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Computational analysis of statics and dynamics of complex structures by finite element method

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Abstract

A finite element model of complex structures is formulated in the paper. Equations of motion are presented. The material and geometrical input data of a suspended bridge, modeled as a cable-beam-shell system, is described and the impact-type loading is assumed. Structural and computational aspects of the system at hand are discussed in detail via illustrating numerical results.

Keywords: Complex structure, dynamics, Rayleigh's damping, finite element.

Analiza komputerowa statyki i dynamiki konstrukcji złożonych metodą elementów skończonych

Streszczenie

W pracy sformułowano model elementów skończonych dla konstrukcji złożonych. Zaprezentowano układ równań ruchu. Opisano materiałowe i geometryczne dane wejściowe dotyczące przykładowego mostu podwieszonego, zamodelowanego jako układ kablowo–belkowo–powłokowy. Przyjęto obciążenie dynamiczne typu nagłego uderzenia. Aspekty konstrukcyjne oraz komputerowe rozpatrywanego układu przedyskutowano szczegółowo przez ilustrujące wyniki liczbowe.

Słowa kluczowe: Konstrukcja złożona, dynamika, tłumienie Rayleigha, element skończony.

1. Introduction

Civil engineering is presently one of the fastest developing areas with respect to modern forms being created as well as materials. An aspiration for obtaining slenderer and more amazing shapes crossing the barriers of the height and span of structural members is observed in architecture for some time nowadays. Bridges are the frontier in this matter, especially suspended ones. The view of thinner and more prone for displacements plates suspended on a set of lines delights and at the same time arouses anxiety for safety. However despite the weak looks, they satisfy strict rules, concerning designing as well as the materials used, that guarantee solid built.

One of those structures that can be designed in Poland in the future may turn out to be similar to the Seri Wawasan Bridge in Putrajaya, Malaysia, with the span of 240 m, suspended on steel cables, the looks resembling a sailing boat. The view of 85-meter pylon supported by monumental arches delights and moves. The object became inspiration for this work, especially the way the finite element method (FEM) can be used to compute state-of-the-art architectural forms.

It is known that FEM is the most effective way of calculation constituting the basis of majority of modern computer codes used for structures. The aim of the text is creating an a FEM model of the object mentioned above and its static and dynamic analysis. It can be treated thus an attempt to 'redesign' such a bridge according to recent structural codes used in Poland.

In the age of modern, unstable times, some terroristic threat is not by all means unreal. Therefore, in this work we defined a dynamic load as a direct hit with a significant intensity, imitating an attempt to destroy the structure. Making the analysis we found the most strained elements and present their vibration caused by the given impulse.

2. Equations of motions in the FEM context

The system is first discretized by a mesh of finite elements. Nodal displacements are determined and internal forces are then calculated. Let us assume $\bar{u}=\{u_1,u_2,u_3\}$ be the displacement vector at any point inside a finite element while $\bar{u}^* = \{u_1^*,u_2^*,u_3^*,...,u_k^*\}$ the vector of nodal displacements, described in the element's local coordinate system as, cf. [1]

$$\bar{u}(\bar{x}) = \widetilde{H}(\bar{x})\bar{u}^*(t), \tag{1}$$

where $\widetilde{H}(\bar{x})$ is the shape function matrix. For the three-dimensional (3D) beam element we have $\bar{u}=\{u,v,\varphi\}$ and $\bar{u}^*=\{u_i,v_i,\varphi_i, u_j,v_j,\varphi_j\}$. The strain vector can be expressed as

$$\bar{\varepsilon}(\bar{x}) = \widetilde{B}(\bar{x})\bar{u}^*(t), \tag{2}$$

where

$$\widetilde{B}(\bar{x}) = \frac{d\widetilde{H}(\bar{x})}{d\bar{x}}. \tag{3}$$

Using the generalized Hooke's law the stress vector can be written in terms of the strain vector via the constitutive relationship in the form

$$\bar{\sigma} = \widetilde{C}\bar{\varepsilon} = \widetilde{C}\widetilde{B}\bar{u}^* \tag{4}$$

$\widetilde{C}$ being the constitutive matrix which regulates dependence between stress and strain.

We transform $\bar{u}(\bar{x})$ into the global coordinate system by

$$\bar{u} = \widetilde{T}\bar{q}, \tag{5}$$

where $\widetilde{T}$ is the transformation matrix, including directional cosine entries and $\bar{q}$ is the nodal displacement vector in the global coordinate system.

In order to formulate equations of motion we use the expression for total energy of a system. We apply the Lagrange's equation of the second type in the form, cf. [2]

$$\frac{\partial L}{\partial \bar{q}} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\bar{q}}}\right) = 0, \tag{6}$$

where $L = E_k - E_p + W$ is total energy of the system, with E_k and E_p being kinetic and potential energy, respectively, and W external force work.

$$E_p = \int_V \frac{1}{2}\bar{\sigma}^T \bar{\varepsilon} dV = \frac{1}{2}\bar{u}^{*T}\ \tilde{K}_L \bar{u}^*. \tag{7}$$

The symbol $\tilde{K}_L$ denotes the stiffness matrix of the considered element in the local coordinate system. We have to transpose it to the global coordinate system to get

$$E_p = \frac{1}{2}\bar{q}^T \tilde{K}_e \bar{q}. \tag{8}$$

The global stiffness matrix of the element can now be expressed as

$$\tilde{K}_e = \tilde{T}^T \tilde{K}_L \tilde{T}. \tag{9}$$

The system global stiffness matrix of the whole structure yields

$$\tilde{K}_G = \sum \tilde{K}_e. \tag{10}$$

Clearly, after imposing boundary conditions matrix $\tilde{K}_G$ is symmetric and positive definite. Its dimension is equal to the number of system degrees of freedom (DOF), denote by N, being number of components entering the vector $\bar{q}_G$.

Further, kinetic energy reads

$$E_k = \int_V \frac{1}{2}\rho \dot{\bar{u}}^T \dot{\bar{u}} dV = \frac{1}{2}\dot{\bar{u}}^T \tilde{M}_L \dot{\bar{u}}^*, \tag{11}$$

where ρ is the mass density and $\tilde{M}_L$ is the mass matrix in the local coordinate system, expressed by the equation

$$\tilde{M}_L = \int_V \rho \tilde{H}^T \tilde{H} dV. \tag{12}$$

That implies kinetic energy in the global coordinates

$$E_k = \dot{\bar{q}}^T \tilde{M}_e \dot{\bar{q}}, \tag{13}$$

where

$$\tilde{M}_e = \tilde{T}^T \tilde{M}_L \tilde{T}. \tag{14}$$

Assuming that $\bar{F}$ is the vector of external forces acting at the nodal points we write external work in the form

$$W = \int_V \bar{u}^T \bar{F} dV = \bar{u}^{*T} \bar{Q}_L, \tag{15}$$

where $\bar{Q}_L$ is the nodal loading vector in the local coordinate system, i.e.

$$\bar{Q}_L = \int_V \bar{H}^T \bar{F} dV. \tag{16}$$

Rewriting external force work into global coordinate system we have

$$W = \bar{q}^T \bar{Q}_e, \tag{17}$$

where

$$\bar{Q}_e = \tilde{T}^T \tilde{Q}_L. \tag{18}$$

Total energy in the matrix form can be express as

$$L = \frac{1}{2}\dot{\bar{u}}^T \tilde{M}_L \dot{\bar{u}} - \frac{1}{2}\bar{u}^{*T}\ \tilde{K}_L \bar{u}^* + \tilde{T}^T \bar{Q}_L. \tag{19}$$

Taking advantage of the Lagrange's equation of the second type we note that

$$-\tilde{K}_L \bar{u}^* + \bar{Q}_L - \tilde{M}_L \ddot{\bar{u}}^* = 0. \tag{20}$$

Rewriting energy to the global coordinate system we get equations of motion in the FEM context as

$$\tilde{M}_e \ddot{\bar{q}} + \tilde{K}_e \bar{q} = \bar{Q}_e. \tag{21}$$

When damping effects are taken into account we have

$$\tilde{M}\ddot{\bar{q}} + \tilde{C}\dot{\bar{q}} + \tilde{K}\bar{q} = \bar{Q}(t). \tag{22}$$

In many cases it is convenient to consider the damping terms as a linear combination of the mass and stiffness terms, so that the Rayleigh's damping matrix can be assumed as

$$\tilde{C} = \alpha \tilde{M} + \beta \tilde{K}, \tag{23}$$

where α and β are the coefficients, obtained in an experimental way.

If the inertial and damping effects are neglected, the first two terms on the left-hand side of Eq. (22) vanish, leading to

$$\tilde{K}_G \bar{q}_G = \bar{Q}_G, \tag{24}$$

where $\bar{Q}_G$ now becomes time-independent. This way we obtain the set of static equilibrium equations in matrix recording.

Recalling Eq. (23) it is pointed out here that in the framework of the superposition method [3] the modal damping coefficients can conveniently be computed. To this end we first note that the coupled equations of motion (22) can be transformed into the base of the normal (modal) coordinates y_n, $n = 1, 2, \ldots, N$, as

$$\bar{q} = \tilde{\Phi}\bar{y}, \tag{25}$$

where $\bar{y} = \{y_1, y_2, \ldots, y_N\}$, $\tilde{\Phi} = \left[\bar{\Phi}_1, \bar{\Phi}_2, \ldots, \bar{\Phi}_N\right]$ is the eigenvector matrix, solved for by the generalized eigenproblem, cf. [3]

$$\left(\tilde{K} - \tilde{\Omega}\tilde{M}\right)\tilde{\Phi} = 0, \tag{26}$$

with $\tilde{\Omega}$ being a diagonal matrix, whose entries are the system natural frequencies squared, $\tilde{\Omega} = [\omega_1^2, \omega_2^2, \ldots, \omega_N^2]$. Taking on account the mass orthonormality and stiffness orthogonality, expressed respectively as

$$\tilde{\Phi}^T \tilde{M}\tilde{\Phi} = \mathrm{I}, \quad \tilde{\Phi}^T \tilde{K}\tilde{\Phi} = \tilde{\Omega}, \tag{27}$$

we arrive at the uncoupled system of the form

$$\ddot{y}_n + 2\lambda_n \omega_n \dot{y}_n + \omega_n^2 y_n = \bar{\Phi}_n^T \bar{Q}, \quad n = 1, 2, \ldots, N. \tag{28}$$

The n-th modal damping factor λ_n is determined from, cf. Eq. (23)

$$C_n = \bar{\Phi}_n^T \tilde{C} \bar{\Phi}_n = \alpha + \beta\omega_n^2 \tag{29}$$

and, by assumption, from

$$C_n = 2\lambda_n \omega_n , \tag{30}$$

implying

$$\lambda_n = \frac{1}{2}\left(\frac{\alpha}{\omega_n} + \beta\omega_n\right), \tag{31}$$

or, generally

$$\lambda = \frac{1}{2}\left(\frac{\alpha}{\omega} + \beta\omega\right). \tag{32}$$

Clearly, if

$$\omega = \sqrt{\frac{\alpha}{\omega}} , \tag{33}$$

then λ tends to minimum. This implies

$$\alpha = \lambda\omega \ , \ \ \beta = \frac{\lambda}{\omega} \tag{34}$$

and

$$\lambda_n = \frac{\lambda}{2}\left(\frac{T_n}{T} + \frac{T}{T_n}\right). \tag{35}$$

With

$$T = \frac{2\pi}{\omega}, \ \ T_n = \frac{2\pi}{\omega_n}. \tag{36}$$

The damping factor λ will be on input applied in numerical analysis below, in Section 4.

3. Structure description – FEM model

The FEM model is inspirited by the most characteristic bridge in Malaysia, in our opinion. It is asymmetric cable-stayed bridge with inverted-Y pylon height 85 m. Its look resembles a sail ship, Fig. 1.

Fig. 1. Suspended bridge
Rys. 1. Most podwieszony

The total length of the bridge is 240 m with the main span 168.5 m. The employed materials are: pylon — reinforced concrete, cables — steel. The number of cables is 102. The deck width is 37.2 m.

As mentioned before, the goal of this work is thus, in a computationally experimental way by the finite element setting, an attempt to 'redesign' the structural model according to the existing Polish rules of civil engineering. The main input data are so assumed as follows: height of the non-forked pylon — 100 m, length of main span — 160 m, total length of the bridge — 220 m, total width of deck is 40 m, including lanes —6x5,0 m, pavements — 2x2,5 m and median part — 5 m. The main span hangs on 62 symmetrical cables connecting a pylon and a plate. The Pylon is supported by back-cables and steel arches. Between the arches and pylon are designed additional steel ties with cross-sectional areas significantly smaller than those of the main cables.

The whole structure is modelled by three 3D element types — truss, beam and shell. Clearly, all the cables are considered as truss elements and only axial forces can be found in them. The pylon, arches and ribs are divided into beam elements. The deck is split into shell elements. The number of particular elements are: trusses — 154, beams — 675 and shells — 510. The total number of system degrees of freedom is equal 3536. Regarding the numerically experimental character of the paper we consider the structure as if it were located in Szczecin, Poland. The structural scheme is shown in Fig. 2, cf. [5]

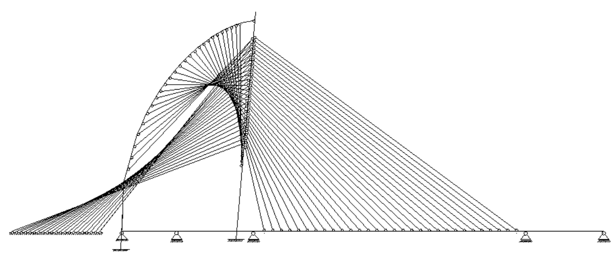


Fig. 2. Structural scheme
Rys. 2. Schemat obliczeniowy

In order to simplify the FEM modelling, creating a forked pylon in the form of letter 'Y' was relinquished and a caisson cross-section, whose size are as in Fig. 3(a) was adopted.

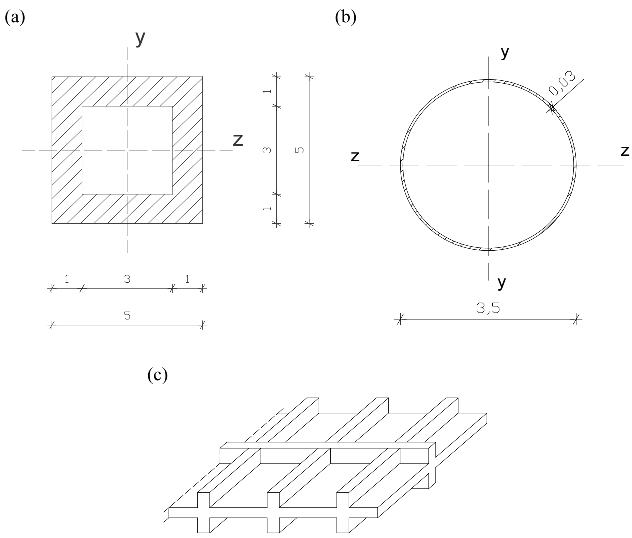


Fig. 3. (a) Pylon's cross-section, (b) Arch's cross-section, (c) Plate model
Rys. 3. (a) Przekrój poprzeczny pylonu, (b) Przekrój poprzeczny łuku, (c) Model płyty

The main span is designed as a plate from special composite material ($\boldsymbol{E}$=10^4 kN/cm^2 and ν=0,25) strengthened with longitudinal and crosswise ribs from steel. Ribs are attached to the bottom of the plate. However, adopting such a scheme in FEM would require complicated and time-consuming calculations, which were avoided by using a simplified model as given in Fig. 3(c).

Assumed plate's thickness of cross-section d = 30 cm. For reducing weight of span we decided to stiffen the plate by using steel against reinforced concrete for ribs. The computer code used to calculate the chosen structure by FEM allows to model the arch as a shell with variable lengthwise cross-sections. For simplification we accept the constant lengthwise pipe cross-section, Fig. 3(b).

In the calculation we assumed the ropes are assumed in the form of seven galvanized wires Φ5 mm which are in the HDPE coat, because they are the most widespread. The following material data are adopted, [4]: yield stress σ_Y = 1670 MPa, breaking strength R_{pk} = 1870 MPa and Young's modulus E_c = 200 GPa. The cross-section of main ropes A_c = 63,02 cm^2 the ties connecting pylon with arches are less strenuous and they have secondary function so we take for them cross-section in the form of single bar from high-performance steel with diameter equal to 36 mm A_{c2} =10,18 cm^2. Accepted Young's modulus for these cables is 210 GPa, in accordance with [4].

4. Numerical results

In static analysis we focused rather on considering the way particular structural members cooperate than on creating the basis for designing. In the calculation we adopted then, for the sake of simplification, only the loads below at their maximum values:

- constant surface load – according to Polish rules 82/B-020010 and 85/S-10030 the system of roadway layers g_{1d} = 14,865 kN/m^2 and system of pavement layers g_{2d} = 19,26 kN/m^2 ,
- constant linear load: kerbstone – 0,891 kN/m, cornice – 4,331 kN/m,
- moving load of road bridge objects - on the basis of Table 5, of Polish rule 85/S-10030 we established: class of object as A, permissible weight of vehicle – 500 kN, load of cars' train – q = 4,0 kN/m^2, K = 800 kN, pressure on axle 200 kN,
- crowd load q_{td} = 3,25 kN/m^2,
- computational value of suction effect of wind on the plate of span with perpendicular horizontal flow of the air – w_{1d} = 0,65 kPa, computational value of horizontal pressure of load on the cross-section of the main girder – w_{2d} = 1,625 kPa, static wind load on the pylon.

Because of the size of the structure the cables' length reach of 150 m, which makes them work like springs and not fulfill their carrying function. Initial tensions are required in order to make them carry load experimentally. This significantly decreases the deflections of suspended part of the span. Simultaneously, it increased the deflection of the adjoining plate rested on the supports. The results of internal forces and nodal displacements in chosen elements are presented in Tables 1 and 2.

Tab. 1. Displacements at the representative nodal points of the pylon
Tab. 1. Przemieszczenia wybranych punktów węzłowych pylonu

Nodal point	X Translation [cm]	Y Translation [cm]	Z Translation [cm]	XX Rotation [rad]	YY Rotation [rad]	ZZ Rotation [rad]
635	9,41E-01	-9,11E-13	-4,45E-01	1,85E-16	3,23E-05	4,78E-16
610	7,06E-01	-4,39E-13	-3,89E-01	1,56E-16	1,01E-04	3,29E-16
580	2,97E-01	-9,37E-14	-2,32E-01	7,47E-17	1,23E-04	1,50E-16

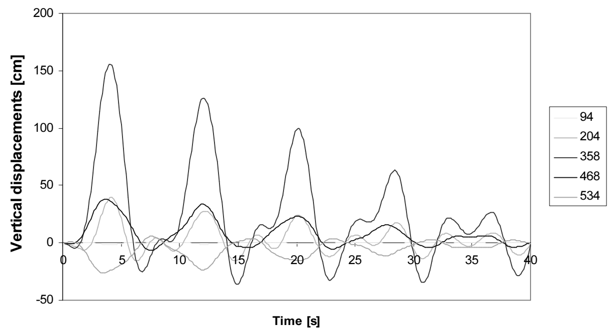
Tab. 2. Displacements of the representative nodal points of the plate
Tab. 2. Przemieszczenia wybranych punktów węzłowych płyty

Nodal point	X Translation [cm]	Y Translation [cm]	Z Translation [cm]	XX Rotation [rad]	YY Rotation [rad]	ZZ Rotation [rad]
309	1,75E-02	4,47E-04	-8,99E-01	-3,45E-02	-8,05E-04	0
310	1,74E-02	3,59E-04	-9,56E+00	-3,41E-02	-8,49E-04	0
311	1,73E-02	2,39E-04	-2,50E+01	-2,78E-02	-6,45E-04	0
312	1,72E-02	1,42E-04	-3,69E+01	-1,87E-02	-3,81E-04	0
313	1,72E-02	4,75E-05	-4,29E+01	-5,26E-02	-3,12E-04	0
314	1,72E-02	-1,19E-15	-4,35E+01	-1,09E-16	-3,21E-04	0
315	1,72E-02	-4,75E-05	-4,29E+01	5,26E-02	-3,12E-04	0
316	1,72E-02	-1,42E-04	-3,69E+01	1,87E-02	-3,81E-04	0
317	1,73E-02	-2,39E-04	-2,50E+01	2,78E-02	-6,45E-04	0
318	1,74E-02	-3,59E-04	-9,56E+00	3,41E-02	-8,49E-04	0
319	1,75E-02	-4,47E-04	-8,99E-01	3,45E-02	-8,05E-04	0

In dynamic analysis we consider the case of a sudden hit of a significant amount on to the top of the pylon. The eigenproblem is solved for the first 12 eigenpairs and converged after 11 iterations. Dynamic analysis is carried out by the mode superposition method with 2000 time steps Δt with 0,02 s, each. The assumed damping factor is λ=0,01. We apply the impulse 10^5 kN at time zero and decreases to 1000 kN after 40 seconds.

What was foreseeable that the influence of the larger vertical vibrations occur in the middle of the suspended part of the plate, the vibrations the others are smaller. The largest vibrations in the X-axis direction are at the point number 635, on the top of the pylon. The nearer the basis the smaller the displacements are. The obtained results are described in Figs. 4.

(a)



(b)

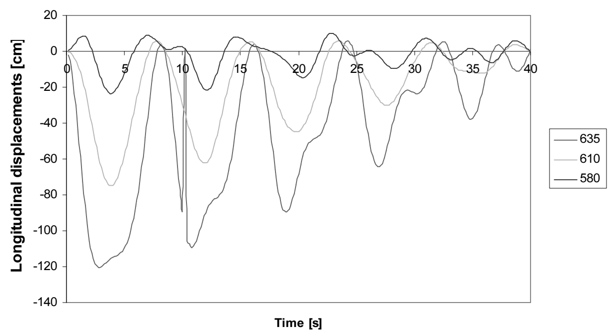


Fig. 4. (a) Displacements in the plate, (b) Displacements in the pylon
Rys. 4. (a) Przemieszczenia na płycie, (b) Przemieszczenia na pylonie

5. Concluding remarks and future work

- Dynamic effects cannot be ignored in designing such modern slender structures. Some options of dynamic analysis should necessarily be given in design rules.
- The maximum value of deflation of the plate is about 44 cm. It may be too large and not satisfy the requirements of Polish rules. Further work at this model would be advisable, considering the way to overcome this through other material solutions for the plate.
- It should be necessary to consider efficient ways for the initial compression of the cables in order to simultaneously determine the value of the tensions on their cross-sections.
- It should be recommended to consider modern, high-quality materials for particular structural members and as well as possible changes in the constructions of structural models.
- This text serves as the starting point for further work in the forthcoming paper.

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