EXTENDED REYNOLDS ANALOGY FOR DIFFERENT FLOW CONDITIONS OF THE HEATED PLATE

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Summary

The investigation of the extended Reynolds analogy of the transitional boundary layer on the heated plate with different onflow condition was carried out. The coefficients of intermittency were determined on the basis of the tree-parametric cumulative distribution function of Weibull using the local shear stress C_f and Stanton number St distributions along the plate which are experimentally measured. For the unsteady flow with wakes the extended Reynolds analogy coefficient s is equal to 1.10 while for the flow without unsteadiness it is equal to 1.0. Finally, using the cumulative distribution function of Weibull the formula was proposed for the Prandtl number distribution in the region of laminar-turbulent transition.

1. INTRODUCTION

In the paper [1] the authors presented the needs in convective heat transfer research for industry and also fo the turbomachinery. An excellent survey of experimental investigations and numerical calculation on this subject is given by Dunn [2], who indicated the basics problems of the heat transfer in the gas turbine blades as follows: external heat transfer with and without film cooling, internal heat transfer with cooling and finally influence of instationarity, turbulence level and laminar-turbulent transition on heat transfer.

The reported investigation is aimed at the improvement of the modelling of heat and momentum transfer phenomena in the region of transition on the blades in the turbomachinery. In the cold boundary layer the velocity profiles in the boundary layer were measured by means of thermoanemometry. The different cold boundary layer characteristics were determined among other also the local shear stress coefficient C_f. The transitions in the aerodynamic and thermal boundary layer were investigated. The heat transfer investigation on the heated plate was carried out and compared with the investigation of a cold boundary layer for different on-flow conditions (small turbulent level, and flow with wakes). The coefficients of intermittency were determined on the basis of the tree-parametric cumulative distribution function of Weibull. The different parameters of Weibull CDF were determined for different onflow conditions. Next the Reynolds analogy coefficient s was established for these different external flow conditions. Finally, the formula was proposed for the Prandtl number in the region of laminar-turbulent transition in which the CDF of Weibull was also applied.

2. EXPERIMENTAL RIG

The investigation was carried out in the wind-tunnel of small turbulence level, Tu < 0.08%, at the Institute of Fluid-Flow Machinery in Gdańsk. The experimental stand and the heated flat plate was described in [3]. Condition for the external flow were generated by the grid wire diameter d = 1 mm and with mesh M = 3 mm and two different wake generators (single cylinder and squirrel cage with eight rods of 3 mm diameter each on its circumference). The frequency of the cylinder and squirrel cage motion was in both cases the same f = 4 Hz. The flat plate of dimensions: 0.7 m long, 0.6 m wide and 0.012 m thick was used for both the cold and heated boundary layer investigations. The plate has six different separated heated sections made of high-resistive foil glued on the plate surface.

The local shear coefficient C_f was find out by means of the velocity gradient in the boundary layer measured with help of hot-wire, while the Stanton number was determined by means of the measurement of the temperature distribution in the heated flat plate by means of the semiconductor sensor mounted below the heated foil. The heating condition can be describes as non-isothermal but constant heat stream q = const.

3. BOUNDARY LAYER MEASUREMENTS

It is well known that the Reynolds analogy describes proportionality between the heat and momentum transfer, so to determine the Reynolds analogy both the momentum transfer and heat transfer is to be measured. Momentum transfer can be calculated by mean of the measurement of the local shear stress in the boundary layer. Thus, the velocity profiles and also velocity fluctuations profiles along the cold plate for different flow conditions were measured by means of the hot-wire. Especially the velocity gradient $\partial u/\partial y$ was determined and the local shear stress τ calculated:

$$\tau_w = \mu \left.\frac{\partial U}{\partial y}\right|_{y=0} \tag{1}$$

and then the local shear stress coefficient C_f determined:

$$C_f = \frac{2\tau_w}{\rho U^2} \tag{2}$$

Furthermore, during the experiment also some other characteristics of boundary layer like its thickness, displacement and momentum thicknesses were measured in laminar, transitional and turbulent boundary layer regions.

4. HEAT TRANSFER MEASUREMENTS

The determination of local heat transfer coefficient α was made by means of well known method of electrocalorymetry:

$$\alpha = \frac{q_{foil} - q_{loss}}{t_w - t_\infty} \tag{3}$$

where t_w – local temperature of the surface (wall), t_∞ - free stream temperature, q_{foil} - density of heat flux generated by alternating electrical current,

$$q_{foil} = \frac{\sqrt{3} IE}{F} \tag{4}$$

where I = current intensity of the foil (heated section), E – voltage difference of the heated section, F – surface of the heated section.
And heat losses q_{loss} are given by the formula

$$q_{loss} = q_{rad} + q_l + q_d \tag{5}$$

where radiation losses q_{rad} are given as:

$$q_{rad} = \varepsilon\sigma\left(T_w^4 - T_\infty^4\right) \tag{6}$$

The emissivity of the painting is equal to ε = 0.3.

The heat losses caused by the heat transfer along the plate can be calculated from the heat balance of the infinitesimal section along the plate and depends on the conductivity of materials composed the plate and their thicknesses:

$$q_l = (\lambda_{pl} d_{pl} + 2\lambda_{foil} d_{foil}) \frac{\partial^2 T_m}{\partial x^2} \tag{7}$$

where λ_{pl}, d_{pl} and λ_{foil}, d_{foil} are the conductivity coefficients and thicknesses of the plate and heat foil, and T_m is average temperature across the plate $T_m = (T_u + T_d)/2$ where T_u and T_d are temperatures on the upper and lower surface (foil) of the plate. These losses are neglected in the calculations of heat flux.

The heat losses across the plate is given by the following formula:

$$q_d = \lambda_{pl} \frac{\partial T}{\partial y} \tag{8}$$

Assuming that the temperature gradient of the plate is constant this formula can be simpler presented:

$$q_d = \lambda_{pl} \frac{\Delta T_d}{d_{pl}} \tag{9}$$

where d_{pl} – is the plate thickness and $\Delta T_d = (T_u - T_d)/2$ – temperature difference across the plate. This time the assumption is made that the temparature across the foil is rather steady because of its thickness and the foil is the heat source.

Finally, the Stanton number is defined by following formula:

$$\mathrm{St} = \frac{\alpha}{\rho C_p U_o} \tag{10}$$

According to the Reynolds analogy, the following formula expresses the relation between local shear stress C_f and the Stanton number St for the laminar boundary layer, where Pr is the Prandtl number and C_{si} is non-isothermality coefficient which for the case of constant heat flux is equal to C_{si} = 1.35 [5].

$$\mathrm{St} = C_{si} \frac{C_f}{2} \mathrm{Pr}^{-2/3} \tag{11}$$

Finally, the Reynolds analogy coefficient s was determined by means of following formula:

$$s = C_{si} \frac{C_f}{2\mathrm{St}} \mathrm{Pr}^{-2/3} \tag{12}$$

Thus the measurement of C_f and St are necessary as mentioned above to determine the Reynolds analogy coefficient.

5. TRANSITION IN THE COLD AND HEATED BOUNDARY LAYER

The laminar-turbulent transition can be well described by the intermittency coefficient γ which is the ratio of the time when the flow is turbulent to the whole time of observation. So for the laminar boundary layer $\gamma = 0$ and for the turbulent $\gamma = 1$.

Using the intermittency coefficient γ the local shear stress coefficient C_f in the transitional region can be given by following formula:

$$C_f = (1-\gamma)\cdot C_{fl} + \gamma \cdot C_{ft} \tag{13}$$

The similar formula is usually used to described Stanton number in the transitional range of boundary layer:

$$\mathrm{St} = (1-\gamma)\cdot \mathrm{St}_l + \gamma \cdot \mathrm{St}_t \tag{14}$$

The cumulative distribution function of Weibull probability distribution was for the first time proposed [4] for the description of the intermittency factor. The formula below shows the three parametric Weibul CDF which was used for further calculations.

$$\gamma = 1 - \exp\left[-\left(\frac{\mathrm{Re}_x - \mathrm{Re}_B}{\mathrm{Re}_\theta - \mathrm{Re}_B}\right)\right]^{\beta} \tag{15}$$

where Re_B is the begin of the transition (begin of the CDF of Weibull distribution), Re_θ is the shape coefficient (i.e. point where the intermittency coefficient is equal to $(e\text{-}1)/e$ where e – natural logarithm basis), and β is the slope coefficient of Weibull distribution.

6. RESULTS OF INVESTIGATION

In Fig. 1 to 3 the C_f coefficient and Stanton number St versus Reynolds number Re_x are shown for different flow condition and the differences in the transitions caused by different flow conditions are easy to see. In Fig. 1 the Stanton number and C_f coefficient for the flow with grid with the mesh M = 3 mm and wire with diameter 1 mm is shown for three oncoming velocities U = 10, 15 and 20 m/s.

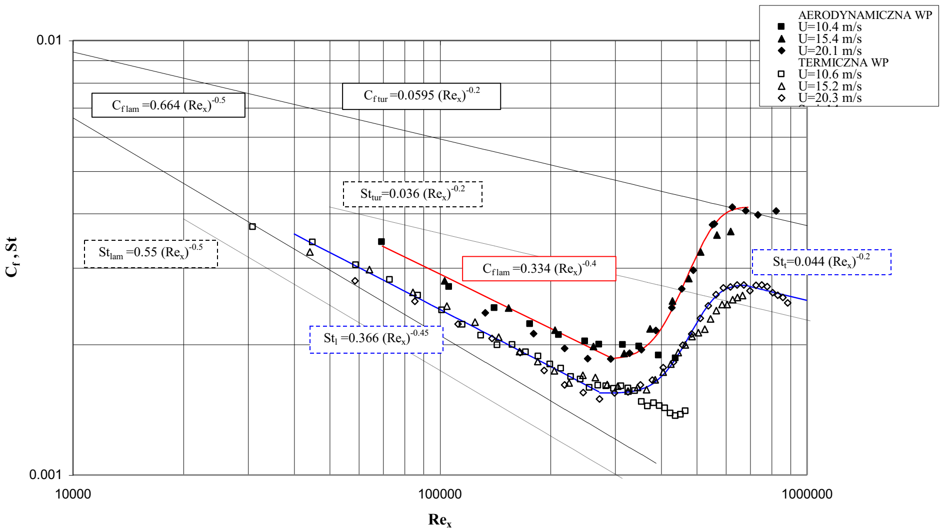

Fig. 1. C_f and Stanton number St versus Re_x for the flow with grid

Next, Fig. 2 shows the local shear stress coefficient C_f and Stanton number St for the flow with one cylinder moving up and down. And finally in Fig. 3 the local shear stress coefficient C_f and Stanton number St for the flow with squirrel cage is showed. In all this diagrams the line of C_f and Stanton number St for the laminar and turbulent flow are appropriately shown.

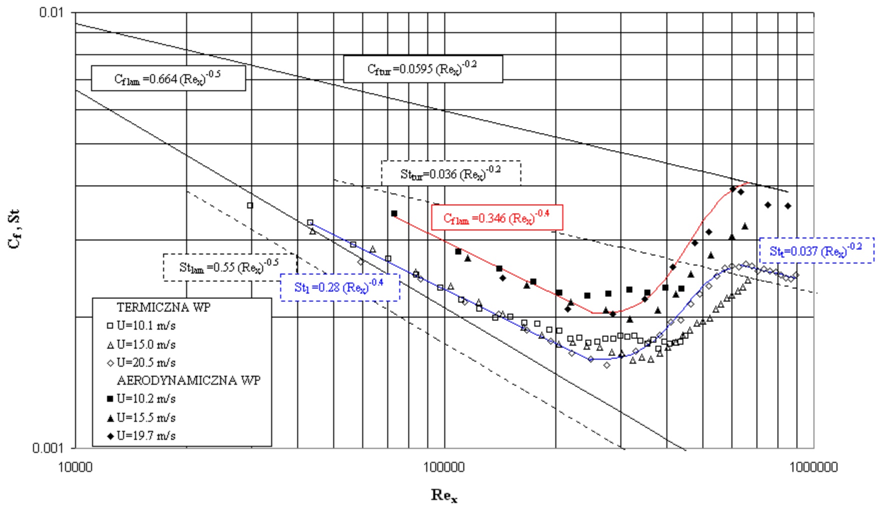

Fig. 2. C_f and Stanton number St versus Re_x for the flow with one cylinder in up and down motion

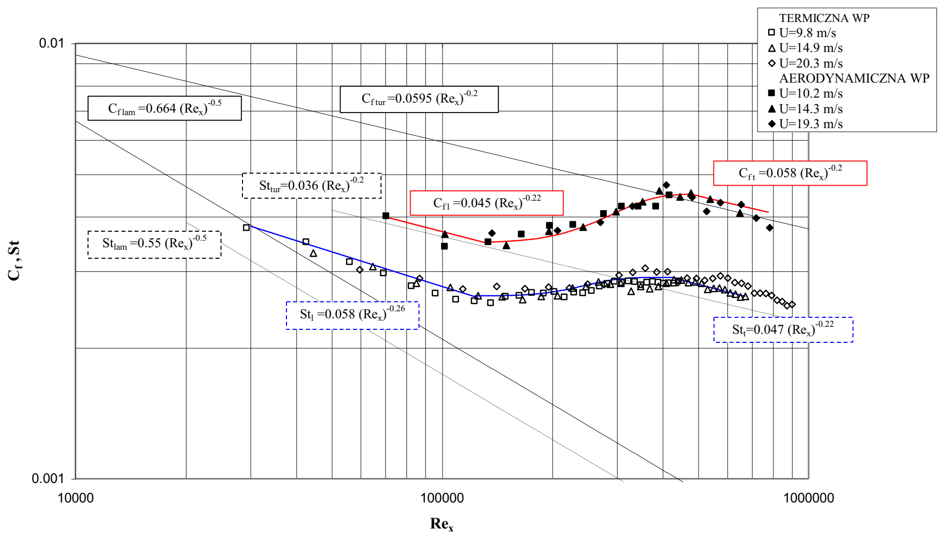

Fig. 3. C_f and Stanton number St versus Re_x for the flow with squirrel cage in motion f = 4Hz.

Basing on both the C_f and Stanton number measurements and using the equations (13) and (14) it is possible to find the intermittency coefficient γ and then taking twice the logarithm of the intermittency factor and matching the experimental data by the least square method it is easy to find the parameters of the Weibull CDF for different flow. In Fig. 4 an example of this calculation is shown for the C_f distribution.

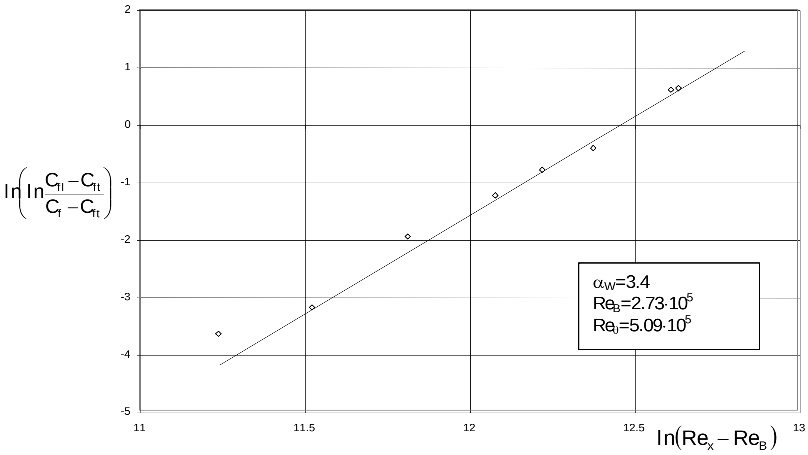

Fig. 4. Determination of the three parameters of the CDF of Weibull distribution [6]

The results of these calculations are given in the Tab. 1 using the Stanton number St_f measurements. Almost the same results for Re_θ, Re_B and α are calculated using the C_f coefficient distribution versus Reynolds number.

The application of the Weibull distribution for the intermittency coefficient was first proposed by Wiercinski [4] who found for the undisturbed flow and natural transition $\alpha = 3.5$. This three parametric Weibull distribution quite good expresses the different reason (turbulence and instationarity of flow) of transition of the flow in boundary layer especially by the slope parameter α. The other parameters of the Weibull distribution are also included in the Tab. 1 [6].

Tab. 1. Parameters of intermittency coefficient by the Weibull CFD [6]

	St_l	St_t	$Re_B \cdot 10^5$	$Re_\theta \cdot 10^5$	β
grid	0.00154	0.00273	2.72	5.1	3.4
cage	0.0024	0.00266	1.12	2.7	2.7
grid +rode	0.0016	0.00264	2.48	4.8	2.7
grid+cage	0.00263	0.0029	1.23	2.6	2.8

And finally the Reynolds analogy coefficient s was determined using the formula (12) and the results of these calculations are given in the Table 2 [6].

Tab. 2. The Reynolds analogy coefficient s for different flow conditions [6]

L.p.	Flow	Pr_l	Pr_t	s
1	undisturbed	0.72		1±0.02
2	grid	0.72	1	1±0.02
3	cage	0.72	1	1.1±0.04
4	grid+cage	0.72	1	1.1±0.04
5	grid +rode	0.72	1	1±0.03

It is found that for the case of unsteady flow with wakes from the cage and cage with grid the Reynolds analogy coefficient is equal to 1.1 while for other flow condition it equals to 1.0.

Finally on the basis of the accomplished investigation the following formula for the Prandtl number in the region of transition is proposed:

$$\Pr = (1-\gamma)\cdot \Pr_l + \gamma \cdot \Pr_t \qquad (16)$$

In this formula the intermittency coefficient γ is described by the above given formula of CFD of Weibull which three parameters were determined by means of reported method and experiment. This formula can be also found useful in the numerical calculation of flows with heat transfer.

7. CONCLUSIONS

Basing on the C_f and Stanton number St measurements for different flow conditions (flow with low turbulence level, flow with 3% turbulence and wakes of single cylinder and of squirrel cage) the Reynolds analogy coefficients s of the heated plate were determined. Only for the flow with squirrel cage wake the Reynolds analogy coefficient was greater than one and equal to 1.1, for other onflow conditions the Reynolds coefficient was equal to one. Moreover, the relationship for the Prandtl number Pr in the transition region as the weighted function of intermittency given as th CDF of Weibull and laminar and turbulent Prandtl number was proposed.

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NOMENCLATURE

C_f – local shear stress
C_p – specific heat
C_{ni} – coefficient of nonisothermality
d – thickness
E – voltage difference of the heated section
F – surface of heated section
q – density of heat flux
i – incidence angle
I – current intensity
Nu – Nusselt number
Pr – Prandtl number
R – resistivity of foil
Re- Reynolds number
s – Reynolds analogy coefficient
St – Stanton number
T – temperature in ºK
Tu – turbulence level
t - temperature in external flow in ºC
U_o – velocity of oncoming flow
(x, y) – coordinates

Greek symbols

α – local heat transfer coefficient
λ – specific heat conductivity
ρ – density; specific resistivity
ε – emissivity coefficient
σ –Stefan – Boltzman constant

Subscripts

d – across plate; lower surface of plate
foil – refers to heated foil
l – along plate
loss – refers to heat losses
m – mean
pl – refer to plate
rad – refers to radiation losses
u – upper surface of plate
w – refers to plate surface

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ROZSZERZONA ANALOGIA REYNOLDSA W WARSTWIE PRZYŚCIENNEJ NA GRZANEJ PŁYCIE W RÓŻNYCH WARUNKACH OPŁYWU

Streszczenie

W pracy przedstawiono rezultaty badań rozszerzonej analogii Reynoldsa w warstwie przyściennej na grzanej płycie w różnych warunkach jej opływu. Na podstawie pomiarów lokalnego współczynnika oporu C_f oraz liczby Stantona wzdłuż płyty w strumieniach o różnym poziomie turbulencji wyznaczono rozkład współczynnika intermitencji jako trójparametrowej dystrybuanty rozkładu Weibulla. Dla niestacjonarnego przepływu ze śladami spływowymi wartość współczynnika rozszerzonej analogii Reynoldsa wynosi 1.10, zaś w pozostałych badanych przypadkach wartość tego współczynnika wynosi 1. Ponadto, zaproponowano zastosowanie rozkładu współczynnika intermitencji w postaci dystrybuanty trójparametrowego rozkładu Weibulla do wyznaczenia rozkładu liczby Prandtla w obszarze przejścia laminarno-turbulentnego w warstwie przyściennej dla przepływów o różnym poziomie niestacjonarności.

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РАСШИРЕННАЯ АНАЛОГИЯ РЕЙНОЛЬДСА В ПОГРАНИЧНОМ СЛОЕ НА ПОДОГРЕВАЕМОЙ ПЛАСТИНЕ В РАЗНЫХ УСЛОВИЯХ ОБТЕКАНИЯ

Резюме

В работе представлены результаты исследования расширенной аналогии Рейнольдса в пограничном слое на подогреваемой пластине в разных условиях обтекания. На основе измерений локального коэффициента сопротивления C_f и числа Стантона вдоль пластины в струях с разным уровнем турбулентности определено распределение коэффициента интермитенции как трехпараметрической дистрибуанты распределения Вейбулла. Для нестационарного течения с следами стекания величина коэффициента расширенной аналогии Рейнольдса составляет 1.10, а в остальных исследуемых случаях величина этого коэффициента составляет 1. Кроме того, предлагается применение распределения коэффициента интермитенции в качестве дистрибуанты трехпараметрического распределения Вейбулла для определения распределения Число Прандтля в зоне ламинарно-турбулентного перехода в пограничном слое для течений с разным уровнем нестационарности.