y.Q., Feliu V., Lanusse P. and Sabatier J.: Proposals for Fractional $PI^{\lambda}D^{\mu}$ Tuning, The First IFAC Symposium on Fractional Differentiation and its Applications, Bordeaux, France (2004).

[12] Hamamci S. E.: An Algorithm for Stabilization of Fractional-Order Time Delay Systems Using Fractional-Order PID Controllers, IEEE Trans. on Automatic Control, 52, 1964-1969 (2007).

[13] Ruszewski A.: Stability regions of closed loop system with time delay inertial plant of fractional order and fractional order PI controller, Bull. Pol. Ac.: Sci. Tech. 56 (4), 329–332 (2008).

[14] Busłowicz M.: Frequency domain method for stability analysis of linear continuous-time fractional systems, In: K. Malinowski and L. Rutkowski (Eds.): Recent Advances in Control and Automation, Academic Publishing House EXIT, Warsaw 2008, 83-92.

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