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Application of SVD decomposition in correction tasks of mathematical models of dynamic objects

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Abstract

In the paper one of the possible approaches to the problem of developing and verifying of mathematical models of dynamic processes and their computer implementation is considered. This problem plays a decisive role in the efficient design of complex control systems of dynamic objects, in the development of computer tutoring systems for operators of such objects, in preparing simulation systems for pilots, helmsmen ships, and in many other issues. The proposed methodology is based on analysis of characteristics of controllability and observability of suitable objects. Mathematical base for this methodology is singular value decomposition of appropriate matrix.

Keywords: mathematical model, controllability, singular value decomposition.

Zastosowanie rozkładu SVD w zadaniach korekcji modeli matematycznych obiektów dynamicznych

Streszczenie

W artykule zaprezentowano podejście do tworzenia i weryfikowania modeli matematycznych procesów dynamicznych. Opisywany problem odgrywa ważną rolę w procesie opracowywania systemów sterowania złożonymi obiektami dynamicznymi, przy tworzeniu komputerowych systemów nauczania dla operatorów obiektów dynamicznych, w przygotowywaniu systemów symulacyjnych dla pilotów, sterników statków i w innych zagadnieniach. W zadaniach korekcji modeli matematycznych ważna jest odpowiedź na pytania w jakim sensie system jest niesterowalny (nieobserwowalny) i jak zmieniać charakterystyki systemu, żeby system stał się systemem sterowalnym (obserwowalnym). W zadaniach walidacji modeli dynamiki podstawą jest wyjaśnienie stopnia wpływu parametrów modelu na badane charakterystyki. Powstaje problem, w jaki sposób zmieniać charakterystyki modelu, aby jego zachowanie odpowiadało zachowaniu obiektu rzeczywistego. Odpowiedzi należy formułować z wykorzystaniem ogólnosystemowych pojęć znanych i zrozumiałych dla matematyków i programistów. Pojęcie miary cech systemu określa nie tylko stwierdzenie faktu, że system jest lub nie jest sterowalny (obserwowalny), ale także pozwala ocenić bliskość granicy utraty sterowalności (obserwowalności) systemu. W artykule przedstawiono zadanie korekcji efektywności sygnałów sterujących matematycznych modeli dynamiki z wykorzystaniem miary sterowalności liniowych systemów dynamicznych. Podstawą matematyczną opisywanej metodologii jest rozkład SVD odpowiednich macierzy.

Słowa kluczowe: model matematyczny, sterowalność, rozkład względem wartości szczególnych.

1. Introduction

Many studies [1, 2] have been devoted to the problem of analysis of structural characteristics of dynamic objects (controllability, observability, identifiability, etc.). Formulated a number of different concepts and definitions of system in the class of continuous and discrete models. In many works, starting at the classical work of Kalman [3], a number of criteria of controllability and observability are formulated. These criteria are threshold and give the answer yes-no: system is controllable (observable) or not. Very often, also in tasks of correction of mathematical model in tutoring computer-based systems, it is very important to know in what sense the system is not controllable (not observable) and what can be done (how to change characteristics of system) to the system was controllable (observable). During validation of computer models of dynamics, it is important to explain level of influence the different parameters of model on characteristics of controllability and observability. The question arises of how to change characteristics of computer model that its behavior corresponds to behavior of real object. The answers to such kind of questions should be worded not only in a special terminology, adopted in practice of research, but also with using the terminology that is know and understood for mathematicians and programmers. In this way the problem of assessment of quantitative system properties of dynamic object has arisen. The concept of assessment of properties of system (controllability, observability) determines not only a statement of fact that the system is or not (controllable, observable) but also allow to determine the nearness of the border of controllability or observability of system. This type of quantitative evaluation is very useful in the tasks of dynamic correction and validation of models. In this paper, the evaluation of this type has been calculated on the basis on singular value decomposition of rectangular numerical matrices [1, 4, 5, 6]. The correction of dynamic characteristics of any object that is described using the linear model, can be carried out in the form of two separate tasks: correction of the eigenvalue of linear operator, which describes the dynamics of the object and correction of the effectiveness of steering components. Both tasks are linked with each other but for each of them there are the appropriate methods and the tasks should be solved separately. In other words, the correction of dynamic characteristics of the object will be rely on change elements of matrix A and B in equation:

$$\dot{X} = AX + BU, \quad X \in R^{n}, U \in R^{m}, \tag{1}$$

where: X – vector of state variables, U – vector of control signals, A and B – matrices with adequate dimension and with fixed coefficients.

For tasks of correction the spectrum of matrix A the most adequate method is the modal control method [7]. However in correction tasks of efficiency of control signals of mathematical models of dynamics it could be used the assessment of controllability of linear dynamic systems.

2. The efficiency correction of control signals

For the first time, quantitative assessment controllability, observability and identifiability of linear stationary systems have been proposed in [8,9]. Proposed in this article quantitative assessment are a continuation of the known concepts of controllability and observability, and of systems proposed by Kalman [3]. Condition of complete controllability of stationary system (1) can be written in the form:

$$\mathrm{rank} W = n \,, \tag{2}$$

where $W = (B \quad AB \quad A^2 B \ldots A^{n-1} B)$ – the controllability matrix of system (1).

The controllability matrix W of dimensionality $\dim W = (n \times nm)$ is always real. So it is possible singular value decomposition of matrix W in the form:

$$W = P \cdot Q \cdot R^T \,, \tag{3}$$

where P and R – orthogonal matrices of dimensionality $\dim P = (n \times n)$ and $\dim R = (nm \times nm)$. If $(n<nm)$ the rectangular matrix Q is in the special form:

$$Q = \begin{pmatrix} \mu_1 & 0 & 0 & \vdots 0 \\ 0 & \ldots & 0 & \vdots 0 \\ 0 & 0 & \mu_n & \vdots 0 \end{pmatrix} = \begin{pmatrix} & \vdots 0 \\ M & \vdots 0 \\ & \vdots 0 \end{pmatrix}. \tag{4}$$

The nonnegative elements from this matrix $\{\mu_1, \mu_2, \ldots \mu_n\}$ are the singular values of the matrix W. They are located decreasing order, that is $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_n > 0$. The value

$$\xi^{\Delta} = \frac{1}{Cond2(T_s)} = \frac{\mu_n}{\mu_1} \,, \tag{5}$$

is the assessment of nearness of the border of transformation the nonsingular matrix M to singular matrix [8, 9].

According to equation (5) the coefficient ξ is quantitative assessment of general controllability of system (1). The assessment of controllability of linear dynamic systems is basic mathematical apparatus in tasks of correction the efficiency of control signals of mathematical models of dynamics.

3. The efficiency correction of control signals – an example

Let's present the efficiency correction control signals on the example of the linear model of longitudinal movement of the aircraft, which can be written in the form of general (6)

$$\dot{X} = AX + BU, \quad X \in R^n, U \in R^m, \tag{6}$$

where: $X = (V_a \quad \alpha \quad \theta \quad q)^T$ – vector of state variables: V_a – speed of the aircraft respect air, α – angle of attack, θ – the pitch angle, q – angular velocity relative to the X-axis, $U = (\delta_e \quad \delta_t)^T$ – control signals: δ_e – amount of rudder, δ_t – position of the throttle control lever. Matrices A and B of the test model, have the following numerical values:

$$A = \begin{pmatrix} -0.1383 & -19.0380 & -8.9846 & -0.1966 \\ -0.0043 & 0.9294 & 0.0612 & 1.1251 \\ -0.0001 & 1.1680 & 0.0005 & 1.0997 \\ 0.0036 & -62.9474 & 0.0281 & -4.4528 \end{pmatrix}, \tag{7}$$

$$B = \begin{pmatrix} -8.4591 & 30.5134 \\ 0.0420 & -0.1517 \\ 0.2316 & -0.8354 \\ -12.6077 & 45.4780 \end{pmatrix}. \tag{8}$$

The task of evaluation and correction of the efficiency of various control signals based on a study of changes in the index of controllability ξ depending on changes of the elements of the matrix B (6). The studies relate to the correction of a single signal - the correction of rudder deflection δ_e. In this case the coefficient of controllability of object ξ will be written as a function of four arguments $\xi = \xi(b_{11}, b_{21}, b_{31}, b_{41})$. These arguments are elements in the first column $b1$ of matrix B, that correspond to control δ_e. For the purposes of the outline and for further analysis is convenient to describe the coefficient ξ in sequence as a function of two arguments: $\xi = \xi(b_{11}, b_{21})$, $\xi = \xi(b_{11}, b_{31})$, etc. The value of the other two arguments do not change. Figure 1 presents the course of the variability of the coefficient of controllability ξ depending on changes elements b_{11} and b_{21} in matrix B of the base linear model (6).

On the contour line graph the coefficient of controllability of the base model has been marked as a black point. When examining Figure 1 it can be seen that the change in the value of the element b_{11}dramatically affects the rate of change of the controllability coefficient. Increase or decrease the value of the element b_{11} from the base value b_{11}=–8.45 causes large increase in the coefficient of controllability. However, change of value of element b_{21} virtually do not effects on change of coefficient of controllability (decreasing the value b_{21} causes a slight decrease of coefficient of controllability and increasing the value b_{21} causes a slight increase of coefficient of controllability ξ).

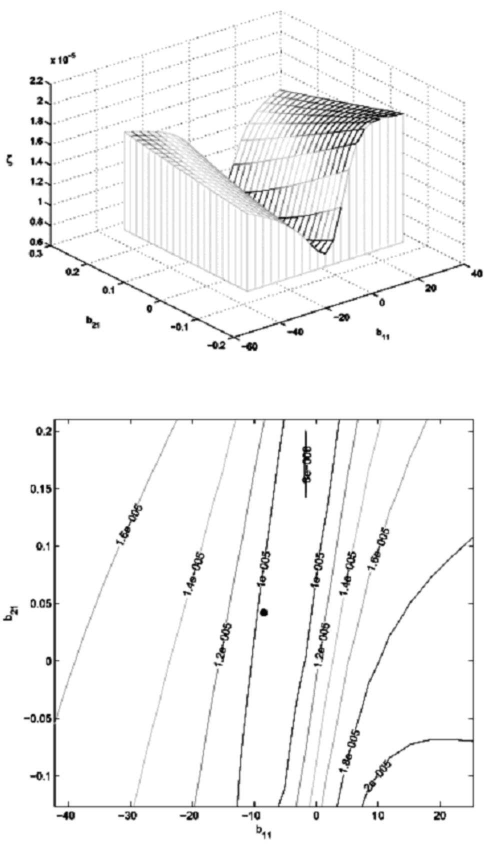

Fig. 1. Changes of the coefficient of controllability of model (6) depending on the changes of elements b_{11} and b_{21} of matrix B

Rys. 1. Zmiana wskaźnika sterowalności modelu (6) w zależności od zmiany elementów b_{11} i b_{21} macierzy B

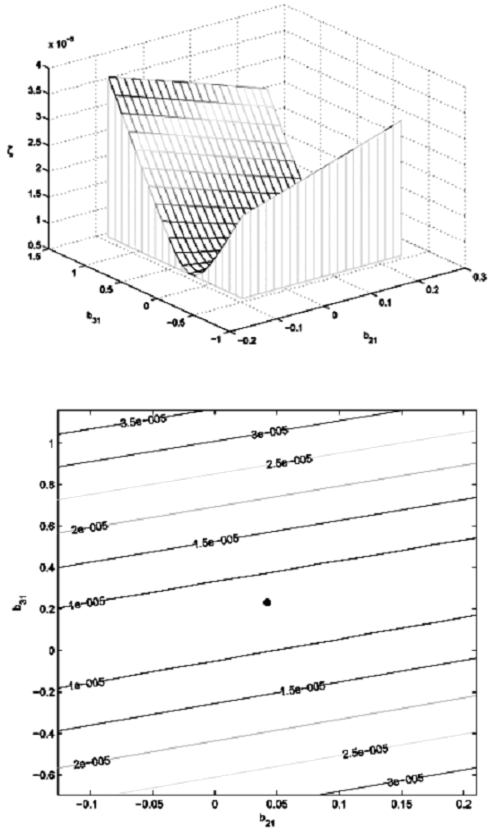

Fig. 2. Changes of the coefficient of controllability of model (6) depending on the changes of elements b_{21} and b_{31} of matrix *B*

Rys. 2. Zmiana wskaźnika sterowalności modelu (6) w zależności od zmiany elementów b_{21} i b_{31} macierzy *B*

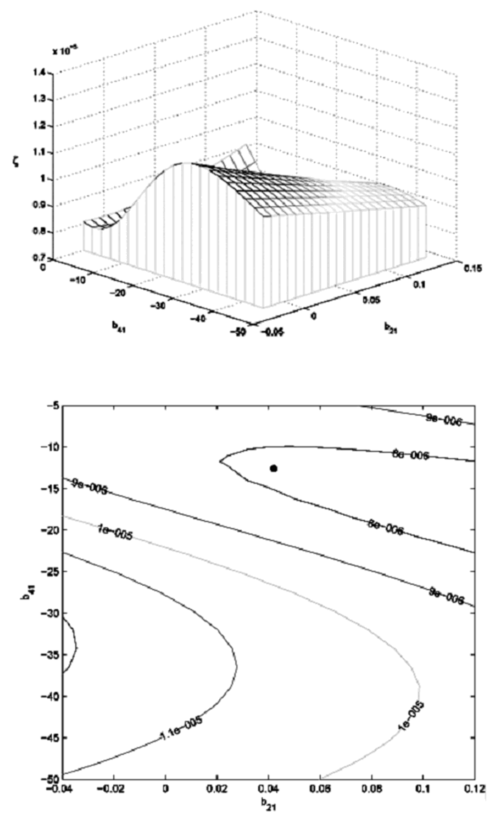

Fig. 3. Changes of the coefficient of controllability of model (6) depending on the changes of elements b_{21} and b_{41} of matrix *B*

Rys. 3. Zmiana wskaźnika sterowalności modelu (6) w zależności od zmiany elementów b_{21} i b_{41} macierzy *B*

This may mean, that during the correction of efficiency of the control signals we can arbitrarily change the parameters, that affecting the value b_{21} and it not deteriorate the index of controllability ξ. The above statement is confirmed by Figure 2, that shows relationship $\xi = \xi(b_{21}, b_{31})$ and by the Figure 3, that shows relationship $\xi = \xi(b_{21}, b_{41})$.

Additionally, Figure 3 shows that reducing the value of element b_{41} will cause initially increase the coefficient of controllability. After exceedance the value b_{41}=–35 the value of coefficient ξ decreases.

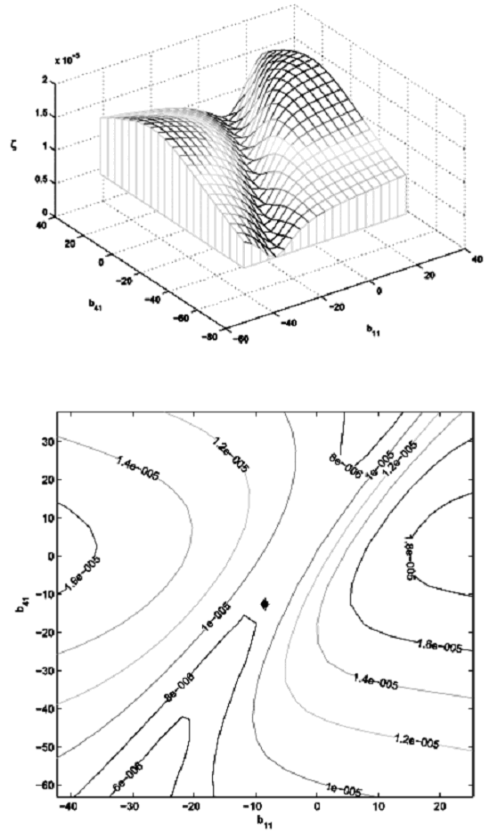

Fig. 4. Changes of the coefficient of controllability of model (6) depending on the changes of elements b_{11} and b_{41} of matrix *B*

Rys. 4. Zmiana wskaźnika sterowalności modelu (6) w zależności od zmiany elementów b_{11} i b_{41} macierzy *B*

On Figure 4 the relationship $\xi = \xi(b_{11}, b_{41})$ is shown. It follows from this, that the simultaneous decreasing or increasing the values of elements b_{11} and b_{41} cause decreasing the coefficient of controllability ξ. In a similar way we can analyze the trajectory of coefficient of controllability ξ for the other elements from the first column of matrix *B* of model (6).

During the analysis of response of base model (6) to various changes in the control signal δ_e, it has been stated that the model poorly reacts to the inclination of rudder δ_e. The correction of model parameters will be conducted on the basis of analysis of the change of numerical value of elements from the first column of matrix *B* of base linear model (6). The Figure 1a shows, as has been previously described, that decreasing the value of element b_{11} causes increase of the coefficient of controllability ξ. Let's assume new value of element b_{11}=–15. After fixing the value of b_{11} we analyze the relationship $\xi = \xi(b_{21})$, from which appears that decreasing the value of element b_{21} we increase the value of coefficient of controllability ξ.

The new value of element b_{21} was set at b_{21}=–0.15.

After the correction, the new numerical value of the first column of the matrix *B* take the form:

$$b1 = (-15 \quad -0.15 \quad 0.2316 \quad -12.6077)^T, \qquad (9)$$

Figure 5 shows the flight trajectory of linear models of aircraft for the same initial value. The base model is marked by continuous line and the changed model is marked by dashed line. Matrix A of a new linear model is the same as for the base model. Its numerical value determines the equation (6).

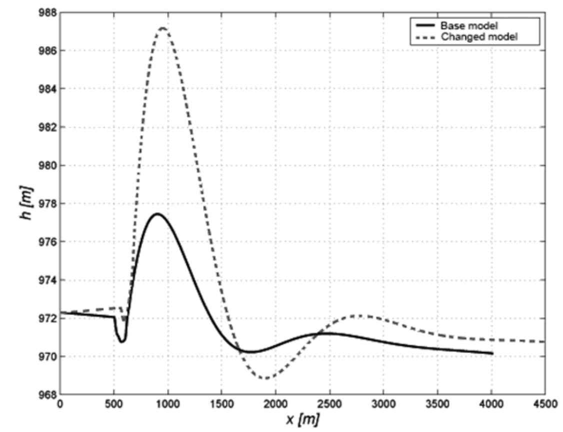

Fig. 5. Trajectory of two linear models: before and after correction of control signal
Rys. 5. Trajektoria dwóch modeli liniowych: przed i po korekcie sygnału sterującego

From Figure 5 shows that the new model responds strongly to changes of the rudder deflection δ_e. The coefficient of controllability of base model has value ξ=7.8634e-006 while for the new model its value is ξ=8.7823e-006. This means that the adjustment did'nt deteriorated conditions of controllability of model.

4. Summary

At the end, it should be pointed out that the order of operations does not allow for the first time on finding the best solution, but the proposed methodology is a useful measure for the target to seek the necessary solutions.

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