Henryk LASKOWSKI, Leszek MIKULSKI
POLITECHNIKA KRAKOWSKA, INSTYTUT MECHANIKI BUDOWLI

Control Theory in Composite Structure Optimizing

Dr inż. Henryk LASKOWSKI

Assistant professor in the Chair of the Fundamentals of Continuum Mechanics of the Civil Engineering Faculty in Cracow University of Technology. Author of 10 publications in the range of theories and uses of the optimal control in the tasks of the strength optimization of the constructions.

e-mail: henryklaskowski@poczta.onet.pl

Prof. dr hab. inż. Leszek MIKULSKI

Head of the Chair of the Fundamentals of Continuum Mechanics of the Civil Engineering Faculty in Cracow University of Technology. Research and scientific interests concern optimal shaping of the constructions and systems problems, author of 50 publications in this range.

e-mail: mikul@optra.wil.pk.du.pl

Streszczenie

The paper deals with applying the optimal control method in design of a composite girder subjected to constant and varying loads. The maximum principle was applied to optimal shaping of the composite structures. The multipoint boundary problem was formulated using the maximum principle. Optimization concerns cross section shaping for different cost functions with constraints resulting from technical rules and standards.

Słowa kluczowe: optimization, optimal control, maximum principle, optimal modeling.

Teoria sterowania w optymalizacji konstrukcji

Abstract

W artykule przedstawiono oryginalną metodę obliczeń konstrukcyjnych opartą na zasadzie maksimum. Zasada maksimum pozwala sformułować warunki konieczne optymalizacji i sprowadzić problem optymalnego kształtowania do wielopunktowego problemu brzegowego, który następnie może być rozwiązany numerycznie. Tę metodę zastosowano w obliczeniach konstrukcyjnych stalowo-betonowego dźwigara zespolonego poddanego działaniu złożonych układów obciążeń stałych i zmiennych z uwzględnieniem stanów montażowych. Metoda umożliwia przyjęcie różnych funkcji celu oraz złożonych ograniczeń wynikających z przepisów technicznych i norm.

Keywords: optymalizacja, optymalne sterowanie, zasada maksimum, optymalne kształtowanie.

1. Introduction

The optimal control theory, and particularly one of its methods called the minimum principle, is being applied in optimizing structural elements and structures. Formulating form optimizing problems consisting in the category of the control theory requires establishing a mathematical model, in which the variables of state and control, the constraints and the objective function are defined. The minimum principle is being successfully applied in optimizing building structures with boundary conditions and internal point conditions. That principle makes it possible to set up the necessary optimization conditions, which essentially are multi-point boundary conditions for a system of ordinary differential equations, or generally speaking - are a differential-algebraic constraints problem.

Correct formulation of optimum shaping problems is particularly important, i.e. selecting the objective function, the deciding control variables and the necessary constraints. Properly assuming these magnitudes enables exhibiting the advantages resulting from optimization. Taking into account a greater number of variables makes it considerably more difficult to achieve the optimum solution. The problems of optimum forming in civil engineering are characterized by many controls and multiple limitations. It is important to assume such a structure of control, which is appropriate for the particular problem, i.e. the sequence in which the controls determined by various conditions come up.

The numerical solution is being achieved by intermediate methods, i.e. the multipoint boundary problem of canonical differential equations is solved numerically for the optimal control structure. The scientific software Dircol-2.1 makes it possible to effectively solve numerically formulated optimizing problems [1, 10].

2. General formulation of the problem

This paper analyzes the problem of optimum control of the Mayer type. Sought is the control variable u, which is the result of the following problem:

$$\begin{aligned} &\min \varphi(y(l)), \quad \varphi : R^{n_y} \to R \\ &y' = f(y,u), \quad f: R^{n_y} \times R^{n_u} \to R^{n_y} \\ &S(y) \ge 0\,, \quad S: R^{n_y} \to R^{n_s} \\ &g(y,u) \ge 0, \quad g: R^{n_y} \times R^{n_u} \to R^{n_g} \\ &\Psi(y(0), y(l), l) = 0\,, \quad \Psi: R^{n_y} \times R^{n_y} \times R \to R_\Psi \end{aligned} \tag{1}$$

with the functions of state $y:[0,l] \to R^{n_y}$ and the control $u:[0,l] \to R^{n_u}$ In some cases instead of the Mayer-type function as the objective function, the Boltz-type function is being considered [9].

$$\min\left\{\varphi(y(l)) + \int_0^l L(y(x),u(x))dx\right\}, \quad \varphi: R^{n_y} \to R, \; L: R^{n_y} \times R^{n_u} \to R \tag{2}$$

Generalization of the problem (1), which is considered in this paper, occurs in cases of discontinuity of state variables at the internal points x_s, of the sectional defined right-hand sides of differential equations f, and of problems clearly dependent on the independent variable x.

We assume that the gradients $\frac{\partial g_i}{\partial u}$ of all active constraints $g_i \ge 0,\, i = 1, ..., n_g$ are linearly independent. The initial and terminal boundary conditions have generally the form $y(0) - y_0 = 0$, $\Psi(y(l), l) = 0$.

The necessary optimality conditions are set up for a representative control structure with the boundary ranges x_w, x_z and the contact point x_s. Setting up the necessary optimality conditions as a multi-point boundary problem, the numerical solution of which is possible, we assume the existence of the boundary range x_w, x_z, in which the constraint $g(y, u) = 0$ is active. The range of admissible controls $\Omega(y)$ is defined by

$$\Omega(y) = \left\{u \in R^{n_u} \mid g(y,u) \ge 0\right\} \tag{3}$$

The algebraic-differential boundary problem resulting on the base of the minimum principle has the general form:

$$
\begin{aligned}
y' = f(y,u)\,, \quad -\lambda' = H_y \\
0 = \lambda^T \cdot f_u(y,u) + \mu^T \cdot g_u(y,u) \\
0 = g_i(y,u)\,, \quad i \in [x_w, x_z] \\
0 = \mu_i\,, \quad i \notin [x_w, x_z] \\
H = \lambda^T \cdot f(y,u) + \mu^T \cdot g(y,u) \\
\lambda^T(x_s^+) = \lambda^T(x_s^-) - v_k S_y(y(x_s))\,, \quad v_k \geq 0
\end{aligned} \tag{4}
$$

with the natural boundary conditions

$$
\begin{aligned}
\lambda^T(0) = -\frac{\partial \Phi}{\partial y(0)}\,, \quad \lambda^T(l) = -\frac{\partial \Phi}{\partial y(l)} \\
0 = -\frac{\partial \Phi}{\partial x} + H(l) \\
\Phi(y(0), y(l), l) = \varphi(l) + \alpha^T \Psi(y(0), y(l), l)
\end{aligned} \tag{5}
$$

The system of differential equations (4) has a special structure in the scalar control function $n_u = 1$. The non-linear state equations depend only on the state y of the system, and the conjugate equations are linear as control depends only on the state of the system. The numerical solution of the problem can be achieved using the software Dircol-2.1 [10].

3. Computational example – optimization of a composite girder

3.1. Description of the girder

The girder to be optimized consists of the two main elements of a triple span road bridge with spans of 60 – 90 – 60 m length. The steel part of the bridge consists of an I-section plate girder. The dimensions of the bottom flange and the web thickness are varying and shall be determined by optimization. The upper flange, having a fixed width, is rectilinear while the bottom flange has a predefined parabolic profile. The girder is a continuous beam (Fig. 1). It is a plate girder combined with the reinforced concrete slab platform, interconnected at the bearings. The cross section of the composite girder, its main dimensions and designations are shown in Fig. 2 [2].

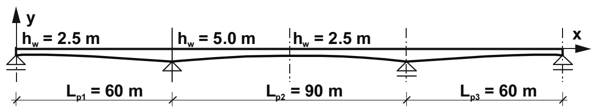
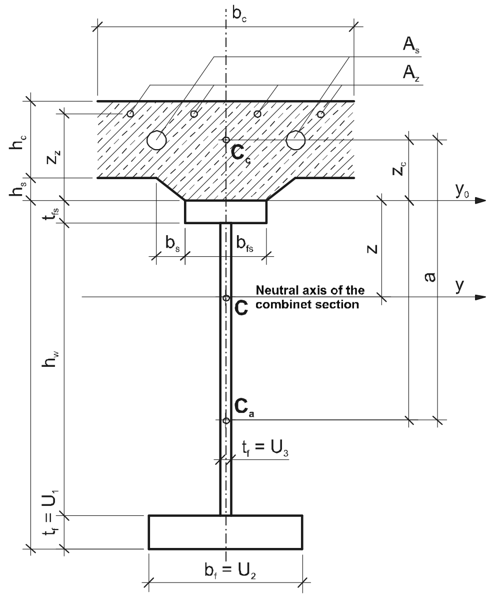


Fig. 1. Diagram of the girder
Rys. 1. Schemat dźwigara

The cross sections determined in the course of optimization shall satisfy the ultimate state criteria of load capacity and serviceability according to mandatory standards. These states shall not be exceeded neither during assembly, nor during the service life of the object. It is therefore essential to anticipate and take into account in the optimization scheme all critical design situations concerning the ultimate states of load capacity and the serviceability limit states, which can occur from the time of starting construction to the end of the service life. The optimization problem shall therefore be preceded by an analysis of the assembly state and the possible loading combinations during the usage of the object.



Fixed dimensions of the cross section

b_{fs}= 0.400 m
t_{fs} = 0.020 m
b_s = 0.120 m
h_s = 0.060 m
b_c = 4.000 m
h_c = 0.250 m
z_z = 0.250 m

Web height:

h_{min} = 2.500 m
h_{max} = 5.000 m

Adoptable variables:

u_1 – Bottom flange thickness
u_2 – Bottom flange width
u_3 – Web thickness

Fig. 2. Cross section of the girder and designations
Rys. 2. Przekrój poprzeczny dźwigara - oznaczenia

3.2. Assembly phases

The condition of the system in the phases depends on two factors, namely the structural system and the load pattern. During assembly, the service phases are determined mainly by considering the structural system, while during the service period only the load pattern is decisive.

Phase 1.
The bearing sections of the steel girder resting on the fixed bearings and the erection bearing (Fig. 3) are assembled during the first phase. Concreting of the deck slab on the assembled parts is also carried out in the course of that phase. Is was assumed that both the geometry and the loading of the object are symmetric during the assembly phases, therefore the half system was considered in the analysis (Fig. 4). In the further parts of this paper the remaining assembly phases are described in a generalized manner only.

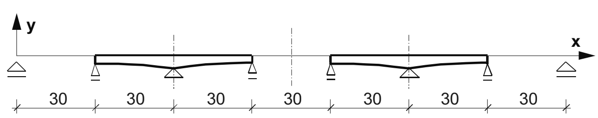


Fig. 3. Schematic diagram of the first assembly phase
Rys. 3. Schemat ideowy montażu w pierwszej fazie montażowej

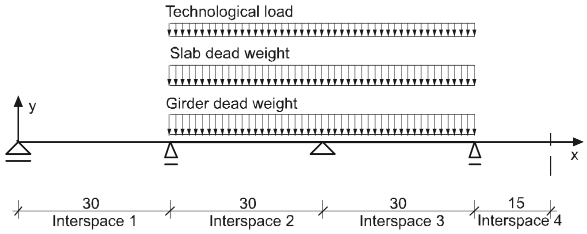


Fig. 4. Load pattern of the girder during the first assembly phase
Rys. 4. Schemat dźwigara w pierwszej fazie montażowej

Phase 2.
Tensioning the deck slab by raising the erection bearings at points 2 and 4.

Phase 3.
Tensioning the bearing sections of the deck slab by means of stressing cables.

Phase 4.
Assembling the steel girder and concreting the deck slab on interspaces 1 and 4.

Phase 5.
Assembling the outfit and bridge flooring.

Appropriate additional load systems resulting from construction work are considered in all assembly phases. The geometrical characteristics of the cross section varying along the girder axis were also considered, taking into account the method of assembly and the rheological phenomena occurring in concrete.

3.3. Service phases

Standard specification loads and their most unfavorable combinations were considered for the service life phases. The following service phases regarding loading were assumed:
- Phase of loading by trucks q and a crowd of pedestrians p - 4 load combinations.
- Phase of loading by a vehicle K - 7 load combinations to determine the influence line of sectional reactions.
- Phase of creeping, in which the redistribution of sectional reactions due to creeping in the service life was considered.
- Phase of concrete contraction in the service life.
- Phase of thermal loads – 2 extreme design situations, in which the steel girder and the concrete slab reach the maximum and minimum temperatures according to the standard code.

The half-section load pattern was assumed in all considered design situations, with the corresponding bearing or corresponding conditions of the state variables at the symmetry axis of the object.

3.4. Equations of state

The paper describes only the general methods of formulating the equations of state applied in the presented example for the types of loading present in the various service phases. The equations of state are formulated separately for each characteristic interspace. Four characteristic interspaces were assumed, determined by the axis of symmetry and the position of the fixed bearings and the assembly bearings (Fig. 4).

Distributed load
The basic system of differential equations of a beam with distributed load has the form (6):

$$y' = \varphi, \quad \varphi' = \frac{M}{EI}, \quad M' = Q, \quad Q' = -q \qquad (6)$$

where: y – deflection, φ - deflection angle, M – bending moment, Q – transverse force, q distributed load.

The point conditions of the state variables are to be formulated for these equations, the number of variables being equal to the product of the number of equations and the number of characteristic interspaces.

Concentrated loads
When the concentrated loads occur within the limits of characteristic interspaces, then the beam is to be described using the basic system of equations (6), the value of the concentrated loads being accounted for in the conditions of state variables. If, in the assumed model of concentrated loads, the loads occur within the characteristic interspaces, then the basic system of equations is reduced to the first three equations of the system (6), in which the transverse force Q is expressed analytically. These equations are supplemented by an adequate number of point conditions.

Geometrical loads
In the analysis phase considering the geometrical loads, the beam can be described by the basic system of equations (6), taking into account also the distributed loads, if they occur at that phase. The geometrical loads are considered in the form of determined dislocations of bearing points.

Slab pretensioning
The equations describing slab pretensioning are the two first equations of the basic system of equations. The bending moment is expressed analytically, including the unknown bearing reactions and the tensioning force. The tensioning force function can be determined applying algorithms used in the design of pre-stressed structures. The unknown bearing reactions are considered in the optimization problem as control parameters, which are then determined by formulating additional point conditions in the form of state variable points and global equilibrium conditions.

Concrete shrinkage
In that case, as in the case of pretensioning, formulated are the first two equations of the basic system of equations, in which the bending moment is analytically expressed. The bearing reactions due to shrinkage in statically indeterminate systems are included in the optimization problem as control parameters, whereas the force caused by shrinkage is being determined assuming flat cross sections and the balance of forces.

Thermal loads
Thermal loads are accounted for by applying the same equations as in the case of concrete shrinkage, replacing the shrinkage force by the force due to the temperature difference between the steel part and the concrete one, assuming the sense of the force proper for the actual situation.

Load combinations
In cases when combinations of forces are to be considered in optimization, then the state equations shall be formulated separately for each type of loading at the step of formulating the constraints. Selection of the most disadvantageous load combination is done applying the maximum or minimum function. Such approach leads in most cases to a smaller number of state equations.

Moving loads
Moving loads can be considered in the optimization process analyzing a finite number of load settings and selecting the most disadvantageous moving load setting at the step of formulating the constraints. This method can be used in the case of small objects. In the case of multi-span bridges, a smaller number of equations is required by the method, which leads to the determination of the envelope of bending moments. That method was also used in this example. The envelope of minimum moments and of maximum moments at the supports is determined by considering $2n$-2

positions of the moving load (n = number of spans), while the envelope of maximum span moments is reached using the influence lines of support reactions determined in the optimization problem.

Concrete creeping

In order to take into account the creeping of concrete at assembly states, the method of the equivalent modulus of elasticity was applied, modifying the Young's modulus of concrete according to its age. At the service states, the Trost method was applied, which made it possible to determine the changes in the sectional reactions in the cross section in the steel and concrete parts during the service life in a statically determinate system. In the optimization problem two additional state equations were introduced, which enable determination of the coefficients in the basic equation of the force method in a single degree statically indeterminate system.

This way was used to determine 79 state equations together with point conditions and additional conditions considering the control parameters. The description of the state variables is presented in Table 1 [2].

Tab. 1. Description of the state variables
Tab. 1. Opis zmiennych stanu

		State variables in the analyzed design situations						
		Deflection	Angle of rotation	Mending moment	Transverse force	δ_{11}	δ_{1P}	V
Assembly stage I		y_1	y_2	y_3	y_4			
Assembly stage II		y_5 y_9	y_6 y_{10}	y_7	y_8			
Assembly stage III		y_{11} y_{13}	y_{12} y_{14}					
Assembly stage IV		y_{15} y_{19}	y_{16} y_{20}	y_{17}	y_{18}			
Assembly stage V		y_{21} y_{25}	y_{22} y_{26}	y_{23}	y_{24}			
Service stage with truck load q an pedestrian crowd p	q1	y_{27}	y_{28}	y_{29}	y_{30}			
	q2	y_{31}	y_{32}	y_{33}	y_{34}			
	q3	y_{35}	y_{36}	y_{37}	y_{38}			
	q4	y_{39}	y_{40}	y_{41}	y_{42}			
Service stage with truck K	K1	y_{43}	y_{44}	y_{45}	y_{46}			
	K2	y_{47}	y_{48}	y_{49}	y_{50}			
	K3	y_{51}	y_{52}	y_{53}	y_{54}			
	K4	y_{55}	y_{56}	y_{57}	y_{58}			
	K5	y_{59}	y_{60}	y_{61}	y_{62}			
	K6	y_{63}	y_{64}	y_{65}	y_{66}			
	K7	y_{67}	y_{68}	y_{69}	y_{70}			
Creeping stage						y_{71}	y_{72}	
Shrinkage stage		y_{73}	y_{74}					
Thermal load stage	T1	y_{75}	y_{76}					
	T2	y_{77}	y_{78}					
Volume								y_{79}

3.5. Formulating the optimization problem

The objective optimum solution consists in determining the cross section of the steel girder, in which the value of the objective function is minimal. The objective function selected in the presented example is the volume of steel used to make the plate part of the composite girder. Precisely this was why the state variable $y_{79} = V_a$ and the following state formula [4] was applied:

$$y_{79}' = A_a\,,\;\; y_{79}(0) = 0 \tag{7}$$

where: a – designation of the steel part of the composite girder without reinforcement.

Applying that variable made it possible to reduce the optimization problem with the general Lagrange functional to the Mayer-type problem (1), (2).

The state equations, constraints, and the optimization objective function constitute parts of the indispensable formal structure enabling the application of the maximum principle, which formulates the necessary condition of optimization.

1. The optimized girder is described by the system of first order differential equations

$$y_i' = f_i[\mathbf{y}(x),\mathbf{u}(x),x]\,,\;\; i = 1 \div 79\;\; (n_y = 79) \tag{8}$$

2. The admissible spaces and the constraints were determined and formulated for the state variables and the decision variables

$$g_s[\mathbf{y}(x),\mathbf{u}(x),x] \geq 0\,,\;\; s = 1 \div 8\;\; (n_s = 8) \tag{9}$$

3. The Hamilton function in the analyzed problem has the form

$$H = \boldsymbol{\lambda}^T \mathbf{f}[\mathbf{y}(x),\mathbf{u}(x),x] + \boldsymbol{\mu}\mathbf{g}[\mathbf{y}(x),\mathbf{u}(x)] \tag{10}$$

4. The system of conjugate variable equations has the form

$$\lambda_i' = -\sum_{k=1}^{79} \lambda_k \frac{\partial f_k(\mathbf{y},\mathbf{u})}{\partial y_i} - \sum_{s=1}^{8} \mu_s \frac{\partial g_s(\mathbf{y},\mathbf{u})}{\partial y_i}\,,\;\; i = 1 \div 79 \tag{11}$$

5. The system of equations following the condition of the Hamilton function maximum has the form

$$0 = \sum_{k=1}^{79} \lambda_k \frac{\partial f_k(\mathbf{y},\mathbf{u})}{\partial u_j} + \sum_{s=1}^{8} \mu_s \frac{\partial g_s(\mathbf{y},\mathbf{u})}{\partial u_j}\,,\;\; j = 1 \div 3\;\; (n_u = 3) \tag{12}$$

The optimal solution is found by way of the differential-algebraic system of equations (7), (9) and (10), respecting the following exceptions:

1. If a decision variable assumes values from the boundary of the defined admissible space, then the equation formulated with this variable is excluded from the system of equations (12).

2. When one of the constraints (9) is active, then the respective equation (12) is excluded from that system and is replaced by the following equation:

$$g_m[\mathbf{y}(x),\mathbf{u}(x),x] = 0\,,\;\; m - \text{ number of the active constraint} \tag{13}$$

3. The problem can be solved only when the number of cases described under item 1 and 2 does not exceed the number of decision variables.

The number of constraints in the presented problem is much higher than the number of equations of the kind (12), which increases the probability that circumstances can arise at which there is no optimal solution.

3.6. Numerical solution

Applying the maximum principle strictly, the optimization problem was reduced to a multi-point boundary problem, which was solved using the software Dircol-2.1. Along this way, the following clauses were determined:
- 79 state variables,
- 79 adjoint variables,
- 3 decision variables,
- 25 control parameters (23 bearing reactions and 2 force equation coefficients),
- 8 Lagrange multipliers related to constraints of the state variables,
- 135 constants, responsible for the jump of state variables at support points and at the symmetry axis,
- 144 constants, responsible for the jump of adjoint variables at support points and at the symmetry axis,
- 5 constants responsible for the jump of Hamilton function at support points and at the symmetry axis.

Altogether 478 magnitudes were assigned. A solution was found, which satisfies the necessary conditions of optimality (Fig. 5, 6, 7).

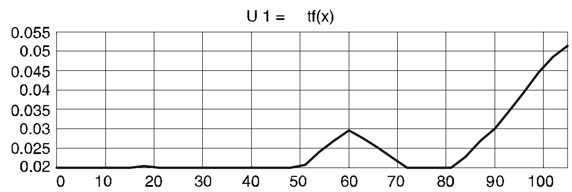


Fig. 5. Optimal thickness of the bottom flange of the plate girder – decision variable u_1 [m]
Rys. 5. Optymalna grubość dolnego pasa blachownicy – zmienna decyzyjna u_1 [m]

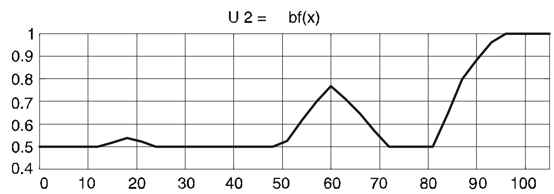


Fig. 6. Optimal width of the bottom flange of the plate girder – decision variable u_2 [m]
Rys. 6. Optymalna szerokość dolnego pasa blachownicy – zmienna decyzyjna u_2 [m]

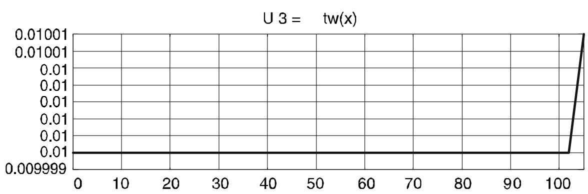


Fig. 7. Optimal thickness of the plate girder web – decision variable u_3 [m]
Rys. 7. Optymalna grubość środnika blachownicy – zmienna decyzyjna u_3 [m]

The computations carried out using the software Dircol-2.1 shall be preceded by loading the starting values of the state variables, which are automatically corrected in the subsequent steps of iteration until achieving the solution, which satisfies the necessary conditions of optimality. Often, depending on the starting values, different solutions are achieved. From these solutions those ones shall be selected, the result of which is characterized by the minimum value of the objective function. There never exists, however, sureness that no better solution is possible.

In the presented example, the same solution was always achieved when different starting values were assumed. It can therefore be supposed that considering the given constraints and adopted assumptions this is the only, i.e. the optimal solution respecting the minimum steel volume criterion.

The figures 5, 6, 7 present the elements of the cross section of the plate girder, determined by the optimizing process. Both dimensions of the bottom flange are increasing at the zone of the intermediate bearing and in the span sections. The third determined dimension – the web thickness - does not undergo any changes practically and achieves the minimum value of the admissible range of variability.

On sections where all decision variables achieve values of the limit of the admissible range (0 ÷ 12 m, 24 ÷ 51 m, 69 ÷ 84 m), all equations (12) were excluded from the optimization process.

On sections where two decision variables achieve values of the limit of the admissible range (12 ÷ 15 m, 21 ÷ 24 m, 96 ÷ 105 m), two equations of the system (12) were excluded. If on these sections one of the constraints g_s is active, then also the third equation is excluded from the optimization process, and that variable, the values of which are not situated at the limit of the admissible range, is determined by the active constraint. The diagrams of constraint functions, which are not included in this presentation of the paper, show that only the first constraint is active on some intervals. They are the intervals 15 ÷ 21 m, 51 ÷ 69 m, and 84 ÷ 105 m.

On sections where only one decision variable achieves a value of the limit of the admissible range (15 ÷ 21 m, 48 ÷ 72 m, 84 ÷ 96 m), one equation of the system (12) is excluded. One of the subsequent two equations is excluded when the first constraint is active. When only one equation is excluded from the system of equations (12), then the variables, the values of which are not situated on the limit of the admissible range, are determined by these equations. Whereas, when due to the activity of the first constraint of the system (12) the second equation is excluded, then the variables, the values of which are not situated on the limit of the admissible range, are determined by the active constraint and by one of the equations (12). It is however impossible to find out, which decision variable is affected directly by the activity of the constraint. The above dependencies in the analyzed problem are presented in form of the structure of control in Table 2.

Tab. 2. Structure of the optimal solution
Tab. 2. Struktura rozwiązania optymalnego

Przedział	u_1	u_2	u_3
0 ÷ 15	$u_{1.min}$	$u_{2.min}$	$u_{3.min}$
12 ÷ 15	$u_{1.min}$	$u_{2.opt} \Leftarrow (12.2)$	$u_{3.min}$
15 ÷ 21	$(u_{1.opt}, u_{2opt}) \Leftarrow \begin{cases} g_1 = 0 \\ (12.1) \cup (12.2) \end{cases}$		$u_{3.min}$
21 ÷ 24	$u_{1.min}$	$u_{2.opt} \Leftarrow (12.2)$	$u_{3.min}$
24 ÷ 51	$u_{1.min}$	$u_{2.min}$	$u_{3.min}$
51 ÷ 69	$(u_{1.opt}, u_{2opt}) \Leftarrow \begin{cases} g_1 = 0 \\ (12.1) \cup (12.2) \end{cases}$		$u_{3.min}$
69 ÷ 84	$u_{1.min}$	$u_{2.min}$	$u_{3.min}$
84 ÷ 96	$(u_{1.opt}, u_{2opt}) \Leftarrow \begin{cases} g_1 = 0 \\ (12.1) \cup (12.2) \end{cases}$		$u_{3.min}$
96 ÷ 105	$u_{1.opt} \Leftarrow g_1 = 0$	$u_{2.max}$	$u_{3.min}$

4. Summary

The theory of optimal control can be applied in problems of optimization of building structures. The paper presents in detail an example of optimizing a girder with composite cross section, in which various loading conditions were considered, and a complex structure of the optimum solution was proposed, in which the multipoint boundary problem is convergent. The solved practical example confirms the possibility of applying the optimal control theory in optimal structure design.

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