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**A study of the effects of the sensor guideway slope on the results of the reference measurement of cylindricity deviations**

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Studia wyższe na Wydziale Elektroniki Politechniki Warszawskiej ukończył w 1981, a w 1996 obronił tam pracę doktorską. W 1996 obronił pracę habilitacyjną w Instytucie Podstawowych Problemów Techniki PAN. Obecnie pracuje na stanowisku profesora w Politechnice Świętokrzyskiej i jest kierownikiem Zakładu Informatyki i Sterowania. Zainteresowania naukowe obejmują teorię sterowania ze szczególnym uwzględnieniem metod identyfikacji i sterowania adaptacyjnego oraz matematyczne podstawy metrologii powierzchni.

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Dr inż. Krzysztof Stępień jest adiunktem w Katedrze Technologii Mechanicznej i Metrologii Politechniki Świętokrzyskiej w Kielcach. W swoich pracach zajmuje się zagadnieniami podstaw metrologii i pomiarów wielkości geometrycznych, a w szczególności pomiarami zarysów okrągłości i walcowości. Jest współautorem 37 prac prezentowanych w czasopismach krajowych i zagranicznych oraz na konferencjach międzynarodowych.

e-mail: [kstepien@tu.kielce.pl](mailto:kstepien@tu.kielce.pl)**Abstract**

The concept of applying the reference method to measure cylindricity deviations under industrial conditions, even on a machine tool, was developed at the Kielce University of Technology. The results of the statistical analysis show that the maximum error of the new method in relation to the radial method characterized by high accuracy is 19% for a level of confidence of 0.95. From the analysis and the experimental verification it is clear that the accuracy of the reference method can be considerably improved but it is necessary to study the potential sources of the method errors. One of the main factors to be analyzed is the sensor guideway slope.

**Keywords:** reference method, cylindricity, measurement error.**Badanie wpływu pochylenia prowadnicy czujnika na wynik pomiaru odchyłki walcowości metodą odniesieniową****Streszczenie**

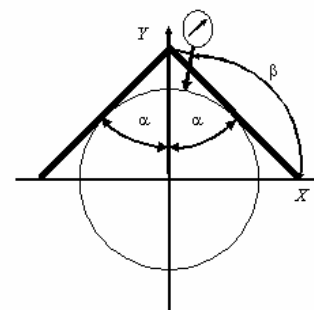
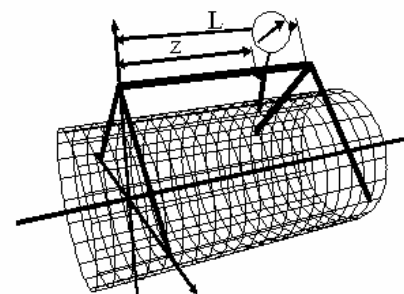
Koncepcja pomiaru odchyłek walcowości metodą odniesieniową, opracowana w Politechnice Świętokrzyskiej, umożliwia pomiar elementów w warunkach przemysłowych, w tym na obrabiarce. Przeprowadzone badania statystyczne pokazały, że maksymalny błąd nowej metody w stosunku do dokładnej metody promieniowej wynosi ok. 19% dla poziomu ufności  $P=0,95$ . Analiza nowej metody oraz wyniki uzyskane podczas eksperymentów pozwoliły na stwierdzenie, że jej dokładność może zostać znacznie zwiększona. Z tego powodu przeprowadzona została analiza potencjalnych źródeł błędów metody. Jednym z analizowanych czynników było pochylenie prowadnicy czujnika pomiarowego, czego dotyczy niniejsza praca.

**Słowa kluczowe:** metoda odniesieniowa, walcowość, błąd pomiaru.**1. Introduction**

As stated in the ISO 12180 standard, a cylindricity error is a complex form error. In the general case, it is a sum of three components: the axis straightness error, the form error in the longitudinal cross-section and the form error in the transverse cross-section. Under laboratory conditions, cylindricity profiles are generally assessed with instruments using the radial (non-reference) methods of measurement based on the measurement of the changes in the radius [1, 2]. Measurements of cylindricity profiles with the radial methods are characterized by a high metrological level. The radial methods, however, are suitable for measuring small elements and are performed mainly under laboratory conditions. Today, industry requires that measurements of cylindricity profiles be conducted during the manufacturing process, and, if possible, directly on machine tools [3, 4]. The reference methods are reported to be more suitable. In a reference measurement, two- or multi-point measurement bases are used.

In the reference method, the degree of detectability of the predominant type of roundness errors or of the harmonic component of an irregular profile depends on the number of points of support, their arrangement and the angle of position of the sensor in relation to those points. The predominant type of errors is termed the predominant harmonic component in the Fourier series expansion of the profile function. The reference methods require using appropriate coefficients for each harmonic component, which are frequently called coefficients of detectability. It should be mentioned that the roundness profile obtained in this way differs considerably from the real profile.

The research initiated by the Kielce University of Technology (research project No 7T07D04008 financed by the State Committee for Scientific Research) described in Ref. [5] focused on the implementation of the reference methods to accurate measurements of cylindricity profiles. A model testing facility was constructed to measure cylindricity deviations with the reference method. The principle of the reference measurement of cylindricity profiles is shown in Fig. 1.

Fig. 1. Principle of the reference measurement of cylindricity profiles ( $L$ ,  $\alpha$  and  $\beta$  - the method parameters)Rys. 1. Zasada pomiaru walcowości metodą odniesieniową (wielkości  $L$ ,  $\alpha$  i  $\beta$  to parametry metody)

In a reference measurement, a cylindrical object is placed in a centre device. Two connected prisms are in contact with the object surface. The element connecting the prisms functions as the guideway, and the inductive sensor moves along it. A measurement of cylindricity involves scanning the object surface with the sensor along a predetermined trajectory by controlling the angle of rotation of the object and the sensor shift. The sensor indications are dependent on the value of the deviation at the point of contact with the object surface, the deviation from the real axis and other instrument errors, such as non-straightness of the sensor shift or unparallelism of the shift axis to the object axis. It was necessary to develop the theoretical fundamentals of the reference cylindricity measurements for the proposed measurement system. This required performing a mathematical transformation of the measured profiles and a mathematical model of the reference assessment of cylindricity. The model was used for computer simulations, on the basis of which it was possible to determine the optimal parameters assuring high accuracy of a cylindricity measurement and verify the proposed concept of mathematical transformation.

The concept was tested in practice by performing a statistical analysis. The results show that the accuracy error of the reference method in relation to the radial method known to be of very high accuracy is approx. 19% for a level of confidence of 0.95. As the accuracy is sufficient, the testing facility can be applied to measurements under industrial conditions.

Due to the fact that the concept has considerable practical importance, extensive research is being carried out on the possibility of improving the accuracy of reference measurements of cylindricity profiles. The analyses focus on the potential measurement errors and the methods of their elimination. One of the potential sources of errors is the slope of the sensor guideway axis in relation to the measured element.

## 2. A theoretical analysis of the guideway axis slope problem

Figure 2 showing a fragment of a cross-section of a cylindrical object being measured was used to illustrate the problem of the guideway axis slope in relation to the nominal cylinder axis.

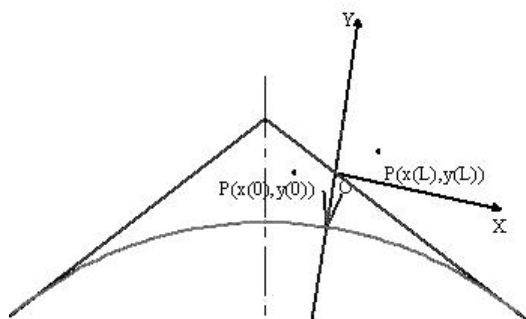


Fig. 2. A system of coordinates related to the sensor guideway  
Rys. 2. Układ współrzędnych związany z prowadnicą czujnika

Let us assume that the point of intersection of the sensor axis and the base prism is the origin of the orthogonal, spatial coordinate system (XYZ), where Y axis coincides with the sensor axis, and the Z axis is parallel to the nominal cylinder axis. In the ideal case, when the guideway axis is not deviated from the nominal cylinder axis, the Z axis is parallel also to the guideway axis. A local coordinate system, associated with the sensor axis, is rotated in relation to the global coordinate system, associated with the nominal cylinder, at angle  $\beta$  being one of the method parameters. If the guideway axis slopes in relation to the Z axis, then, if the orientation coordinates of the axis in the initial (for

$z = 0$ ) and the final (for  $z = L$ ) cross-sections are known, we can calculate the orientation coordinates of this axis in the XYZ system for any given  $z$  using the following formula:

$$x(z) = \frac{x(0)(L-z) + x(L)z}{L}, \quad (1)$$

$$y(z) = \frac{y(0)(L-z) + y(L)z}{L}. \quad (2)$$

In the general case, the guideway axis slopes in relation to the nominal cylinder axis both in the XY plane and in the YZ plane. For better clarity, the two cases will be considered separately.

### 2.1. Axis slope in the XZ-plane

If the guideway axis slopes in relation to the nominal cylinder axis in the XZ plane, then  $y$  coordinates equal zero. Thus, only the  $x$ -coordinates will be taken to account in further considerations. The influence of the guideway axis slope in the XZ plane will be studied basing on Fig. 3.

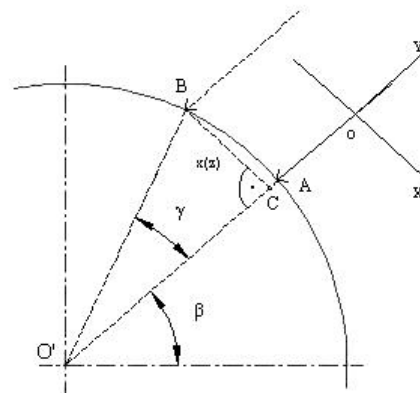


Fig. 3. Guideway axis slope in the XZ-plane  
Rys. 3. Pochylenie prowadnicy w płaszczyźnie XZ

Let the centre of the nominal cylinder cross-section be denoted by  $O'$ . Point A is the contact point of the workpiece surface and the tip of the sensor, displaced from its nominal position by a certain value  $x(z)$ , which can be calculated from relationship (1). Point C is the point of intersection of the line perpendicular to the nominal sensor axis going through point B and the line perpendicular to the sensor slope axis in the nominal position.

Thus:

$$\overline{OB} = \overline{OA} = R_0, \quad (3)$$

where  $R_0$  is the radius of the nominal cylinder, and

$$\overline{BC} = x(z). \quad (4)$$

We can see that  $\overline{AC}$  is the value of the change in the sensor indications caused by the slope of the axis in the XZ plane. For better clarity, let us denote  $\overline{AC}$  by  $\Delta F$ . This value can be calculated from the following relationship:

$$\Delta F(z) = \overline{OA} - \sqrt{\overline{OB}^2 - \overline{BC}^2} = R_0 - \sqrt{R_0^2 - x(z)^2}. \quad (5)$$

From relationship (5) it is clear that the change in the sensor indications caused by the axis slope depends on the  $z$  coordinate. Therefore, the cylindricity profile for which the measurement

results are influenced by the guideway axis slope in the XZ plane can be defined by:

$$F_{XZ}(\varphi, z) = F(\varphi + \gamma, z) - \Delta F(z), \quad (6)$$

where  $F_{XZ}(\varphi, z)$  is the cylindricity profile when the guideway axis slopes in the XZ-plane,  $F(\varphi, z)$  is the nominal cylindricity profile,  $\Delta F(z)$  is the value of the change in the sensor indications caused by the guideway axis slope in the XZ-plane, and  $\gamma$  is the phase shift of the  $F(\varphi, z)$  profile.

A computer simulation was performed to analyze the influence of the guideway axis slope in the XZ-plane. The coordinates of the guideway slope in the XZ plane in a cross-section were assumed to be:  $z = 0$  and  $z = L$ :  $x(0) = -0.1$  mm and  $x(L) = 0.1$  mm. The other simulation parameters were as follows: the cylinder length,  $L = 100$  mm; the angular method parameters,  $\beta = 90^\circ$  and  $\alpha = 60^\circ$ ; the number of cross-sections,  $K = 11$ ; the number of measurement points on the circumference of the cylinder,  $N = 1024$ ; and the radius of the nominal cylinder,  $R_0 = 20$  mm.

The simulation involved generating the nominal cylinder profile and then determining the influence of the assumed guideway axis slope on the nominal cylinder using relationships (1)-(6). Figure 4 presents the simulated ideal cylindricity profile recorded by the sensor moving along the guideway sloping in relation to the nominal cylinder in the XZ plane. As can be seen, the measured profile has the shape of a barrel.

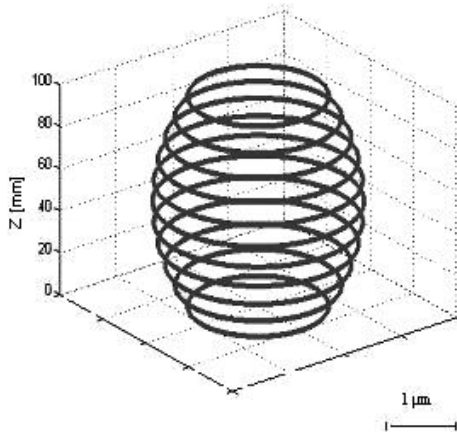


Fig. 4. Barrelity deviation caused by the axis slope in the XZ-plane  
Rys. 4. Odchyłka baryłkowości spowodowana pochyleniem osi w płaszczyźnie XZ

It was found that the barrelity deviation between the nominal profile and the one resulting from the axis slope is  $0.25 \mu\text{m}$  for the predetermined simulation parameters. This value was obtained for a very large axis slope. In practice there is little probability of such a large slope, so it can be assumed that the guideway axis slope in relation to the axis of the nominal element is negligible and does not affect the measurement results.

## 2.2. Axis slope in the YZ-plane

Even a cursory analysis reveals that the guideway axis slope in the YZ-plane has a direct impact on the measurement results. Using relationship (2), we can calculate the  $y(z)$  coordinate of the guideway axis slope in the YZ-plane. The  $y(z)$  coordinate is equal to the difference between the sensor indications caused by the guideway axis slope in the YZ plane. Basing on this relationship, we can write the equation of the cylindricity profile which is influenced by the guideway axis slope in the YZ plane:

$$F_{YZ}(\varphi, z) = F(\varphi, z) + y(z) \quad (7)$$

where:  $F_{YZ}(\varphi, z)$  is the cylindricity profile measured when the guideway slopes in the YZ-plane,  $F(\varphi, z)$  is the nominal cylindricity profile, and  $y(z)$  is the coordinate of the guideway axis slope in the plane YZ; ( $y(z)$  is also the value of the change in the sensor indications caused by the guideway axis slope in the YZ plane.

As  $y(z)$  is linearly dependent on  $z$ , it will have a linear influence on the value of the cylindricity profile in consecutive cross-sections of the cylinder. As a result, the ideal profile measured with an instrument whose guideway axis slopes in the YZ-plane contains an apparent error of conicity, which was confirmed by the computer simulations. The coordinates of the guideway axis slope in the YZ-plane in the cross-section were assumed to be as follows:  $z = 0$  and  $z = L$ :  $y(0) = -0.1$  mm and  $y(L) = 0.1$  mm. The other simulation parameters were the same as in the case of the axis slope in the XZ-plane. The simulation results are shown in Fig. 5.

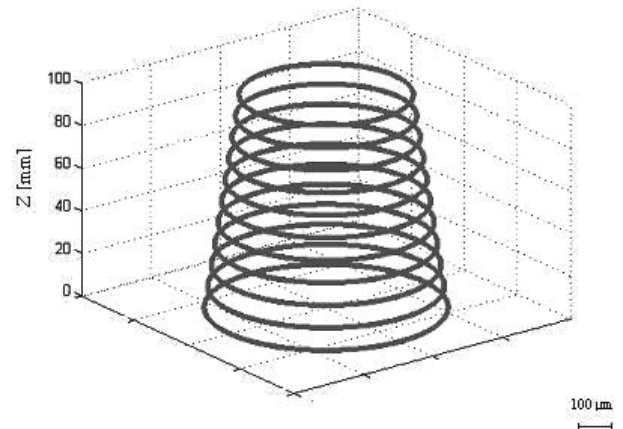


Fig. 5. Conicity deviation caused by the guideway axis slope in the YZ-plane  
Rys. 5. Odchyłka stożkowości spowodowana pochyleniem osi w płaszczyźnie YZ

If  $y(z)$  changes linearly in relation to  $z$ , then it will have a linear influence on the value of the profile in the consecutive cross-sections. This will result in the occurrence of virtual (not real) conicity of the profile, as shown in Fig. 5. The value of the slope in the YZ-plane will affect the observed conicity in a direct way, in accordance with relationship (7).

## 3. A method of identification of the guideway slope on the basis of the measurement data

As the guideway slope in the YZ-plane may considerably influence the measurement accuracy, it was necessary to develop a procedure for its identification and elimination. Basing on relationship (7), we can write that:

$$F(\varphi, z) = F_{YZ}(\varphi, z) + \lambda_p \cdot z, \quad (8)$$

where:  $F(\varphi, z)$  is the nominal cylindricity profile,  $F_{YZ}(\varphi, z)$  is the cylindricity profile registered when the guideway axis slopes in the YZ-plane, and  $\lambda_p$  is the coefficient of correction of the guideway axis slope in the YZ-plane. From relationship (8) it is clear that the value of coefficient  $\lambda_p$  needs to be determined so that the guideway axis slope in the YZ-plane can be identified and eliminated. This can be achieved by using the factor of quality described by relationship (9):

$$J(\lambda_p) = \sum_{i=1}^N (F_i - F_{YZi} - \lambda_p \cdot z_i)^2, \quad (9)$$

where:  $F_i$  is a sample of the standard profile,  $F(\varphi, z)$  for the  $(\varphi_i, z_i)$  coordinates,  $F_{YZi}$  is a sample of the  $F_{YZ}(\varphi, z)$  profile for the  $(\varphi_i, z_i)$  coordinates, and  $\lambda_p$  is the above mentioned coefficient of correction of the guideway axis slope in the YZ-plane.

Assuming that all the data are written in the form of vectors, we can write relationship (9) as:

$$J(\lambda_p) = (D_F - \lambda_p Z)^T (D_F - \lambda_p Z), \quad (10)$$

while:

$$D_F = F - F_{YZ}, \quad (11)$$

where:  $F$  is a column vector containing selected values of the nominal profile,  $F_{YZ}$  is a column vector containing selected values of the profile determined for a guideway axis slope in the YZ-plane,  $Z$  is the column vector containing the properly grouped  $z$ -coordinates of the profile points, and  $\lambda_p$  is the coefficient of correction of the guideway axis slope in the YZ-plane. Coefficient  $\lambda_p$  can be determined by minimizing the factor of quality described by relationship (10). Then, we obtain:

$$\lambda_p = (Z^T Z)^{-1} Z^T D_F. \quad (12)$$

A computer procedure was developed in the MATLAB program basing on relationships (9)–(12). The procedure can be used to determine the coefficient of correction of the sensor guideway slope in the YZ-plane when the standard profile and the measured profile are known. The correctness of the procedure was verified using computer simulations and the results were positive.

#### 4. Conclusion

Basing on the concept of reference measurement applied to cylindricity profiles, which was developed at the Kielce University of Technology, it is possible to assess the quality of cylindrical elements during the manufacturing process, even directly on a machine tool. All the requirements concerning industrial processes are met. The statistical verification of the method shows that the maximum error of the new method in relation to the radial method is approximately 19% for a predetermined level of confidence of 0.95.

The results of the analysis and the experiments confirm that the accuracy is much higher. It was necessary to determine and analyze the potential sources of errors for the reference method of measurement. One of the factors that may have an effect on the results was the slope of the sensor guideway in relation to the object measured. The slope was divided into two component cases: slope in the plane determined by the axis and the sensor shift direction and the slope in the plane perpendicular to it. As a result, it was possible to derive mathematical relationships describing the influence of the guideway slope on the change in the value of a signal registered by the sensor. These relationships were used for computer simulations. The experiments show that the guideway slope causes the occurrence of two types of errors: barrelity and conicity. Barrelity was reported to be a negligible error with no effect on the measurement results in practice. Conicity, on the other hand, had a significant influence on the measurement results. It was, thus, vital to develop a procedure for the identification of the guideway slope basing on the measurement data using the optimization method. Computer simulations show that the procedure is suitable for identifying and eliminating the effects of the guideway slope resulting from the application of the reference method to measure cylindricity profiles.

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*Artykuł recenzowany*

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