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Pole placement controller for linear continuous-time system with dead-time

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Abstract

Pole placement feedback controller for linear continuous-time system with dead-time is presented. Properties of the closed loop systems with the controller are discussed. Necessary and sufficient conditions for the controller existence are given. Numerical example illustrates behavior of the control system with proposed controller.

Keywords: dead-time systems, linear systems, pole placement.

Przesuwanie biegunów ciągłego układu liniowego z opóźnieniem transportowym

Streszczenie

Przedstawiono syntezę regulatora przesuwającego bieguny wielowymiarowego ciągłego układu z opóźnieniem transportowym. Przedyskutowano własności układu zamkniętego z proponowanym regulatorem. Podano warunki konieczne i dostateczne dla istnienia regulatora. Przedstawiono przykład liczbowy ilustrujący działanie układu regulacji z proponowanym regulatorem

Słowa kluczowe: układy z opóźnieniem transportowym, układy liniowe, przesuwanie biegunów.

1. Introduction

Time-delay systems one meets in biology, chemistry, economics and also in engineering. The continuous-time models of time-delay system are in the form of differential-difference equations. Their analysis is rather difficult because time-delay and derivative have quite different nature, the equation can be alternatively considered as infinitely dimensional functional differential equation. Review of most important results concerning analysis and synthesis of time-delay control systems one can find in [1].

The most important for engineers are models with transportation delays (dead-time, input delay). Unfortunately, there is no simple method for controller design for system with dead-time, one can easily check that in classical textbooks for control engineers the control of time-delay systems is omitted or reduced to PID type control, e.g. [3, 4, 5, 6, 7]. A well known approach for design of controller for dead-time system was proposed in [2]: adding to the control system so called Smith's predictor one can design controller like for system without dead-time. In this paper we present a controller for pole placement for system with dead-time described by linear continuous-time model. The proposed controller is in a form of classical state feedback, analogous to systems without dead-time.

Organization of the paper is as follows: in section 2 the considered problem is presented. In section 3 the controller design is proposed, then in section 4 control system with disturbances is also considered and in section 5 robustness of the proposed control system is discussed. Next, in section 6, numerical examples are presented. Finally, concluding remarks are given.

2. Problem formulation

Given multivariable system with dead-time (transportation delay, input delay) described by linear continuous-time model

$$\begin{aligned} \dot{x} &= Ax + Bu(t-T_0) \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$ is a state vector, $u \in R^m$ is an input vector, $y \in R^p$ is an output vector, T_0 denotes dead-time, and A, B and C are real matrices of appropriate dimensions.

Then, the considered problem can be formulated as follows: find a feedback controller for dead-time system (1) such that the closed loop system will be asymptotically stable. Moreover, it is desired that the closed loop system will have required poles.

Model (1) is basic one for control engineers. Many industrial processes can be described by this model. Therefore, synthesis of stabilizing controller for system described by the model is rather important.

3. Controller design

If the pair (C,A) is observable one can design state observer for system (1)

$$\dot{v} = Gv + Hu(t-T_0) + Ly \tag{2}$$

where $v \in R^n$ is an observer state vector, such that

$$\lim_{t\to\infty}(v - x) = 0 \tag{3}$$

Indeed, calculating observer error $e(t) = v(t) - x(t)$ one obtains

$$\dot{e} = \dot{v} - \dot{x} = Ge + (G - A + LC)x + (H - B)u(t-T_0) \tag{4}$$

Clearly, if observer matrices are calculated the following way

$$G = A - LC \quad \text{and} \quad H = B \tag{5}$$

we have

$$\dot{e} = Ge$$

Hence, it is obvious that if matrix L is chosen such that all eigenvalues of matrix G have negative real part condition (3) will be satisfied.

Next, using estimate $v(t)$ of state vector $x(t)$ one can calculate in time t estimate $x(t+T_0|t)$ of state $x(t+T_0)$ based on model (1) as follows

$$x(t+T_0|t) = e^{AT_0}v(t) + \int_t^{t+T_0} e^{A(t+T_0-\tau)}Bu(\tau - T_0)d\tau = x(t+T_0) + e^{AT_0}e(t) \tag{6}$$

One easily notes that $x(t+T_0|t) \to x(t+T_0)$ for $t \to \infty$ if observer (2) is properly designed. It is also easy to see that $x(t+T_0|t)$ is a solution of the following system

$$\dot{x}_x = Ax_x + Bu(t-T_0) \tag{7}$$

with initial condition $x_x(t)=v(t)$, in time $t+T_0$:

$$x(t+T_0|t) = x_x(t+T_0).$$

Next, applying to system (1) the following control input

$$u(t) = Kx(t+T_0\,|\,t) = K[x(t+T_0) + e^{AT_0}e(t)] \tag{8}$$

one obtains from (1) and (2)

$$\begin{aligned}\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} &= \begin{bmatrix}A & 0\\ 0 & G\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix} + \begin{bmatrix}B\\ H\end{bmatrix}K[x(t) + e^{AT_0}e(t-T_0)] + \begin{bmatrix}0\\ L\end{bmatrix}Cx\\ &= \begin{bmatrix}A+BK & 0\\ LC+HK & G\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix} + \begin{bmatrix}B\\ H\end{bmatrix}Ke^{AT_0}e(t-T_0)\end{aligned} \tag{9}$$

Then, after state transformation $T_e\begin{bmatrix}x\\ v\end{bmatrix} = \begin{bmatrix}x\\ e\end{bmatrix}$ where

$$T_e = \begin{bmatrix}I & 0\\ -I & I\end{bmatrix} \text{ and } T_e^{-1} = \begin{bmatrix}I & 0\\ I & I\end{bmatrix}$$

we obtain the following equation of the closed loop control system

$$\begin{aligned}\begin{bmatrix}\dot{x}\\ \dot{e}\end{bmatrix} &= \begin{bmatrix}A+BK & 0\\ LC+HK-A-BK+G & G\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix}Ke^{AT_0}e(t-T_0)\\ &= \begin{bmatrix}A+BK & 0\\ 0 & G\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix} + \begin{bmatrix}BKe^{AT_0}\\ 0\end{bmatrix}e(t-T_0)\end{aligned} \tag{10}$$

It is well known that if the pair (A,B) is controllable one can calculate matrix K such that all eigenvalues of matrix $A_K = A + BK$ have negative real part. In this case, we see that all eigenvalues of the closed loop system state matrix have negative real part. It is also easy to note that the observer error e tends to zero independently on system state vector x. Thus, system state x tends to zero, too. Therefore, the whole closed loop system is asymptotically stable.

By appropriate choice of matrices K and L one can choose eigenvalues λ of matrix A_K and μ of matrix G, i.e. eigenvalues of the closed loop system state matrix, eigenvalues deciding about the closed loop system properties similarly as in linear systems without time-delay – $e(t–T_0)$ in (9) and (10) can be treated as disappearing disturbance. For the reason we call the proposed control algorithm pole placement of the dead-time system, however, real eigenvalues λ_r of the closed loop system are slightly different than λ chosen for calculation of the control gain matrix K. Nevertheless, it is easy to see that if observer error tends to zero then eigenvalues λ become real system poles with respect to system behavior. It can be also noted that eigenvalues μ are real roots of the characteristic equation and they are real system poles

$$\det\left(Is - \begin{bmatrix}A+BK & 0\\ 0 & G\end{bmatrix} + \begin{bmatrix}0 & BKe^{AT_0}\\ 0 & 0\end{bmatrix}e^{-T_0 s}\right) = 0 \quad \text{for } s=\mu \tag{11}$$

Finally, one simple finds that the asymptotically stable closed loop system (9) for $t\to\infty$ tends to the following system without dead-time

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix}A+BK & 0\\ LC+HK & G\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix}$$

where eigenvalues of the state matrix are equal to λ and μ.

After one notes that

$$\int_t^{t+T_0} e^{A(t+T_0-\tau)}Bu(\tau-T_0)d\tau = \int_0^{T_0} e^{A\tau}Bu(t-\tau)d\tau$$

we can summarize presented investigation in the following theorem.

Theorem 1

If the pair (A,B) of dead-time continuous-time system (1) is controllable and the pair (C,A) is observable then the control feedback

$$u(t) = Kw(t) \tag{12}$$

where

$$w(t) = e^{AT_0}v(t) + \int_0^{T_0} e^{A\tau}Bu(t-\tau)d\tau \tag{13}$$

and

$$\dot{v} = Gv + Hu(t-T_0) + Ly \tag{14}$$

and gain matrix K is calculated in such a way that all eigenvalues λ_i, i=1,...,n, of the matrix $A_K = A + BK$ have negative real part, matrix L is calculated in such a way that all eigenvalues μ_i, i=1,...,n, of the matrix $A_L = A – LC$ have negative real part, and $G = A_L$, $H = B$, stabilizes the closed loop system (1) and (12).

Moreover, the closed-loop control system for $t \to \infty$ tends to the following one

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix}A+BK & 0\\ LC+HK & G\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix}$$

$$y = [C \quad 0]\begin{bmatrix}x\\ v\end{bmatrix}$$

which have poles in given λ_i and μ_i, i=1,...,n.

Remarks

1. It is easy to see that calculation of matrices K and L, G and H, i.e. synthesis of control gain and state observer, can be done independently.
2. The controller can be calculated similarly as in the case of the system without dead-time.
3. The controller consists of state observer (14) and state predictor (13).
4. One can calculate estimate $w(t)$ of state $x(t+T_0)$ using for instance the following rectangular approximation of integral (12)

$$w(t) = e^{AT_0}v(t) + \int_0^{T_0} e^{A\tau}Bu(t-\tau)d\tau \approx (e^{AT_p})^h v(t) + T_p\sum_{i=0}^{h-1}(e^{AT_p})^i Bu(t-iT_p)$$

where T_p is a sampling time and $h = \dfrac{T_0}{T_p}$, $h\in I$.

5. Stability of the closed loop system (1) and (12) doesn't depend on time delay T_0.

□

4. Control system with disturbances

Now consider continuous-time system (1) with unmeasurable disturbance

$$\begin{aligned}\dot{x} &= Ax + Bu(t-T_0) + Ez(t)\\ y &= Cx + Fz\end{aligned} \tag{15}$$

where $z\in R^q$ is an unmeasurable disturbance vector and E and F are real matrices of appropriate dimensions.

In this case using state observer (2) we obtain instead of (4)

$$\dot{e} = \dot{v} - \dot{x} = Ge + (G - A + LC)x + (H - B)u(t-T_0) + (LF - E)z = Ge + (LF - E)z \tag{16}$$

Obviously, if z doesn't disappear then observer error e doesn't tend to 0.

Next, calculating in time t estimate $x(t+T_0|t)$ of state $x(t+T_0)$ one has analogously to (6)

$$x(t+T_0|t)=e^{AT_0}v(t)+\int_t^{t+T_0}e^{A(t-\tau)}Bu(\tau-T_0)d\tau=x(t+T_0)-\int_t^{t+T_0}e^{A(t-\tau)}Ez(\tau)d\tau+e^{AT_0}e(t)$$
$$=x(t+T_0)-\int_0^{T_0}e^{A\tau}Ez(t+\tau)d\tau+e^{AT_0}e(t) \tag{17}$$

Now, instead of (8) we apply to system (15) the following control input

$$u(t)=Kx(t+T_0|t)=K\left(x(t+T_0)-\int_0^{T_0}e^{A\tau}Ez(t+\tau)d\tau+e^{AT_0}e(t)\right) \tag{18}$$

Then, analyzing the whole control system one obtains from (15) and (2) similarly to (9)

$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix}=\begin{bmatrix}A & 0\\ 0 & G\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix}+\begin{bmatrix}B\\ H\end{bmatrix}K\left(x(t)-\int_0^{T_0}e^{A\tau}Ez(t+\tau)d\tau+e^{AT_0}e(t-T_0)\right)+\begin{bmatrix}0\\ L\end{bmatrix}[Cx+Fz]+\begin{bmatrix}E\\ 0\end{bmatrix}z$$
$$=\begin{bmatrix}A+BK & 0\\ LC+HK & G\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix}+\begin{bmatrix}B\\ H\end{bmatrix}Ke^{AT_0}e(t-T_0)-\begin{bmatrix}B\\ H\end{bmatrix}K\int_0^{T_0}e^{A\tau}Ez(t+\tau)d\tau+\begin{bmatrix}E\\ LF\end{bmatrix}z \tag{19}$$

From the above equation it follows that characteristic equation of the system is the same as for system without disturbances. Thus, the system is asymptotically stable. However, in the case of not disappearing disturbance z neither state vector x nor observer error e doesn't tend to zero.

Thus, we can state the following theorem

Theorem 2

The asymptotically stable closed-loop continuous-time system (1) and (12) is asymptotically stable independently of disturbances acting on the system as in (15).

□

5. Robustness of the control system

Considering robustness of the closed loop control system we have assumed that state observer (G,H,L) and gain matrix K are calculated based on uncertain model (1) with (A_m,B_m,C_m,T_{0m}) = $(A+\Delta_A,B+\Delta_B,C+\Delta_C,T_0+\Delta_{T0})$, i.e. $(G,H,L)=(G_m,H_m,L_m)$ and $K=K_m$. In this case one finds observer error of system (1) as follows

$$\dot{e}_m=\dot{v}_m-\dot{x}=G_me_m+(\Delta_A-L_m\Delta_C)x+B[u(t-T_{0m})-u(t-T_0)]+\Delta_Bu(t-T_{0m})$$

where $G_m=A_m-L_mC_m$ and $H_m=B_m$. Clearly, e_m doesn't tend to zero. However, if x and u tend to zero for $t\to\infty$ then also e_m tends to zero.

Then, calculating in time t estimate $x_m(t+T_{0m}|t)$ of state $x(t+T_0)$ based on uncertain model (1) we obtain

$$x_m(t+T_{0m}|t)=e^{A_mT_{0m}}v_m(t)+\int_t^{t+T_{0m}}e^{A_m(t+T_{0m}-\tau)}B_mu(\tau-T_{0m})d\tau$$
$$=x(t+T_0)+e^{A_mT_{0m}}v_m(t)-e^{AT_0}x(t)+z_m(t,u)$$

where

$$z_m(t,u)=\int_t^{t+T_{0m}}e^{A_m(t-\tau+T_{0m})}B_mu(\tau-T_{0m})d\tau-\int_t^{t+T_0}e^{A(t-\tau+T_0)}Bu(\tau-T_0)d\tau$$
$$=\int_0^{T_{0m}}[e^{A_m\tau}B_m-e^{A\tau}B]u(t-\tau)d\tau-\int_{T_{0m}}^{T_0}e^{A\tau}Bu(t-\tau)d\tau$$

It is easy to see that $z_m(t,u)\to 0$ if model (A_m,B_m,C_m,T_{0m}) tends to system (A,B,C,T_0). It can be also noted that, similarly to (7), $x_m(t+T_{0m}|t)$ is a solution of the following system

$$\dot{x}_u=A_mx_u+B_mu(t-T_{0m})$$

with initial condition $x_m(t)=v_m(t)$ at time $t+T_{0m}$:

$$x_m(t+T_{0m}|t)=x_u(t+T_{0m}).$$

Then, calculating control input

$$u(t)=K_mx_m(t+T_{0m}|t)=K_m[x(t+T_0)+e^{A_mT_{0m}}v_m(t)-e^{AT_0}x(t)+z_m(t,u)]$$

where K_m is a state feedback gain calculated based on uncertain system model (A_m,B_m,C_m), we find analogously to (9)

$$\begin{bmatrix}\dot{x}\\ \dot{v}_m\end{bmatrix}=\begin{bmatrix}A & 0\\ 0 & G_m\end{bmatrix}\begin{bmatrix}x\\ v_m\end{bmatrix}+\begin{bmatrix}B\\ H_m\end{bmatrix}K_m[x(t)+e^{A_mT_{0m}}v_m(t-T_0)-e^{AT_0}x(t-T_0)+z_m(t-T_0,u)]+\begin{bmatrix}0\\ L_m\end{bmatrix}Cx$$
$$=\begin{bmatrix}A+BK_m & 0\\ L_mC+H_mK_m & G_m\end{bmatrix}\begin{bmatrix}x\\ v_m\end{bmatrix}+\begin{bmatrix}B\\ H_m\end{bmatrix}K_m[e^{A_mT_{0m}}v_m(t-T_0)-e^{AT_0}x(t-T_0)+z_m(t-T_0,u)] \tag{20}$$

Since $z_m\to 0$ if model tends to plant then for small model uncertainty, i.e. $\Delta_A,\Delta_B,\Delta_C,\Delta_{T0}\approx 0$, magnitude of $z_m(t,u)$ is much smaller than magnitude of $u(t)$. Therefore, in this case, one can assume that $z_m(t-T_0,u)$ is a small structural disturbance additive to control input.

We say that z_m is a structural disturbance since it depends on system input and influence closed-loop system dynamics if control input does it. However, for small model uncertainty its influence is much smaller than the influence of control input and therefore we can ignore this influence analyzing system properties, like in the case $u(k)=Kx(k)+K_1x(k)$ one can omit term $K_1x(k)$ if $||K_1||<<||K||$.

From the proof of theorem 1 it follows that all roots of the characteristic equation of closed loop systems (9) and (19)

$$W(s)=\det\left(Is-\begin{bmatrix}A+BK & 0\\ LC+HK & G\end{bmatrix}+\begin{bmatrix}-BKe^{AT_0} & BKe^{AT_0}\\ -HKe^{AT_0} & HKe^{AT_0}\end{bmatrix}e^{-T_0s}\right) \tag{21}$$

have negative real part.

For small structural disturbance z_m one can approximate the characteristic equation of the control system (20) as follows

$$W_m(s)\approx\det\left(Is-\begin{bmatrix}A+BK_m & 0\\ L_mC+H_mK_m & G_m\end{bmatrix}+\begin{bmatrix}-BK_me^{AT_0} & BK_me^{A_mT_{0m}}\\ -H_mK_me^{AT_0} & H_mK_me^{A_mT_{0m}}\end{bmatrix}e^{-T_0s}\right)$$

Since roots of the characteristic equation $W_m(s)$ are continuous with respect to equation parameters A, B, C, T_0, A_m, B_m, C_m and T_{0m}, we find that roots of the characteristic equation of control system designed based on uncertain model (1) have negative real part for small uncertainties if all roots of equation (21), i.e. poles of control system designed based on certain model, have negative real part. Henceforth, the control system with controller calculated based on uncertain model is asymptotically stable with respect to small system uncertainties.

Thus, one can formulate the following theorem.

Theorem 3

The asymptotically stable closed-loop continuous-time system (1) with feedback control (12) is robustly asymptotically stable with respect to small uncertainties in system and observer parameters, delay-time and control gain.

Remarks

1. Depending on system properties, e.g. stable or unstable plant, and designed control system poles λ and μ small uncertainties can be very small or quite big.
2. It is well known that usually one obtains larger stability margin when negative real parts of the poles of the closed-loop system are close to zero.

□

6. Examples

We illustrate the proposed controller considering system described by the model (1) with dead time T_0=20 and

$$A=\begin{bmatrix}0.08 & -0.04\\ 0.25 & 0\end{bmatrix},\quad B=\begin{bmatrix}0.25\\ 0\end{bmatrix},\quad C=[0 \quad 0.32]$$

It is easy to check that the pair (A,B) is controllable and the pair (C,A) observable. The transfer functions of the system is follows

$$G(s)=\frac{2}{100s^2-8s+1}e^{-20s}=\frac{2}{10^2s^2-2\cdot 0.4\cdot 10s+1}e^{-20s}$$

It is easy to see that the system is a representation of the unstable oscillatory plant (3) with complex poles s_1=0.0400+0.0917j and s_2=0.0400–0.0917j.

Then, there are presented in fig. 1 control system response of the plant with controller calculated in such a way that poles λ and μ of the system and observer were equal to –0.8927 and in fig. 2 with poles λ of the system equal to –0.1015 (close to 0) and poles μ of observer equal to –0.4218. Note, that we wanted to obtain closed-loop system response without oscillations. There are also presented control inputs to the system.

In all simulations initial state of the plant was $x(0)=\begin{bmatrix}0\\ 6.25\end{bmatrix}$ which is a steady state of the system for u=1 and y=2, initial input was $u(t)$=0 for $t\in[-50,0)$, and initial state of the observer was equal to zero: $v(0)$=0.

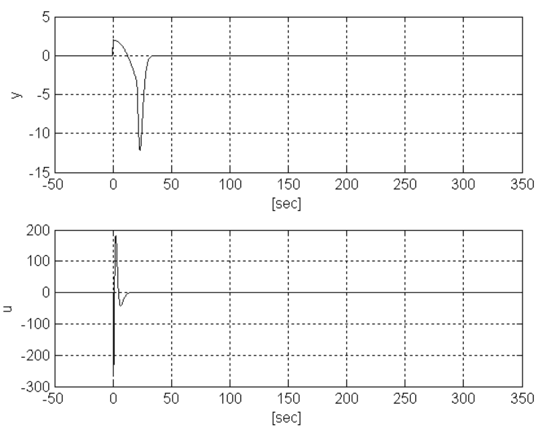

Fig. 1. Response of control system based on exact model of the plant with poles close to –0.9

Rys. 1. Odpowiedź układu regulacji zbudowanego na podstawie dokładnego modelu obiektu regulacji z biegunami w okolicy –0.9

It is easy to see that changing poles of the closed loop system one changes the control time and also magnitudes of system output and control input similarly as for system without dead-time.

One also easily finds that control system output is no aperiodic in spite of the chosen roots λ and μ of the closed-loop system stability polynomial. However, it can be shown that this occurs because the feed-back is realized based on state vector estimation not exact value of the state vector. The estimation starts from zero whereas initial system state is quite different than zero.

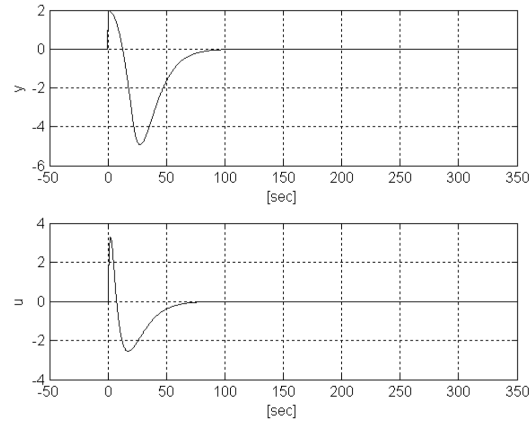

Fig. 2. Response of control system based on exact model of the plant with poles close to –0.1

Rys. 2. Odpowiedź układu regulacji zbudowanego na podstawie dokładnego modelu obiektu regulacji z biegunami w okolicy –0.1

Then, we have used for controller calculation an uncertain model (1) with T_{0m}=22 and

$$A_m=\begin{bmatrix}0.06545 & -0.03306\\ 0.25 & 0\end{bmatrix},\quad B_m=\begin{bmatrix}0.25\\ 0\end{bmatrix},\quad C_m=[0 \quad 0.238]$$

It is easy to check that the pair (A_m,B_m) is controllable and the pair (C_m,A_m) observable. The transfer functions of the model is as follows

$$G_m(s)=\frac{1.8}{121s^2-7.92s+1}e^{-22s}=\frac{1.8}{11^2s^2-2\cdot 0.36\cdot 11s+1}e^{-22s}$$

It is easy to see that model inaccuracy is equal to ±10% of transfer function parameters: gain, time constant, damping coefficient and dead-time, whereas uncertainty of the transfer function coefficients is up to 21% and the state space model parameters up to 34%. Model has the following poles: s_1=0.0327+0.0848j and s_2=0.0327–0.0848j. Differences between the step responses of the system and model one can see in fig. 3.

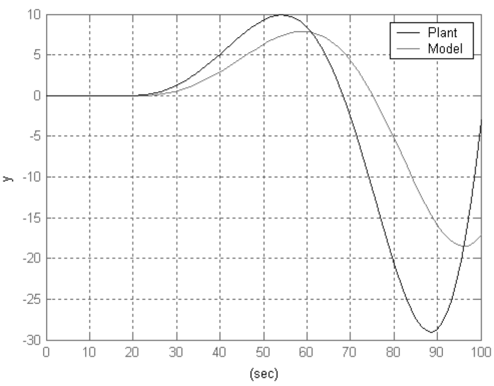

Fig. 3. Step response of the plant and model used for control system design

Rys. 3. Odpowiedź skokowa obiektu regulacji i modelu obiektu regulacji wykorzystanego do syntezy regulatora

In fig. 4 there are presented control system response of the plant and control input to the system. The plant was as before and discrete-time controller was calculated based on uncertain model for the system the same way as for the second case based on exact model, i.e. roots of the matrix $A_{Km} = A_m + B_m K_m$ were equal to –0.1015 (close to 0) and roots of observer state matrix G_m were equal to –0.4218.

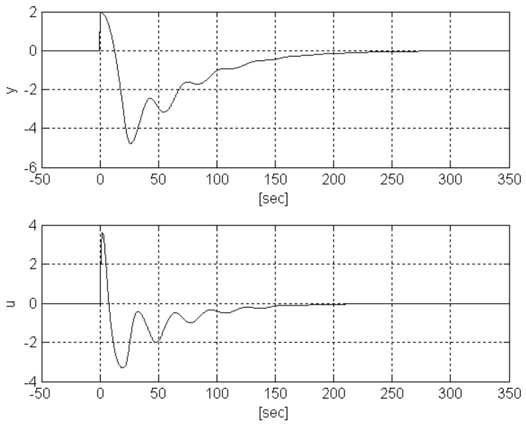

Fig. 4. Response of control system based on uncertain model of the plant with poles close to –0.1
Rys. 4. Odpowiedź układu regulacji zbudowanego na podstawie niedokładnego modelu obiektu regulacji z biegunami w okolicy –0.1

Finally, there are presented responses of the control system designed based on uncertain model with unmeasurable disturbance additive to control input: $u(t)=u(t)+0.2z(t)$, fig. 5, and additive to system output: $y(t)=y(t)+0.2z(t)$, fig. 6, where z is a stochastic random signal with normal distribution with mean equal to zero and variance and standard deviation equal to one. There are also presented control input and disturbance. Controller and observer and system initial conditions were the same as for system without disturbances.

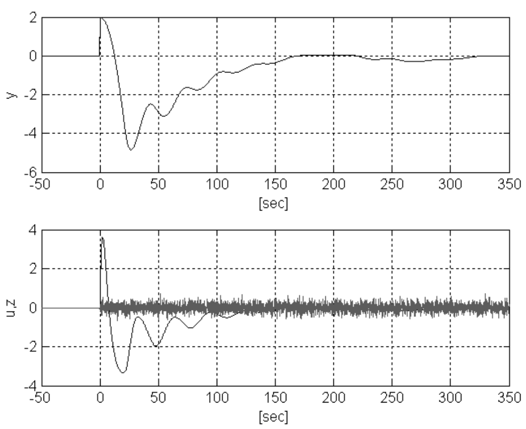

Fig. 5. Response of control system based on uncertain model of the plant with additive input disturbance and poles close to –0.1
Rys. 5. Odpowiedź układu regulacji zbudowanego na podstawie niedokładnego modelu obiektu regulacji z zakłóceniem addytywnym do sygnału wejściowego i biegunami w okolicy –0.1

From the presented examples we see that controller works very well, according to assumption. It stabilizes the closed-loop system with rather big dead-time in both cases, when design was based on certain as well uncertain process model.

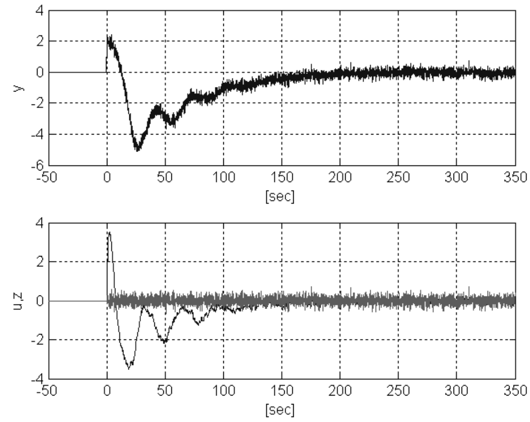

Fig. 6. Response of control system based on uncertain model of the plant with additive output disturbance and poles close to –0.1
Rys. 6. Odpowiedź układu regulacji zbudowanego na podstawie niedokładnego modelu obiektu regulacji z zakłóceniem addytywnym do sygnału wyjściowego i biegunami w okolicy –0.1

7. Concluding remarks

The controller for pole placement of continuous-time linear system with dead-time has been presented. The controller is simple, it is similar to respective controller for system without dead-time. The main advantage of the presented controller is that it gives ones possibility to design stable dead-time control system in a simple manner like for system without dead-time. Moreover, changing poles of the closed-loop system one can change dynamics of the control system. Comparing with the known controllers for systems without dead-time the proposed controller is additionally equipped with predictor of the future system state $x(t+T_0)$.

The proposed controller can find practical applications, particularly for systems with big dead-time. The control systems with the controller is robust with respect to model uncertainties as well system and measurement disturbances. It has nice property: behavior of the control system tends to the behavior of a system without dead-time. In many practical applications it could be also important that one can easily design control system with aperiodic system response.

Finally, it should be noted that the proposed continuous-time controller can be difficult for real analogue realization, however the discrete-time realization can be easily implemented on microprocessor based controller.

8. References

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