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Parallel implementation of a steady state thermal and hydraulic analysis of heat exchanger networks in OpenMP

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Abstract

The considerable computation time of a practical application of sequential algorithms for simulating thermal and flow distribution in heat exchanger networks was the motivating factor to study their parallel implementation. The mathematical model formulated and studied in the paper requires the solution of a set of nonlinear equations, which are solved by Newton-Raphson methods. An object-oriented solver automatically formulates the equations for the network of an arbitrary topology. The hydraulic model that was chosen as a benchmark consists of nodal flows and loop equations. A decomposition algorithm is presented, and results of speedup of its parallel implementation are demonstrated.

Keywords: heat exchanger network, steady state analysis, flow, temperature, parallel implementation.

Implementacja równoległa w OpenMP analizy statycznej przepływów i temperatur w sieci wymienników ciepła

Streszczenie

Motywacją do badania praktycznych zadań statycznej symulacji przepływów i temperatur w sieci wymienników ciepła jest duża czasochłonność algorytmów sekwencyjnych, prowadząca do konieczności ich zrównoleglenia. Sformułowany w artykule model matematyczny sprowadza się do układu nieliniowych równań algebraicznych. Następnie została dokonana dekompozycja modelu matematycznego na dwa modele: model hydrauliczny oraz model temperatur. Model hydrauliczny składa się z równań przepływów w oczkach oraz węzłach. Pozwala to na wykorzystanie istniejących podejść do symulacji natężenia przepływów, które pośrednio są funkcją średnich temperatur. Średnie temperatury podczas symulacji natężenia przepływów przyjmuje się jako stałe. Zależność współczynników oporów od liczby Reynoldsa, będące z kolei funkcją natężenia przepływu, określa się za pomocą metody punktu stałego względem tych współczynników, tj. każda iteracja metody punktu stałego wymaga symulacji przepływów przy zadanych wartościach współczynników oporów. Model symulacji przepływów rozwiązywany jest metodą Newtona-Raphsona. Symulacja modelu temperatur sprowadza się do rozwiązania układu równań liniowych, ponieważ wartości natężenia przepływów oraz średnich temperatur na tym etapie algorytmu przyjmuje się jako stałe. Struktura równań modelu temperatur zależy od kierunków przepływów i formułowana jest po każdym rozwiązywaniu modelu hydraulicznego. Zmienność współczynników modelu temperatur oraz natężenia przepływów od średniej temperatury uwzględnia się za pomocą metody punktu stałego, która w przypadku braku zbieżności wymusza ponownie symulację przepływów. Obiektowo zorientowany pakiet programów implementujący dany algorytm, w sposób zautomatyzowany formułuje równania i przeprowadza symulację sieci o dowolnej topologii. Sekwencyjna implementacja programowa algorytmu posłużyła jako benchmark do badania przyspieszenia tego algorytmu po jego zrównoleglonej implementacji w OpenMP. W artykule pokazano także, że nawet w przypadku drobnoziarnistej równoległości, udaje się uzyskać przyspieszenia rzędu 2.

Słowa kluczowe: sieć, wymiennik ciepła, analiza statyczna, przepływ, temperatura, implementacja równoległa.

1. Introduction

The domain of application of a pipeline network analysis is very wide, e.g. airplane hydraulic, fuel or environmental control systems, district heating systems, air-conditioning systems of buildings, trains or ships, water or gas distribution systems, and so on. The task of a pipeline network system is to convert the magnitudes of parameters fixed at certain boundary system points in the magnitudes prescribed at the other system boundary points. Such parameters are called boundary conditions. The conversion is a result of mutual transformations of different forms of energies. The moving forces of the transformations are finite temperature and pressure differences. To answer the question how such transformations are realized by the system, analysis problem must be solved. To do this, we have additionally to fix magnitudes of geometrical parameters (e.g. for tube they are tube diameters, lengths and roughness, wall thickness) and boundary conditions (temperatures, pressures). The result of the analysis is a vector of the thermodynamic parameters (pressures, temperatures, enthalpies, etc.) and flow rates. Practical simulations of aircraft environmental control systems [1] demonstrate the dependence of flow on temperature. It follows from the fact that a Reynolds number depends on dynamic viscosity being a function of temperature, in turn. For fluids, change of temperature on 20 K changes the dynamic viscosity almost twice, which follows from the water property table [2]. Being a criterion parameter the Reynolds number makes impact on the values of resistances and heat exchange intensity. The impact of the Reynolds number change due to temperature is more essential for laminar or smooth pipe flows [3]. If the pipe network is controllable [4], then any kind of flow can take place in its various parts. To account the temperature impact, the joint thermal and hydraulic simulation should be performed. Individually, the analysis methods for the simulation of flow distribution and temperatures have been addressed in papers. For the flow steady state analysis the most common methods are those in which independent variables are expressed in terms of the link or chord flows, the loop flow correction and the nodal heads. In [5], a lack of stability of the nodal method [5, 6, 7] is reported. In [8, 9] the numerical superiority of the flow method over the nodal head method has been proved. The comprehensive analysis, history and examples of the use of these methods are available in [1, 10-16]. The methods of the second order are described in [16, 17]. In [18] the value of a full-set equation approach is demonstrated which lends itself to the technique of introducing additional equations to describe modified or added network characteristics meeting specified conditions. Being more general formulation, the full-set approach is also used in the present paper. The thermal model is a particular form of the first thermodynamics law. The matrix formulation of thermal model is studied, in detail, in [19, 20], and later in [21]. The general methodology of the thermal-hydraulic simulation is addressed in [22]. The use of it for more and more large networks was becoming more and more time-consuming. It was the motivating factor to study a parallel implementation of the sequential methodology [22]. The OpenMP standard [23] was chosen as a tool for paralleling, because its use is very straightforward.

2. The mathematical model

In the present paper, we consider the stationary thermal and flow simulation of incompressible fluid in a pipe network. We do not explicitly consider pumps in the network, but they could be

easily incorporated as well as the given discharges from the interior nodes. Three basic conservation physical principles are necessary to formulate the model: continuity, momentum and energy. The first one determines a flow rate in the pipe and nodal equations

$$F_i = \sum_{k=1}^{|E_i|} \dot{m}_k = 0, \quad i = \overline{1,v} \tag{1}$$

implying that at each node i, the sum of flows in pipes incident to i, which numbers belong to the subset E_i, is zero. In (1), v is a number of thermal system nodes for which $|E_i| \geq 2$, $\dot{m}_k$ is the flow rate in pipe k. The second principle expresses pressure differences between the pipe section 1 ($P_{k,1}$) and the pipe section 2 ($P_{k,2}$)

$$F_{|N|+k} = P_{k,1} - P_{k,2} - K_k |\dot{m}_k| \dot{m}_k = 0, \quad k = \overline{1,e} \tag{2}$$

through the flow rates and pipe resistances K_k. These resistances in (2) are defined as follows

$$K_k = \left(\frac{L\lambda}{2\rho D_h A_c^2} \right)_k . \tag{3}$$

where: D_h – hydraulic diameter, A_C – cross-sectional area, ρ – density, λ– Darcy-Weisbach friction factor, L – pipe length. The form of the friction factor formula to be used depends on the Reynolds number being a criterion parameter

$$\mathrm{Re} = \frac{\dot{m} D_h}{A_c \mu} . \tag{4}$$

where μ is a dynamic viscosity. According as $\mathrm{Re} < 2300$ (a laminar flow) or $\mathrm{Re} \geq 2300$ (a turbulent flow) the following formulas for computing the friction factor are used [24]

$$\lambda = \frac{64}{\mathrm{Re}}, \ \mathrm{Re} < 2300 , \tag{5}$$

$$\lambda = 0.11 \left(\frac{\Delta}{D_h} + \frac{68}{\mathrm{Re}} \right)^{0.25}, \ \mathrm{Re} \geq 2300 , \tag{6}$$

where Δ is an inner pipe surface roughness. The third principle yields two equations

$$\Phi_k = T_{k,1} - e^{\left(\frac{UA_s}{C\dot{m}}L\right)_k} T_{k,2} + \left[e^{\left(\frac{UA_s}{C\dot{m}}L\right)_k} - 1 \right] T_{k,w} = 0, \quad k = \overline{1,e} , \tag{7}$$

$$\Phi_{e+i} = \sum_{k=1}^{|E_i|} C_{k,i} \dot{m}_k T_{k,i} = 0, \quad i = \overline{1,v} , \tag{8}$$

where $T_{k,1}$, $T_{k,2}$, $T_{k,w}$ are temperatures at the pipe ends and their boundaries; U is an overall heat transfer coefficient, C is a heat capacity, A_s is an inner heat transfer area. The overall heat transfer coefficient is obtained from the following formula

$$UA_s = \frac{1}{\dfrac{1}{hA_s} + \dfrac{D_{h,1} \ln\left(\dfrac{D_{h,2}}{D_{h,1}}\right)}{A_s k_w}} , \tag{9}$$

which is the integral of the differential equation of heat conduction through a cylindrical wall with the given temperature on the tube outer surface as the boundary condition. In (8), $D_{h,1} \equiv D_h$, but $D_{h,2}$ is an outer tube diameter, h is a heat transfer coefficient, k_w is a thermal conductivity. The heat transfer coefficient h is obtained from the definition of the Nusselt number

$$Nu = \frac{hD_h}{k} , \tag{10}$$

which is determined as follows [25]

$$Nu = 0.021 \mathrm{Re}^{0.8} \mathrm{Pr}^{0.43} \left(\frac{\mathrm{Pr}}{\mathrm{Pr}_w} \right)^{0.14}, \ \mathrm{Re} \geq 10^4 , \tag{11}$$

$$Nu = \frac{Nu_l^{6.267}}{Nu_t^{5.267}} \mathrm{Re}^{0.68 \ln \frac{Nu_t}{Nu_l}}, \ 2300 \leq \mathrm{Re} < 10^4 , \tag{12}$$

$$Nu = 1.55 \left(\mathrm{Pr}\,\mathrm{Re} \frac{D_h}{L} \right)^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}, \ \mathrm{Re} < 2300 , \tag{13}$$

for turbulent, transitional and laminar flows, respectively. Into (11)-(13) the Prandtl number quantity enters. It is defined as follows

$$\mathrm{Pr} = \frac{C\mu}{k} . \tag{14}$$

In (12) Nu_l is computed from (13) at Re=2300, and Nu_t is computed from (11) at Re=10^4. All working fluid properties (C, k, μ) are assumed to be functions of temperature, and evaluated at average temperatures for every pipe network element. Hence, all the criterial parameters are also functions of this temperature. The above model (1)-(14) is a nonlinear set of algebraic equations, which is solved with Newton-Raphson method, in this paper. The computer representation of (1) is somewhat different

$$\left[A_{i,j} \right]_{v+1, e+v_b} \left[\dot{m}_j \right]_{e+v_b} = 0 , \tag{15}$$

Equation (15) shows that a pseudo-node is introduced to which all the nodes with the given pressures are connected with a pseudo-edges v_b in number. In this case (2) takes the form

$$\left[A_{i,j} \right]^T_{e+v_b, v+1} \left[P_j \right]_{v+1} = \underset{(e+v_b)\times(e+v_b)}{diag[K]} \ \underset{(e+v_b)\times(e+v_b)}{diag|\dot{m}|} \ [\dot{m}]_{e+v_b} , \tag{16}$$

Comparing (15) with (16) manifests the orthogonality of matrices that is used when applying the chord flow method. In the full-set equation approach (16) (its right part) is transformed to the loop form when excluding the same pressure variables from its left part, which is transformed to a vector of pressure differences at the bounds of pseudo-loops or zeros for loops. Formal procedures of obtaining the loop equation form can be different. We can make eliminations directly on matrix $\left[A_{i,j} \right]^T_{e+v_b, v+1}$, then the introduction of pseudo-links is not needed. The number of the exclusions is equal to the rank of $\left[A_{i,j} \right]^T_{e+v_b, v+1}$. Being generated by the graphic editor, this matrix represents undirected graph, at first. Then, with the depth-first algorithm the directions are formed, and, finally, the above-mentioned eliminations are carried out to eliminate pressure variables from the pipe network model. As a result the model (1)-(14) depends on flow rate and temperature, only. Experience shows [1, 22, 26] the solution of the model can be

decomposed into several layers (see Fig.1): hydraulic, thermal and working fluid property ones.

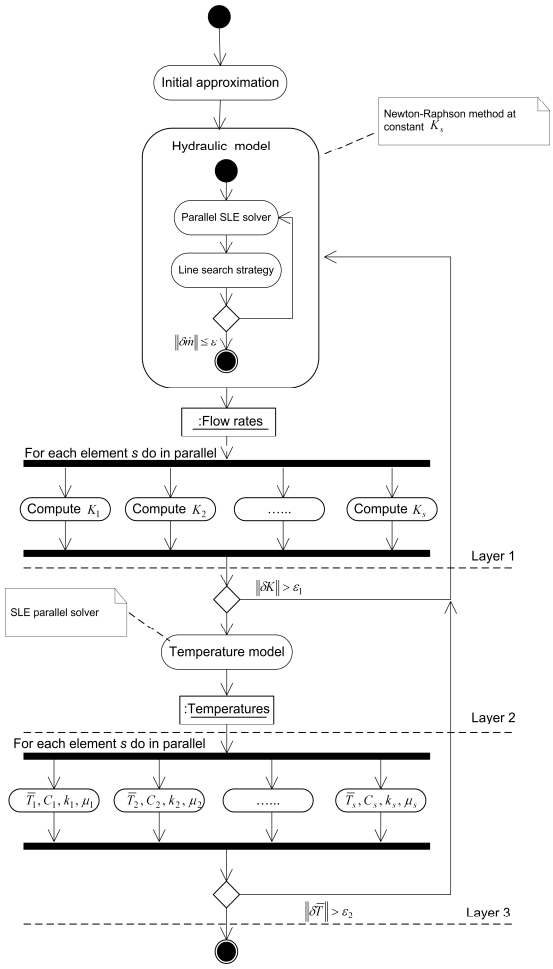

Fig. 1. The general solution algorithm
Rys. 1. Ogólny algorytm rozwiązania

The first level is computed at fixed average temperatures. The hydraulic model computes flow rates at constant resistances, followed by parallel computation of new resistances being functions of flow rates at this level. As a component of the Newton-Raphson method, the parallel LU solver is implemented. At each Newton's iteration, the LU solver solves the following set of linear equations (SLE)

$$J\left(\dot{m}^i\right)\Delta\dot{m}^i = -F\left(\dot{m}^i\right), \tag{17}$$

$$\dot{m}^i = \dot{m}^i + \Delta\dot{m}^i, \tag{18}$$

relative to the full-step flow correction vector, $\Delta\dot{m}^i$. The Newton's method has local quadratic convergence. To guarantee the global convergence, the full-step length is corrected with the line search backtrack procedure. The correction coefficient α is computed by solving the following minimizer

$$\frac{1}{2}\left\|F\left(\dot{m}^i + \alpha_i^k \Delta\dot{m}^i\right)\right\| \to \min. \tag{19}$$

Then, the global convergence step is the following

$$\Delta\dot{m}^i = \alpha_i^k \Delta\dot{m}^i. \tag{20}$$

Equations (17)-(20) describe a typical strategy of solving nonlinear algebraic equations [27]. The second level is computed at fixed flow rate flows from the first level and average temperatures. In this case, (7)-(8) constitute an SLE with constant coefficients (9)-(14). The third level computes average temperatures. Each level iterates relative to one type of parameter vector using the fixed-point method.

3. Results and discussion

In Fig. 2 the pipe network is shown used as a benchmark to answer the question: how many times faster we can solve (3)-(16) in parallel than sequentially. With this end in view, a number of pipe networks configurations followed from the benchmark network (see Fig. 2) are solved sequentially and in parallel. The configurations of pipe networks with the pipe number less than 66 are not depicted. Adding incrementally blocks of 66 pipes, the maximum pipe number network that was solved has 528 pipes. Tests were performed on a computer with 4 processors under the Windows Server 2003 operation systems. The network solver is developed and compiled in the Visual Studio Team 2008 environment. The parallel implementation of the sequential algorithm was coded with the OpenMP standard, which substantially simplifies studies on parallelization, because programming with this standard is very straightforward. From Fig. 1 we can see that the decomposition lead to solving at layer 2 the temperature model described by (7)-(14), which doesn't have a full rank.

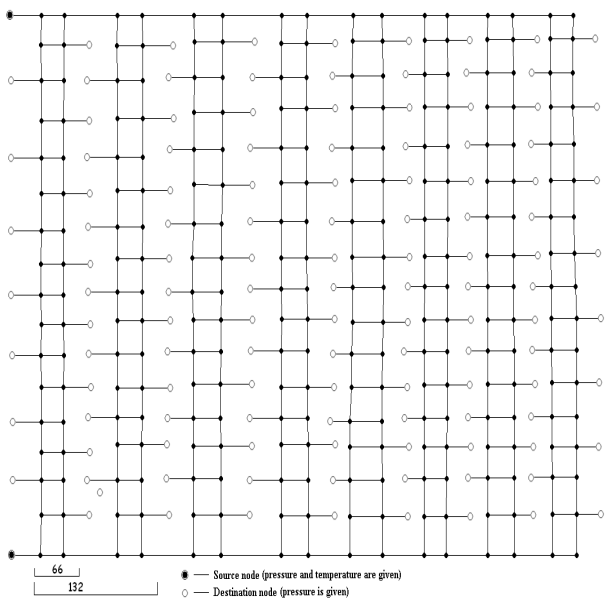

Fig. 2. The benchmark pipeline network
Rys. 2. Układ rurociągu jako benchmark

Consequently, some additional equations must be automatically formed and added to the model [20], [22]. To do this, we analyze the flow configuration obtained from level 1 and find out outflow pipes for each node. After mixing, each outflow pipe has the same temperature. Then, if $\overline{E}$ is a set of outflow pipes of node *i*, then their $\left|\overline{E}\right|-1$ pairs form a set of equations of the following form for each node

$$\Phi_{e+v+l} = T_{\overline{E}_i(l)} - T_{\overline{E}_i(l+1)} = 0, \quad l = \overline{1, n_i - 2}, \quad n_i = \left|\overline{E}_i\right|. \tag{21}$$

Together, SLE (7)-(8), (21) has a full rank. One of the basic activities of the algorithm (Fig.1) is to solve SLE. In Fig. 3 matrix portraits of flow and thermal equations of the pipeline network are shown in Fig. 2. These SLEs are unsymmetrical that requires implementation of the solution algorithm for this general case of SLE. Experiments have shown that the speedup of solving SLE of

size of order 1000 by the LU factorization method doesn't exceed 2, for the given computer. Therefore, we can expect that the speedup of the whole algorithm will be of the similar order. Curve 1 in Fig. 4 demonstrates speedups of the hydraulic model activity shown in Fig. 1. We can note, that the speedup takes place for networks sizes greater than or equal to 330 (see Fig. 2). The flow rate error that was taken is $\varepsilon \leq 10^{-9}$, and $\varepsilon_1 \leq 10^{-6}$ and $\varepsilon_1 \leq 10^{-3}$ for errors in resistances and average temperatures, respectively.

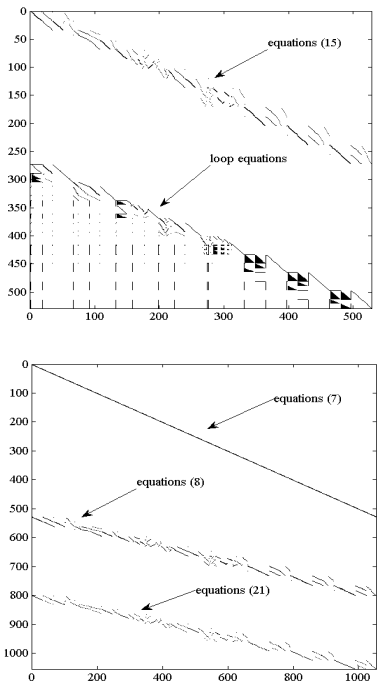

Fig. 3. Flow and thermal equation Matrix portraits of the benchmark pipeline network
Rys. 3. Obrazy macierzowe równań przepływu i ciepła benchmarku

The value of the speedup is less than 2, because the SLE solution is embedded into the sequential Newton's procedure (17), (19), (20). As a result communications concerned with servicing threads at each Newton's step decrease the speedup. Besides, the speedup is decreased by loops, which accompany the Newton's procedure such as preparing SLE, computing minimal values and vector sums, and so on. As a result the efficiency of the hydraulic model activity is of order of 0.325 for the given computer characteristics. Curves 2 and 3 demonstrate that the speedup can be substantially increased by the model decomposition. The curve 2 corresponds functionality to layer 1, while curve 3 corresponds functionality to the complete model.

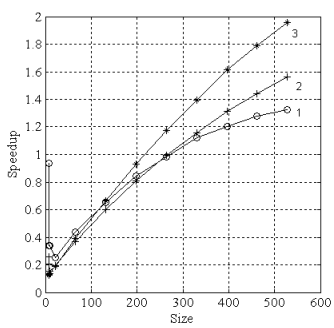

Fig. 4. Speedups of models corresponding different layers of the algorithm from Fig.1
Rys. 4. Przyspieszenie modeli odpowiadające różnym warstwom algorytmu przedstawionego na rys. 1

The speedups, we can observe, occur due to the lack of data communication between the fixed-point methods' iterations, which correct resistances (layer 1), and average temperatures and working fluid properties (layer 3). In the latter case, the efficiency increases almost up to 0.5. It will increase even greater in simulation heat exchanger networks owing to more complex procedures for computing resistances and average temperatures. To support the truth of this, the heat exchanger network benchmark scheme shown in Fig. 5 was simulated. The heat exchanger network has 120 edges. For simplicity we assume all the heat exchangers to be one-pass compact crossflow with both flows unmixed.

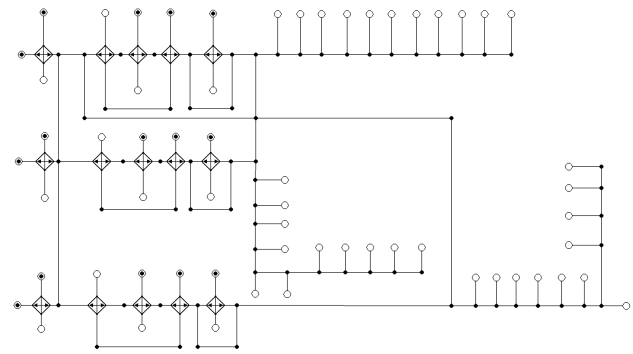

Fig. 5. Flow and thermal equation matrix portraits of the benchmark pipeline network
Rys. 5. Portrety macierzowe równań przepływu i ciepła benchmarku

The model of a heat exchanger was the following [28]

$$\Delta P = \frac{\dot{m}|\dot{m}|}{2\rho A_c^2}\left[\left(K_c + 1 - \sigma^2\right) + \lambda\frac{A_s}{A_c} - \left(1 - \sigma^2 - K_l\right)\right], \quad (22)$$

$$\varepsilon = \frac{W_h\left(T_{h,1} - T_{h,2}\right)}{W_{\min}\left(T_{h,1} - T_{c,1}\right)} = \frac{W_c\left(T_{c,2} - T_{c,1}\right)}{W_{\min}\left(T_{h,1} - T_{c,1}\right)}, \quad (23)$$

where

$$\varepsilon = \frac{1}{W \cdot NTU}\sum_{n=0}^{\infty}\left[1 - e^{-NTU}\sum_{m=0}^{n}\frac{NTU^m}{m!}\right]\cdot\left\{1 - e^{-C\cdot NTU}\sum_{m=0}^{n}\frac{(W\cdot NTU)^m}{m!}\right\}, \quad (24)$$

$$W = \frac{W_{\min}}{W_{\max}},\ W_{\min} = \min(W_c, W_h), \quad (25)$$

$$W_c = C_c\dot{m}_c,\ W_h = C_h\dot{m}_h, \quad (25)$$

$$NTU = \frac{UA_s}{C_{\min}},\ \frac{1}{UA_s} = \frac{1}{h_h\eta_{0h}A_h} + \frac{\delta_w}{A_w k_w} + \frac{1}{A_c h_c \eta_{0c}}, \quad (26)$$

$$h = \frac{St\, C\dot{m}}{A_c}, \quad (27)$$

$$St\,\mathrm{Pr}^{2/3} = \Phi_1(\mathrm{Re}), \quad (28)$$

$$\eta_0 = 1 - \frac{A_f}{A_s}\left[1 - \frac{th(m\cdot l_f)}{m\cdot l_f}\right], \quad (29)$$

$$A_s = A_w + A_f,\ l_f = b/2,\ m = \sqrt{\frac{2h}{k_f\delta}}, \quad (30)$$

where: σ – ratio of the free flow area to the frontal area on one side of the exchanger, K_c, K_l – entrance and exit loss coefficients, A_s – heat transfer surface on one side of the exchanger, A_f – fin

area, A_w – separation plate area, η_0 – overall fin efficiency, b – separation plate distance, St – the Stanton number, ε – heat exchanger effectiveness, δ_w – separation plate thickness, δ – fin thickness, 1 – inlet, 2 – outlet, c – cold, h – hot.

Formula (28) is obtained from an experiment (see Fig. 6), as well as λ. Equations (22), (23) complement model (2), (7). Simulations have shown that the speedup for the heat exchanger network was a little greater than 1, whereas speedup for a pipe network (see Fig. 2) with the same edge number is still less than 1 (see Fig. 4).

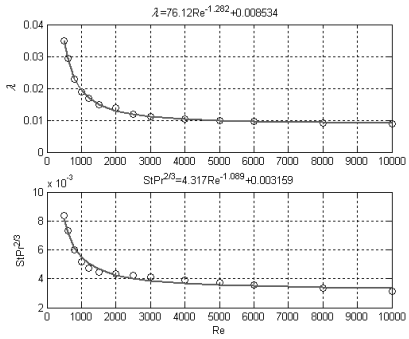

Fig. 6. Friction and heat transfer factors approximations
Rys. 6. Przybliżenia współczynników tarcia i ciepła

It sustains the assumption made above (due to more complex procedure of computation of a heat exchanger effectiveness with (14), (24)-(30) than with (9)-(14) in case of calculation of a heat transfer coefficient of a pipe).

4. Conclusions

In the paper the parallel implementation of the steady-state thermal-hydraulic analysis in OpenMP is presented. The mathematical model studied contains all typical tasks inherent to such an application domain. To demonstrate the influence of each task on the final speedup, numerical experiments were carried out for different sets of subtasks and sizes of pipe networks on computer with two Quad Core Processors (Intel® Xeon E5405 Quad Core Processor) under the Windows Server 2003 operation system. The object-oriented code of the thermal-hydraulic analysis solver was compiled in Visual Studio Team 2008 environment. The paper demonstrates that the parallel implementation of the algorithm used results in speedup, which can reach values equal to 2. Besides, there exists potential in speedup for larger sizes of pipe networks that could be mainly achieved at the expense of decomposition of the whole nonlinear model into hydraulic (computing flow rates at fixed resistances) and temperature (computing network temperatures) ones. Two fixed-point iteration procedures that compose these sub-models into the whole are the sources of earned speedup value as compared to the hydraulic model, where solving SLE is a dominant functionality. For larger networks speedup will increase mainly thanks to greater number of its elements, which parameters are corrected by fixed-point iterations, and which, in turn, do not introduce additional data dependences and, as result, communication; while efficiency of the SLE parallelization will grow slowly. The results also demonstrate that further increase in speedup can be achieved by implementing more advanced symbolic matrix manipulations and decomposition methods resulting in minimal communications, provided that they altogether decrease the computation time of the sequential implementations of these algorithms.

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5. References

[1] Kondrashenko V., Vinnichuk S., Fedorov M.: Simulation of Gas and Liquid Distributing Systems. Naukova Dumka, Kiev, 1990, [in Russian].
[2] Massoud M.: Engineering Thermofluids. Springer-Verlag, Berlin Heidelberg New York, 2005.
[3] Idelchik I.E.: Handbook of Hydraulic Resistances. Machynostrojenije, Moscow, 1992, [in Russian].
[4] Fedorov M.: On Software Design of Thermal Systems Steady State Control. Polish J. Environm. Stud. 4C, 2008.
[5] Donachie R.P.: Digital Program for Water Network Analysis. J. Hydraulics Div., Vol.100, HY3, ASCE, 1974.
[6] Chandrashekar M.: Extended Set of Components in Pipe Networks. J. Hydraulic Div. Vol. 106, HY1. ASCE, 1980.
[7] Nogueira A.C.: Steady-State Fluid Network Analysis. J. Hydraulic Eng., Vol.119, 3, 1993.
[8] Nielsen B.N.: Methods for Analyzing Pipe Networks. J. Hydraulic Eng., Vol.115, 2, ASCE, 1989.
[9] Altman T., Boulos P.F.: Convergence of Newton Method in Nonlinear Network Analysis. Mathl. Comput. Modelling, Vol.21, 4, Elsevier, 1995.
[10] Ohtmer O.: Nonlinear Flow Analysis in Pipe Networks. Int. J. Numer. Meth. Engng., Vol.19, 1983.
[11] Chandrashekar M.: Extended Set of Components in Pipe Networks. J. Hydraulic Div. Vol. 106, HY1, 1980.
[12] Evdokimov A.G.: Optimal Problems of Engineering Networks. Wyzsha Shkola, Charkov, 1976, [in Russian].
[13] Merenkov A.P., Chasilev V.J.: Theory of Hydraulic Circuits. Nauka, Moscow, 1985, [in Russian].
[14] Larock B. E., Jeppson R. W., Watters G. Z.: Hydraulics of Pipeline Systems. CRC Press, Boca Raton London New York Washington, D.C., 2000.
[15] Osiadacz A.J.: Steady-state Simulation of Gas Networks. Fluid systems Sp., Warszawa, 2001, [in Polish].
[16] Sennikova E.V., Sidler V.G.: Mathematical Simulation and Optimization of Evolving Heat Supply Systems. Nauka, Novosibisk, 1987, [in Russian].
[17] Stevanovic V.D., Prica S., Maslovaric B., Zivkovic B., Nikodijevic S.: Efficient Numerical Method for District Heating System Hydraulics. Energy Conversion & Management, Vol. 48, 2007.
[18] Boulos P.F, Wood D.J.: Explicit Calculation of Pipe Network Parameters. J. Hydraulic Eng., Vol. 116, ASCE, 1990a.
[19] Fedorov M.: Automation of Mathematical Model Construction for the Analysis of Heat Regimes of Heat Exchanger Systems by the Method of Gauss Convolution. Electronic modeling, Vol.22, 6, 2000, [in Russian] (Engineering Simulation Vol.18, 2001, [English translation])
[20] Fedorov M.: Steady-State Simulation of Heat Exchanger Networks. Electronic modeling, Vol.24, 1, 2002, [in Russian].
[21] Filho L.O.F., Queiroz E.M., Costa A.L.H.: A Matrix Approach for Steady-State Simulation of Heat Exchanger Networks. Applied Thermal Eng, 27, 2007.
[22] Fedorov M.: Thermal Modes Simulation of Heat Exchanger Networks Having Turbo-machines. Proceedings of the Int. Conf. On Marine Technology IV, WIT Press, Southampton, 2001.
[23] http://openmp.org. OpenMP Standard.
[24] Altschul A.D.: Hydraulic Resistances. Nedra, Moscow, 1970, [in Russian].
[25] Kanevets G.E.: Heat Exchangers and Heat Exchanger Systems. Naukowa Dumka, Kiev, 1982, [in Russian].
[26] Bialecki R.A., Kruczek T.: Frictional Diathermal Flow of Steam in a Pipeline. Chem. Eng. Sc., Vol.51, 19, 1996.
[27] Nocedal J., Wright S.J.: Numerical Optimization. Springer-Verlag, New York Berlin Heidelberg Hong Kong London Milan Paris Tokyo, 1999.
[28] Kays, W. M., London, A. L.: Compact Heat Exchangers, 3rd ed., McGraw-Hill, New York, 1984.

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