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AGMM application in fault diagnosis and manoeuvre detection

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Abstract

Mixed multiple-additive Gauss-Markov models (AGMM) of parameters or structure changes which describe a broad variety of system failures or radar target manoeuvres are presented. Recursive algorithms for solving joint detection-identification problems in the presence of noise are obtained using the generalized likelihood ratio (GLR) approach. The proposed algorithms have relatively moderate computational requirements in a comparison with the multiple model approach. The results of simulation of the proposed algorithms are presented. The method can be used for failure detection-identification or manoeuvre detection in radar systems.

Keywords: failure detection, manoeuvre detection, nonlinear filtering, Markov processes.

Zastosowanie AGMM do detekcji uszkodzeń i wykrycia manewrów

Streszczenie

W artykule przedstawiono addytywne modele Gausa-Markowa (AGMM – *ang. Additive Gauss-Markov Models*). Wykorzystanie AGMM pozwoliło na stworzenie metody, która dzięki wprowadzeniu dodatkowego układu dynamicznego modelującego nagłe zmiany umożliwia objęcie opisem szerokiego zakresu niestacjonarności i pozwala na oddanie właściwego ich charakteru (można je przedstawić w formie procesu losowego, procesu zdeterminowanego, ale o losowym momencie zaistnienia lub procesu typu mieszanego). Zaletą AGMM jest możliwość opisu nawet złożonych zmian dynamiki systemu za pomocą nieskomplikowanego aparatu matematycznego. Modele te umożliwiają stworzenie rekursywnych algorytmów wykrywania uszkodzeń i śledzenia manewrujących obiektów. Struktura systemu ma formę adaptacyjnego filtra dopasowanego do bieżącej dynamiki obiektu i stanu systemu pomiarowego, co zapewnia minimalny błąd pomiaru. Do wykrycia zmian stosowany jest bank filtrów dodatkowych dopasowanych do różnych rodzajów i momentów zaistnienia zmian oraz procedura decyzyjna oparta o metodę uogólnionego stosunku wiarygodności. Zastosowano metodę analitycznego wyznaczania progów decyzyjnych z wykorzystaniem aproksymacji rozkładu prawdopodobieństwa logarytmu uogólnionego stosunku wiarygodności. Zmienna wartość progów decyzyjnych pozwoliła na utrzymanie prawdopodobieństwa fałszywego alarmu na stałym poziomie. Proponowana metoda charakteryzuje się niskim obciążeniem obliczeniowym pozwalającym na stosowanie w systemach czasu rzeczywistego.

Słowa kluczowe: detekcja uszkodzeń, wykrycie manewrów, filtracja nieliniowa, procesy Markowa.

1. Introduction

Methods for detection and estimation of the structure or parameters of abrupt changes in dynamic systems play an important role for solving a number of problems encountered in practice. They have an important significance in different fields of telecommunications and control applications, such as radar tracking of manoeuvring targets, fault diagnosis and identification (FDI), speech analysis, signal processing in geophysics and biomedical systems [1, 2, 6, 7]. Most of these applications belong to a wide class of systems with abrupt random jumps of parameters or structure.

Among wide variety of failure detection and isolation methods an important role play classical approaches based on testing of the innovation process properties. For instance in nominal conditions, innovation process of the filter matched with a process model is zero mean white noise. When the system changes have occurred, the innovation process changes its statistical properties and carries information about the system changes. Fig. 1 illustrates changes of the probability density function (pdf) of the innovation process $z(k/k\text{-}1)$ in a case of an additive bias in measurement channel arising at k_i.

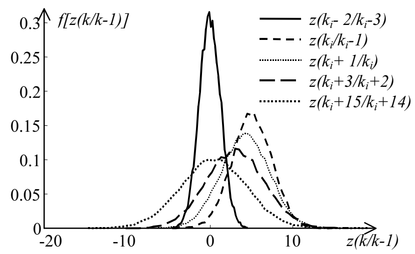

Fig. 1. An example of $z(k/k\text{-}1)$ pdf changes in a case of an additive bias in measurement channel arising at k_i

Rys. 1. Przykład zmian rozkładu gęstości prawdopodobieństwa procesu innowacyjnego spowodowanych zaistnieniem (od momentu k_i) skokowej zmiany addytywnej

As can be seen in Fig. 1 the $z(k/k\text{-}1)$ is zero mean for $k<k_i$ (solid line). Next for $k\geq k_i$ the mean value rises and then goes back to zero. The variance value also changes and retain higher than in nominal state. The character of the changes depends on type and parameters of system nonstationarity. Tracking the $z(k/k\text{-}1)$ fluctuations would allow detection of the fault onset and its identification. Using analytical approach it is desirable to have such a mathematical model of the system and its change which would be quite adequate and allow derivation of $z(k/k\text{-}1)$ analytical description. Model system fulfilling that can be obtained by using additive Gauss-Markov models (AGMM). The method allows description of a wide range of failures (both abrupt and incipient). Moreover it enables to design a multimodel filtering algorithm suitable for FDI or tracking issues.

2. The system and failures model

A model of a system changes can be obtained using a parameter vector $\vartheta(k,k_i)$, which can be written as:

$$\vartheta(k+1,k_i)=\varphi(k+1,k)\vartheta(k,k_i)+\xi(k)\,, \qquad (1)$$

where $\vartheta(k,t_i)$ is an unknown Gauss-Markov state vector of a system that models changes in the system after the jump at the time k_i, $\varphi(k+1, k)$ is a transition matrix, $\xi(k)$ is a white Gaussian sequences with zero mean and covariance matrix $Q(k)$.

Then the system state equation with taking into account possible changes can be presented as follows:

$$x(k+1) = \Phi(k+1,k)x(k) + w(k) + \\ + G_S(k+1)\vartheta(k+1,k_i)1(k+1,k_i), \\ y(k) = H(k)x(k) + v(k), \tag{2}$$

where $x(k)$ is the state vector, $w(k)$, $v(k)$ are white Gaussian sequences with zero mean and covariance matrices $Q(k)$ and $R(k)$ respectively, $y(k)$ is the observation vector, $H(k)$ is the observation matrix and $1(k, k_i)$ is the unit step function that is zero when $k<k_i$.

In a case of modeling changes in the measurement channel, process (1) should be added to the measurement equation:

$$x(k+1) = \Phi(k+1,k)x(k) + w(k), \\ y(k) = H(k)x(k) + v(k) + H_0(k)\vartheta_j(k,k_i)1(k,k_i). \tag{3}$$

The modeling method based on AGMM is highly flexible. Depending on the nature of the parameter vector $\vartheta(k,k_i)$ the model of changes may be classified [2] as deterministic ($\xi(k=0)$), stochastic ($\varphi(k+1, k)=0$) or mixed ($\varphi(k+1,k)\neq 0$, $\xi(k\neq 0)$).

3. Synthesis of the detection-estimation algorithm

Let us consider the system for which state and measurement equations are given by the model (2). Then, calculating the propagation of all signals through the Kalman filter that is matched with a system without jumps, we see that the innovation process $z(k/k-1)$ of the filter and estimates $\hat{x}(k/k)$ can be presented in the following form [3, 5]:

$$z(k/k-1,k_i) = z_1(k/k-1) + \Psi_{zx}(k,k_i)\phi(k_i,k_i-1)\vartheta(k_i-1,k_i) + \\ + \sum_{n=0}^{k-k_i}\Psi_{zx}(k,k_i+n)\xi(k_i+n-1), \tag{4}$$

$$\hat{x}(k/k,k_i) = \hat{x}_1(k/k) + F_{zx}(k,k_i)\phi(k_i,k_i-1)\upsilon(k_i-1,k_i) + \\ + \sum_{n=0}^{k-k_i}F_{zx}(k,k_i+n)\xi(k_i+n-1), \tag{5}$$

where $z_1(k/k-1)$ is the innovation process (zero mean white noise) related to the unchanged system and the remaining elements represent the influence of specific system change on the residuals of the filter matched to the undisturbed model.

All elements $\Psi_{zx}(k,k_i)$ depend on the system matrices, onset time and filter gain and can be calculated in a recursive way as follows:

$$\phi_{zx}(k,k_i) = \phi(k,k-1)\phi_{zx}(k-1,k_i), \tag{6}$$

$$\Phi_{zx}(k,k_i) = G_s(k)\phi_{zx}(k,k_i) + \Phi(k,k-1)\Phi_{zx}(k-1,k_i), \tag{7}$$

$$\Psi_{zx}(k,k_i) = H(k)[\Phi_{zx}(k,k_i) - \Phi(k,k-1)F_{zx}(k-1,k_i)], \tag{8}$$

$$F_{zx}(k,k_i) = K(k)\Psi_{zx}(k,k_i) + \Phi(k,k-1)F_{zx}(k-1,k_i) \tag{9}$$

with initial conditions: $\phi_{zx}(k_i-1,k_i) = \phi^{-1}(k_i,k_i-1)$, $\Phi_{zx}(k_i-1,k_i) = 0$, $F_{zx}(k_i-1,k_i) = 0$ and $\Psi_{zx}(k_i-1,k_i) = \mathrm{I}$ where I is the identity matrix.

Considering equation (4) the detection problem can be formulated as a statistical test of two hypotheses (H_0, H_1), the first one (H_0) is intended to test the presence of the white noise $z_1(k/k-1)$, and the second (H_1), the presence of the signal $\Psi_{zx}(k,k_i)\upsilon_{0\phi}$ to $z_{1\xi}(k/k-1)$ noise background.

$$H_0: z(k/k-1) = z_1(k/k-1), \\ H_1: z(k/k-1) = z_{1\xi}(k/k-1) + \Psi_{zx}(k,k_i)\vartheta_{0\phi}, \tag{10}$$

where $\vartheta_{0\phi} = \phi(k_i,k_i-1)\vartheta(k_i-1,k_i)$ and $z_{1\xi}(k/k-1)$ represent all noise components from equation (4).

Since the distribution of the onset time k_i is unknown a priori, the generalized likelihood ratio (GLR) test should be used:

$$\lambda(k,\hat{k}_i) = \frac{\max_{k_i} f[Z_{k_i}^k / H_1(k_i)]}{f[Z_{k_i}^k / H_0]} \tag{11}$$

where $f[*]$ is the conditional probability density function and $Z_{k_i}^k = \{z(k_i/k_i-1), \ldots, z(k/k-1)\}$.

The decision procedure has the form (12) where the generalized likelihood logarithm $\Lambda(k,\hat{k}_i)$ is compared with the threshold $\Lambda_p(k,\hat{k}_i)$. A variable threshold level is applied.

$$\Lambda(k,\hat{k}_i) \underset{H_0}{\overset{H_1}{\gtrless}} \Lambda_p(k,\hat{k}_i), \quad \hat{k}_i = \arg\max_{k_i}\left(\Lambda(k,k_i)\right), \quad k-M+1 \le \hat{k}_i \le k, \tag{12}$$

where $\Lambda(k,\hat{k}_i)$ is the logarithm of $\lambda(k,\hat{k}_i)$, M is the width of the moving window used to avoid an increasing number of additional filters matched to successive onset moments.

The performance of the decision procedure is essential to the efficiency of detection and so to the quality of estimation. The general principles of the applied GLR method are well established [4, 7, 8]. Unfortunately, the use of the GLR approach requires knowledge of the resulting probability distributions and application of a variable threshold level. Methods proposed in the literature are based on simplified statistics (not GLR) or experimental determination of constant threshold level.

The choice of a decision threshold $\Lambda_p(k,k_i)$ can be obtained using the Neyman - Pearson criterion, where a probability of false alarm P_{FA} (i.e. the probability of taking the decision that a fault has occurred while the system is in a normal state) is assumed.

$$P_{FA} = 1 - F_{\Lambda(k,k_i)/H_0}(\Lambda_p(k,k_i)), \tag{13}$$

where $F_{\Lambda(k,k_i)/H_0}(\Lambda_p(k,k_i)) = \int_{-\infty}^{\Lambda p(k,k_i)} f(\Lambda(k,k_i) = \Lambda_o / H_0)d\Lambda_o$ is the conditional probability distribution function of $\Lambda(k,k_i)$.

As seen in (13), the decision threshold can be determined with the use of $F_{\Lambda(k,k_i)/H_0}(\Lambda_p(k,k_i))$. It can be shown [3] that the GLR logarithm can be computed in the following way:

$$\Lambda(k,t_i) = \frac{1}{2}\sum_{l=k_i}^{k}\Big\{[z(l/l-1)]^T P_{z1}^{-1}(l/l-1)[z(l/l-1)] - \\ - [z(l/l-1) - \bar{z}_{H_1}(l/l-1,t_i)]^T P_z^{-1}(l/l-1) \times \\ \times [z(l/l-1) - \bar{z}_{H_1}(l/l-1,t_i)] + \ln[\det(P_{z1}(l/l-1)] - \\ - \ln[\det(P_z(l/l-1)]\Big\} \tag{14}$$

where $P_{z1}(l/l-1)$, $P_z(l/l-1,k_i)$, and $\bar{z}_{H_1}(l/l-1,k_i)$ are covariance matrixes and the expected value of the following conditional probability distributions:

$$f[z(l/l-1)/Z_{k_i}^{l-1},H_0]=N[0,\ P_{z1}(l/l-1)]\ ,$$
$$f[z(l/l-1)/Z_{k_i}^{l-1},H_1]=N[\bar{z}_{H_1}(l/l-1,k_i),\ P_z(l/l-1)]\ . \quad (15)$$

Unfortunately, as follows from (14) the GLR logarithm $\Lambda(k,k_i)$ has a distribution not easy do define, so an appropriate approximation should be applied. It can be shown that for M=1 the $\Lambda(k,k_i)$ has the non-central χ^2 distribution. It leads to the idea of using this distribution as the approximation for M>1. In this case, the non-centrality parameter, the number of degrees of freedom and the scaling coefficient must be determined. Calculation of these parameters is performed by matching three statistical moments: the first non-central, second and third central [5].

The performance of the proposed method was tested by means of numerical simulations which demonstrated the effectiveness of the proposed probability distribution approximations. As was mentioned, a constant probability of false alarm causes a change in the threshold value. An example is shown in Fig. 2. It should be added, that the character of changes depends on system and failure parameters and can vary from that presented.

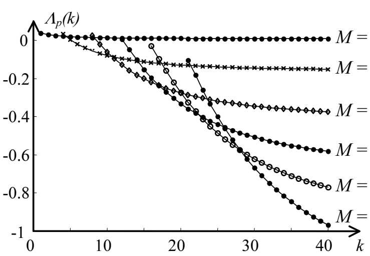

Fig. 2. An ex ample of the threshold level variation in the case of constant P_{FA}
Rys. 2. Przykład zmian wartości progu decyzyjnego przy stałym poziomie P_{FA} i różnej szerokości ruchomego okna obserwacji

Final check of validity of the thresholding algorithms was performed by testing the outcoming probability of false alarm. As can be seen from example shown in Fig. 3, the proposed method demonstrates high accuracy. Mean values $\overline{P_{FA}}$ are very close to the assumed P_{FA}.

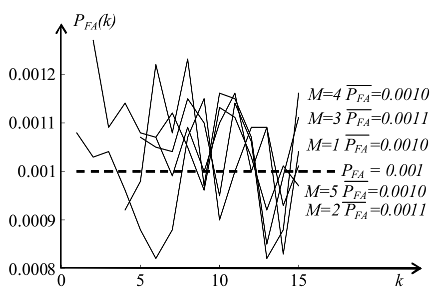

Fig. 3. An example of obtained P_{FA} variation in time when thresholds were calculated for P_{FA}=0.001
Rys. 3. Przykład zmian w czasie prawdopodobieństwa fałszywego alarmu przy zakładanym stałym poziomie P_{FA}=0.001 i różnej szerokości ruchomego okna obserwacji M

Thus the system of joint detection - estimation of jump changes in a dynamic system consists of the basic Kalman filter, which calculates values $z(k/k-1)$, the bank of Kalman filters, which compute the likelihood ratios $\Lambda(k,\ k_i)$ at different moments $k_i=k-M+1,\ldots,k$, the threshold circuit for detection of abrupt changes and the logic circuit, which selects the maximum value $\Lambda(k,t_i)$. As can be seen in Fig. 4 the maximum value of $\Lambda(k,\ k_i)$ on each timestep $k>k_i$ allows detection of the onset time ($k_i=k-M$). It should be noted that the proposed structure also makes it possible to isolate failures. This can be realised by comparing the likelihood ratios $\Lambda_j(k,k_i)$ from all filters matched to possible nonstationarity types and successive onset moments.

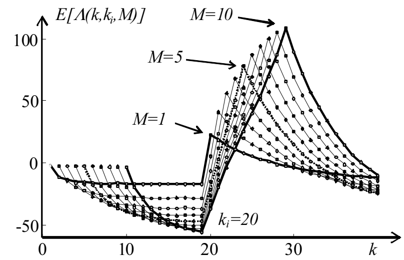

Fig. 4. An example of obtained $\Lambda(k,t_i)$ expected value for different M in the case of nonstationarity arising at k_i=20
Rys. 4. Zmiana wartości oczekiwanej $\Lambda(k,t_i)$ w przypadku zaistnienia niestacjonarności w momencie k_i=20 przy różnej szerokości ruchomego okna M

4. Conclusion

In the paper we have presented a new recursive algorithm for joint detection and estimation of jump changes in the dynamics and measurements of linear discrete-time systems. The jumps were modelled as Gauss-Markov biases in state and observation equations. The proposed models describe a wide class of dynamic systems with jump parameters. The algorithm has been developed on the basis of the GLR method. The method of threshold determination for GLR test was developed. It allows a constant rate of the probability of false alarm to be obtained in the non-stationary state of the object or filter. The structure of the algorithm is sufficiently simple to enable it to be applied in real-time systems with a relatively limited computational burden. The detection-estimation algorithm developed, was successfully applied to the problem of radar manoeuvering target tracking and fault-tolerant signal processing equipment. Simulation results revealed good estimation properties of the algorithm.

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5. References

[1] Blackman S., Popoli R.: Design and analysis of modern tracking systems. Artech House, 1999.

[2] Grishin Yu., Janczak D.: Joint Adaptive Detection - Estimation Algorithm for Manoeuvering Target Tracking. Proc.of the Int. Radar Symposium IRS 2006, Krakow, 2006, pp. 375-378.

[3] Grishin Yu.: An Application of the Additive Gauss-Markov Models of Discrete-Time Dynamic Systems to the Problem of Abrupt Changes Detection. Proc. of Int. AMSE Conf. Systems: Analysis, Control & Design, Lyon, 1994, vol. 1, pp. 211-220.

[4] Gustafsson F.: Adaptive Filtering and Change Detection, Wiley, 2000.

[5] Janczak D.: The GLR thresholding for fault detection algorithm based on additive Gauss-Markov model of changes. Proc. of the 8th IEEE Int. Conf. on Methods and Models in Automation and Robotics, MMAR'2002, Szczecin, 2002, vol. 1, pp. 587-592.

[6] Patton R., Frank P., Clark R.: Fault diagnosis in dynamic systems. Theory and applications. Prentice Hall, N.Y., 1989.

[7] Sage A. P., Melsa J. L.: Estimation theory with application to communication and control. McGraw-Hill, New York, 1972.

[8] Willsky A.: A Survey of Design Methods for Failure Detection in Dynamic Systems. Automatica, 1976, vol. 12, pp. 601-611.

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