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# Calculating the parameters of the associated circle for interrupted roundness profiles

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**Abstract**

Measurements of roundness deviations constitute an important area of geometrical metrology. As high accuracy is frequently a major requirement, it is essential that an analysis involve determining the position of the centre of the associated circle. Extensive studies have been conducted on the accuracy of measurements of closed roundness profiles. Many machine parts, however, have interrupted profiles. The relationships presented in this paper can be used for calculating the centre of the associated circle for interrupted profiles by applying the least squares method.

**Keywords:** associated circle, roundness profile, interrupted profile.

## Obliczanie parametrów okręgu skojarzonego dla przerywanych zarysów okrągłości

**Streszczenie**

Pomiary odchyłek okrągłości stanowią bardzo ważny obszar metrologii wielkości geometrycznych. Bardzo często, w pomiarach tego typu wymagana jest duża dokładność. Z tego względu dokładnie obliczenie położenia środka okręgu skojarzonego jest zagadnieniem szczególnej wagi. Problem ten jest dobrze rozpoznany w odniesieniu do zamkniętych zarysów okrągłości. Jednak wiele elementów części maszyn ma zarysy okrągłości przerywane (niedomknięte). W artykule przedstawiono zależności umożliwiające obliczenie środka okręgu skojarzonego metodą najmniejszych kwadratów dla tego typu zarysów.

**Słowa kluczowe**: okrąg skojarzony, zarys okrągłości, zarys przerywany.

## 1. Introduction

Rotary elements constitute a large and very important group of machine parts. They are commonly found in such industries as automotive, power engineering or paper making. High accuracy of measurement of roundness deviations is one of the priorities of industrial metrology. If a roundness profile is to be measured using an instrument with a rotary spindle or table, the object needs to be centred in such a way that its axis coincides with the axis of rotation [1]. It should be mentioned that such measurements can be performed at a high amplification rate, which reduces the level of measurement-related noise as well as errors resulting from the signal quantization. The centring, however, is a laborious procedure which requires good manual skills, especially if an interrupted profile, i.e. a fragment or several fragments of a closed profile, is to be measured. High non-centricity of a profile may occur in a measurement of concentricity of one profile in relation to another if both profiles are clearly nonconcentric.

Rapid developments in electronics have led to the improvement in the design of measuring amplifiers. They are now characterized by high measurement resolution and a low level of noise. Accurate centring of an object is not required [2]. If the object centre does not coincide with the centre of rotation, it is important that the analysis of the measurement results take account of the nonlinearity of the function linking the real and the measured profiles.

## 2. Determining the centre of the associated circle for closed profiles [3]

Let us consider an *XY* coordinate system whose origin *O* is the point of rotation of the table or spindle (see Fig. 1)

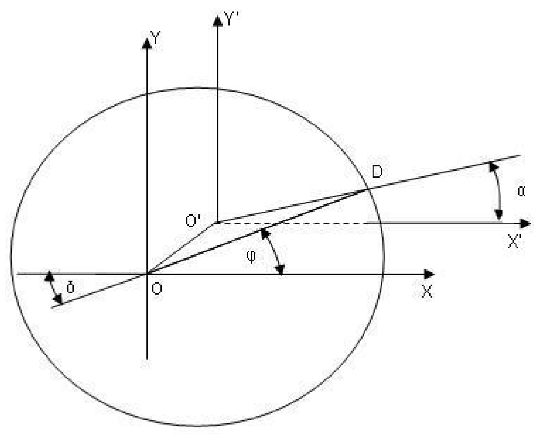


Fig. 1. Reciprocal position of the centre of rotation, *O*, and the centre of the nominal profile, *O'*
Rys. 1. Wzajemnie położenie środka obrotu *O* oraz środka zarysu nominalnego *O'*

Let the equation of the real profile in the polar system with origin *O* be equal to $R_m(\varphi)$. Let *O'* be an arbitrary point in the *XY*-plane. Let the equation of the profile in the polar system with origin *O'* have the form of $R'(\alpha)$. The equation depends on the coordinates of position of point *O'* in relation to point *O*. The coordinates of point *O'* in the *XY* system and in the polar system will be denoted by $(e_x, e_y)$ or $(e, \delta)$, respectively, where $e = \sqrt{e_x^2 + e_y^2}$, $\delta = \arctan(e_x, e_y)$. Coordinates $(e_x, e_y)$ and $(e, \delta)$ will be used interchangeably depending on the circumstances.

When the object centre is determined on the basis of relative readings of the instrument, the absolute value of the distance of the object profile from the centre of rotation, $O$, is not known. We know, however, that:

$$R_m = R_a + \Delta R_m \tag{1}$$

for certain $R_a$, but the value is not exactly known [4]. In the majority of cases, we know the nominal value of the object radius, $R_o$. The value can be identified basing on the value of the mean radius. The task of determining the centre of the associated circle (hereafter referred to as the mean circle) using the least-squares method can be formulated as follows: determine coordinates $(e, \delta)$ of point $O'$ and the mean value of the measured profile $R_a$, in such a way that for a given value of $R_o$, the integral:

$$J(e,\delta,R_a) = \frac{1}{2}\int_0^{2\pi}(R'(\alpha) - R_o)^2 \mathrm{d}\alpha \tag{2}$$

reaches a minimum in relation to variables $e$, $\delta$ and $R_a$.

The solution of the task simplifies considerably when the assumption that the roundness deviation is much smaller than the nominal radius $R_o$ is fulfilled. Figure 1 will be used to illustrate the problem in the considerations. As was assumed, the coordinates of point $O'$ in the $XY$-system are equal to $e_x$, $e_y$, while the distance of point $O'$ from the system origin and the angle of inclination between segment $\overline{OO'}$ and the $X$-axis is $e$ and $\delta$, respectively (thus: $e_x = e\cos\delta$, $e_y = e\sin\delta$). The angles between segments $\overline{OD}$ and $\overline{O'D}$ and the $X$-axis will be denoted by $\varphi$ and $\alpha$, respectively. The relationship between segments $\overline{OD}$ and $\overline{O'D}$ and angle $\varphi$ will be denoted by $R_m(\varphi)$ and $R(\varphi)$, respectively, while the distance of segment $\overline{O'D}$ in the function of angle $\alpha$ will be denoted by $R'(\alpha)$. The following relationships can be derived from Fig. 1:

$$R_m(\varphi) = e\cos(\varphi - \delta) + \sqrt{R^2(\varphi) - e^2\sin^2(\varphi - \delta)}, \tag{3}$$

$$\sin\alpha = \frac{R_m(\varphi)\sin\varphi - e\sin\delta}{R(\varphi)}, \tag{4}$$

$$\cos\alpha = \frac{R_m(\varphi)\cos\varphi - e\cos\delta}{R(\varphi)}. \tag{5}$$

If the roundness deviation is much small than the nominal radius, it can be assumed that:

$$\begin{aligned} R_m(\varphi) &= e\cos(\varphi - \delta) + \sqrt{R^2(\varphi) - e^2\sin^2(\varphi - \delta)} \cong \\ &\cong R_0(\varepsilon\cos(\varphi - \delta) + \sqrt{1 - \varepsilon^2\sin^2(\varphi - \delta)} \overset{\mathrm{df}}{=} \\ &= R_0 f(\varepsilon,\delta,\varphi) \end{aligned} \tag{6}$$

where:

$$f(\varepsilon,\delta,\varphi) \overset{\mathrm{df}}{=} \varepsilon\cos(\delta - \varphi) + \sqrt{1 - \varepsilon^2\sin^2(\varphi - \delta)}, \quad \varepsilon = \frac{e}{R_o}. \tag{7}$$

Then, the mean value of profile $R_m(\varphi)$ in the range $(0.2\pi)$ can be determined using the following relationship:

$$\begin{aligned} R_a(e) &= \frac{1}{2\pi}\int_0^{2\pi} R_m(\varphi)\mathrm{d}\varphi \cong R_o \frac{1}{2\pi}\int_0^{2\pi} f(\varepsilon,\delta,\varphi)\mathrm{d}\varphi = \\ &= R_o \frac{1}{2\pi}\int_0^{2\pi}\sqrt{1 - \varepsilon^2\sin^2(\varphi - \delta)}\mathrm{d}\varphi = \frac{2R_o}{\pi}\kappa(\varepsilon) \end{aligned}, \tag{8}$$

where $\kappa(\varepsilon)$ is a complete elliptic integral of the first kind [5].

Since component $\sqrt{1 - \varepsilon^2\sin^2(\varphi - \delta)}$ of function $f(\varepsilon,\delta,\varphi)$ is periodic with period $\pi$ in relation to variable $\varphi$, we can write that:

$$\begin{aligned} &\int_0^{2\pi}\sqrt{1 - \varepsilon^2\sin(\varphi - \delta)}\sin\varphi\mathrm{d}\varphi = 0, \\ &\int_0^{2\pi}\sqrt{1 - \varepsilon^2\sin(\varphi - \delta)}\cos\varphi\mathrm{d}\varphi = 0. \end{aligned} \tag{9}$$

Thus, using the assumption that the roundness deviation is much smaller than the nominal radius again, we get

$$\frac{1}{\pi}\int_0^{2\pi} R_m(\varphi)\cos\varphi\mathrm{d}\varphi \cong \frac{1}{\pi}\int_0^{2\pi} e\cos(\delta - \varphi)\mathrm{d}\varphi = e_x, \tag{10}$$

$$\frac{1}{\pi}\int_0^{2\pi} R_m(\varphi)\sin\varphi\mathrm{d}\varphi \cong \frac{1}{\pi}\int_0^{2\pi} e\sin(\delta - \varphi)\varphi\mathrm{d}\varphi = e_y. \tag{11}$$

From relationships (10) and (11) it is clear that the coefficients of the first harmonic component of the measured profile are approximately equal to the coordinates of the centre of the profile mean circle, despite the complex nonlinear relationship describing the measured profile.

From the above it is clear that the centre of the mean circle and the real profile can be determined basing on the measured profile, $\Delta R_m$, in the following way:

1. Calculate the coordinates of the mean circle from formulas (10) and (11):

$$e_x \cong \frac{1}{\pi}\int_0^{2\pi}\Delta R_m(\varphi)\cos\varphi\mathrm{d}\varphi, \quad e_y \cong \frac{1}{\pi}\int_0^{2\pi}\Delta R_m(\varphi)\sin\varphi\mathrm{d}\varphi.$$

2. Calculate the profile measured in absolute values

$$R_m = R_a(e) + \Delta R_m = \frac{2R_0}{\pi}\kappa\left(\frac{e}{R_a}\right) + \Delta R_m$$

3. Calculate the real profile from relationship (3), then

$$R(\varphi) = \sqrt{R_m^2(\varphi) - 2R_m(\varphi)e\cos(\delta - \varphi) + e^2},$$

$$\alpha(\varphi) = \arctan(R_m(\varphi)\cos\varphi - e\cos\delta, R_m(\varphi)\sin\varphi - e\sin\delta).$$

## 3. Determining the centre of the associated circle for interrupted profiles

Interrupted profiles are recorded, for instance, in the measurement of tooth wheels, transverse cross-sections of bearing rings, etc.. The solution of the problem of determining the centre of the associated circle for interrupted profiles was divided into two parts. In the first stage, the problem was solved for the case when quotient $\varepsilon = \dfrac{e}{R_0}$ is negligible. It was necessary to develop a special algorithm to determine the centre of the associated circle for an arbitrary value of $\varepsilon = \frac{e}{R_0}$:

a) the case of a low value of coefficient $\varepsilon$

Denote the set of angles for which the profile $\Delta R_m$ is known by $\Phi$. Linearizing Eq. (3) around $e = 0$, we obtain:

$$R_m(\varphi) = R(\varphi) + e\cos(\varphi - \delta) = R(\varphi) + e_x\cos\varphi + e_y\sin(\varphi). \quad (12)$$

Thus, the unknown coordinates of the centre of the mean circle, $R_o$, $e_x$, $e_y$, will be determined by minimizing the index number:

$$J(e_x, e_y, R_o) = \int_\Phi (\Delta R_m(\alpha) - R_o - e_x\cos\varphi - e_y\sin\varphi)^2 \mathrm{d}\varphi. \quad (13)$$

Comparing the partial derivatives $\frac{\partial J}{\partial R_o}$, $\frac{\partial J}{\partial e_x}$, $\frac{\partial J}{\partial e_y}$ to zero, we get:

$$\begin{bmatrix} \int_\Phi \cos^2\varphi \mathrm{d}\varphi & \int_\Phi \cos\varphi\sin\varphi \mathrm{d}\varphi & \int_\Phi \cos\varphi \mathrm{d}\varphi \\ \int_\Phi \cos\varphi\sin\varphi \mathrm{d}\varphi & \int_\Phi \sin^2\varphi \mathrm{d}\varphi & \int_\Phi \sin\varphi \mathrm{d}\varphi \\ \int_\Phi \cos\varphi \mathrm{d}\varphi & \int_\Phi \sin\varphi \mathrm{d}\varphi & \int_\Phi 1\, \mathrm{d}\varphi \end{bmatrix} \cdot \begin{bmatrix} e_x \\ e_y \\ R_o \end{bmatrix} = \begin{bmatrix} \int_\Phi \Delta R_m(\varphi)\cos\varphi \mathrm{d}\varphi \\ \int_\Phi \Delta R_m(\varphi)\sin\varphi \mathrm{d}\varphi \\ \int_\Phi \Delta R_m(\varphi) \mathrm{d}\varphi \end{bmatrix} \quad (14)$$

b) the case of an arbitrary value of coefficient $\varepsilon$

For a given point *O'* with coordinates $(e, \delta)$, let $R'(\alpha): \alpha \in A$ be the profile equation in the polar system with origin *O'*. We need to determine the values of $e$, $\delta$ and $R_a$ in such a way that the integral:

$$J(e, \delta, R_a) = \int_A (R'(\alpha) - R_o)^2 \mathrm{d}\alpha \quad (15)$$

can reach a minimum in relation to variables $e$, $\delta$ and $R_a$. Angle $\alpha$ is a function of angle $\varphi$ and it depends on the values of $\varepsilon$, $\delta$, thus (Eqs.(3-5)):

$$\begin{aligned} \alpha = g(\varepsilon, \delta, \varphi) = \arctan(f(\varepsilon, \delta, \varphi)\sin\varphi - \\ - e\sin\delta, f(\varepsilon, \delta, \varphi)\cos\varphi - e\cos\delta) \end{aligned} \quad (16)$$

Set $A$, on the basis of which the profile $R'(\alpha)$ is determined, is equal to $g(\varepsilon, \delta, \Phi)$ for a certain set of $\Phi$.

This method is iterative in nature. Let $\hat{e}$, $\hat{\delta}$ and $\hat{R}_a$ be certain estimates of unknown values of $e$, $\delta$ and $R_a$. In the first step, assume that $\hat{e} = 0$, $\hat{\delta} = 0$ and $\hat{R}_a = R_0$. Determine integral (15). As the measured profile is given by the function of angle $\varphi$, variable $\alpha$ will be substituted with $\varphi$ in relationship (15). First, we will determine the relationship between differentials $\mathrm{d}\alpha$ and $\mathrm{d}\varphi$. If we assume that the roundness deviation is smaller than the radius, then from relationships (4) and (5), we get:

$$\sin\alpha = f(\varepsilon, \delta, \varphi)\sin\varphi - \varepsilon\sin\delta, \quad (17)$$

$$\cos\alpha = f(\varepsilon, \delta, \varphi)\cos\varphi - \varepsilon\cos\delta. \quad (18)$$

By differentiating Eqs. (17) and (18), we obtain:

$$\cos\alpha \mathrm{d}\alpha = \left(f_\varphi(\varepsilon, \delta, \varphi)\sin\varphi + f(\varepsilon, \delta, \varphi)\cos\varphi\right)\mathrm{d}\varphi \quad (19)$$

$$-\sin\alpha \mathrm{d}\alpha = \left(f_\varphi(\varepsilon, \delta, \varphi)\cos\varphi - f(\varepsilon, \delta, \varphi)\sin\varphi\right)\mathrm{d}\varphi \quad (20)$$

After squaring the two equations, adding to both sides and extracting roots, we finally obtain:

$$\mathrm{d}\alpha = \sqrt{f_\varphi^2(\varepsilon, \delta, \varphi) + f^2(\varepsilon, \delta, \varphi)}\mathrm{d}\varphi \stackrel{df}{=} h(\varphi)\mathrm{d}\varphi. \quad (21)$$

If definition (7) is applied, then:

$$h(\varphi) = 1 + \frac{\varepsilon\cos\varphi}{\sqrt{1 - \varepsilon^2\sin^2\varphi}} \quad (22)$$

Substituting variable $\alpha$ in (15) with $\varphi$, we get:

$$\begin{aligned} J(e, \delta, R_a) &= \int_A (R'(\alpha) - R_o)^2 \mathrm{d}\alpha = \int_\Phi (R(\varphi) - R_o)^2 h(\varphi)\mathrm{d}\varphi \\ &= \int_\Phi \left(\sqrt{\left(R_m(\varphi)\cos\varphi - e_x\right)^2 + \left(R_m(\varphi)\sin\varphi - e_y\right)^2} - R_o\right)^2 h(\varphi)\mathrm{d}\varphi \end{aligned} \quad (23)$$

where: $R_m(\varphi) = R_a + \Delta R(\varphi)$. Let:

$$e_x = \hat{e}_x + \Delta e_x, \quad (24)$$

$$e_y = \hat{e}_y + \Delta e_y, \quad (25)$$

$$R_a = \hat{R}_a + \Delta R_a. \quad (26)$$

where $\hat{e}_x$, $\hat{e}_y$ and $\hat{R}_a$ are the current estimates of $e_x$, $e_y$ and $R_a$, while $\Delta e_x$, $\Delta e_y$ and $\Delta R_a$ are the corrections of the current estimates. Let us denote by $p(e_x, e_y, R_a) = \sqrt{\left(R_m(\varphi)\cos\varphi - e_x\right)^2 + \left(R_m(\varphi)\sin\varphi - e_y\right)^2}$.

After linearizing the $p(e_x, e_y, \varphi)$ function around point $(\hat{e}_x, \hat{e}_y, \hat{R}_a)$, we get:

$$\begin{aligned} p(e_x, e_y, \varphi) \cong\ & p(e_x, e_y, \varphi) \\ & + \frac{\partial p(e_x, e_y, \varphi)}{\partial e_x}\Delta e_x + \frac{\partial p(e_x, e_y, \varphi)}{\partial e_y}\Delta e_y + \frac{\partial p(e_x, e_y, \varphi)}{\partial R_a}\Delta R_a \Bigg|_{(e_x, e_y, R_a) = (\hat{e}_x, \hat{e}_y, \hat{R}_a)} \end{aligned} \quad (27)$$

Thus:

$$\begin{aligned} J_1(\Delta e_x, \Delta e_y, \Delta R_a) &= J(\hat{e}_x + \Delta e_x, \hat{e}_y + \Delta e_y, \hat{R}_a + \Delta R_a) \\ &= \int_\Phi \left(w(\varphi) - \mathrm{v}^T(\varphi)[\Delta e_x \quad \Delta e_y \quad \Delta R_a]\right)^2 \hat{h}(\varphi)\mathrm{d}\varphi \end{aligned} \quad (28)$$

where

$$w(\varphi) = \hat{R}(\varphi) - R_o, \quad (29)$$

$$\mathrm{v}(\varphi) = \frac{1}{\hat{R}(\varphi)} \begin{bmatrix} \hat{R}_m(\varphi)\cos\varphi - \hat{e}_x \\ \hat{R}_m(\varphi)\sin\varphi - \hat{e}_y \\ \hat{R}_m(\varphi) - \hat{e}\cos(\varphi - \hat{\delta}) \end{bmatrix}, \quad (30)$$

$$\hat{h}(\varphi) = 1 + \frac{\hat{\varepsilon}\cos(\varphi - \hat{\delta})}{\sqrt{1 - \hat{\varepsilon}^2\sin^2(\varphi - \hat{\delta})}}, \quad (31)$$

$$\hat{R}_m(\varphi) = \hat{R}_a + \Delta R_m(\varphi), \quad (32)$$

$$\hat{R}(\varphi) = \sqrt{\left(\hat{R}_m(\varphi)\cos\varphi - \hat{e}_x\right)^2 + \left(\hat{R}_m(\varphi)\sin\varphi - \hat{e}_y\right)^2}. \quad (33)$$

By minimizing the functional $J_1$ after $\Delta e$, $\Delta\delta$ and $\Delta R_a$, we finally have:

$$\begin{bmatrix}\Delta e_x\\ \Delta e_y\\ \Delta R_a\end{bmatrix} = \left(\int_{\Phi} \mathrm{v}(\varphi)\mathrm{v}^T(\varphi)h(\varphi)\mathrm{d}\varphi\right)^{-1} \int_{\Phi} w(\varphi)\mathrm{v}(\varphi)\mathrm{d}\varphi \cdot \quad (34)$$

Basing on the derived relationships, we can define the algorithm for determining the unknown parameters of the associated circle $e$, $\delta$ and $R_a$. The algorithm is iterative in nature and we can describe it in the following way:

1. Assume that $\hat{e}=0$, $\hat{\delta}=0$ and $\hat{R}_o = R_a$.
2. From relationships (32), (33), (30), (29), (31), determine $\hat{R}_m(\varphi)$, $\hat{R}(\varphi)$ and $\mathrm{v}(\varphi)$, $w(\varphi)$, $\hat{h}(\varphi)$.
3. From relationship (34), determine the corrections $\Delta e_x$, $\Delta e_y$ and $\Delta R_a$ of the current estimates, $\hat{e}_x$, $\hat{e}_y$ and $\hat{R}_a$.
4. Assume that $\hat{e}_x := \hat{e}_x + \Delta e_x$, $\hat{e}_y := \hat{e}_y + \Delta e_y$, $\hat{R}_a := \hat{R}_a + \Delta R_a$.
5. Stop the algorithm if the stop condition is satisfied $|\Delta e_x| + |\Delta e_y| + |\Delta R_a| \le \xi$ for a properly selected small number of $\xi$. Otherwise, move on to point 2 of the algorithm.

It should be noticed that the first step of the defined algorithm coincides with that of the traditional method described in the previous section.

## 4. Simulation results of the algorithm for interrupted profiles

Consider the following form of the real profile: $R'(\alpha) = 1 + k(\sin(3\alpha) + 0.5\cos(10\alpha) + 0.2\sin(50\alpha + \pi/2)$, $k = 0.001$ defined in the range of angle $\alpha \in [\alpha_1, \alpha_2] = A$. Assume that $\varepsilon = 0.1$, $\delta = \pi/4$, $\alpha_1 = -\pi/2$, $\alpha_2 = \pi/2$.

Figure 2 shows some other samples of the measured profile.

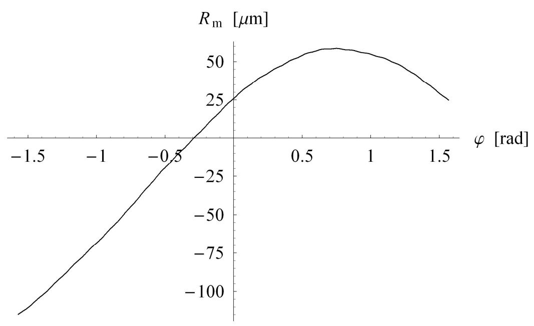


Fig. 2. Other samples of the measured profile $R_m(\varphi)$ obtained by simulation

Rys. 2. Kolejne próbki zarysu zmierzonego $R_m(\varphi)$ uzyskanego w wyniku symulacji

By applying the above mentioned algorithm, we obtained the following values of the corrections $(\Delta e_x, \Delta e_y, \Delta R_a)$:

1. First iteration:
   $(\Delta e_x = 0.0706799, \Delta e_y = 0.0727162, \Delta R_a = 0.0449252)$.
2. Second iteration:
   $(\Delta e_x = 0.0001163442, \Delta e_y = -0.00208172, \Delta R_a = -0.00239817)$.
3. Third iteration:
   $(\Delta e_x = -3.006325 \cdot 10^{-6}, \Delta e_y = 2.49451 \cdot 10^{-7}, \Delta R_a = -2.80133 \cdot 10^{-6})$.
4. Fourth iteration:
   $(\Delta e_x = 1.4454 \cdot 10^{-9}, \Delta e_y = -1.45832 \cdot 10^{-9}, \Delta R_a = 8.57539 \cdot 10^{-10})$.

As can be seen, we obtain the exact values of the parameters $(\Delta e_x, \Delta e_y, \Delta R_a)$ after the first two iterations. Figure 3 shows a graph of the $\Delta R(\alpha)$ profile deviation after the first and second iterations.

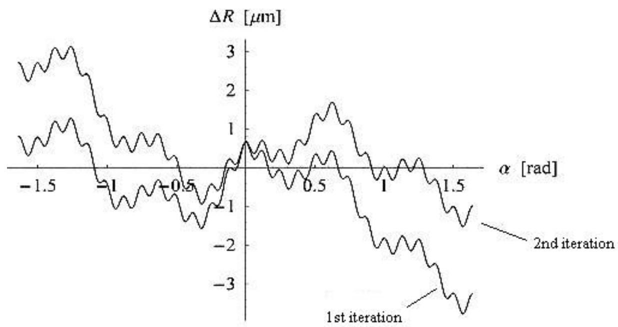


Fig. 3. Deviation of the $\Delta R'(\alpha)$ profile after the first and second iteration

Rys. 3. Odchyłka zarysu $\Delta R'(\alpha)$ po wykonaniu pierwszej i drugiej iteracji

## 5. Conclusion

Measurements of roundness deviations constitute an important area of geometrical metrology. As high accuracy is frequently a major requirement, it is essential that an analysis involve determining the position of the centre of the associated circle. Extensive studies have been conducted on the accuracy of measurements of closed roundness profiles. Many machine parts, however, have interrupted profiles. An example is the transverse cross-section of a bearing ring. Basing on the relationships derived above, one is able to calculate the position of the centre of the associated circle for interrupted profiles by applying the least squares method. It is important to note that the assumption that the ratio of non-centricity to the nominal radius is negligible does not have to be fulfilled. The concept of calculating the centre of the associated circle for interrupted profiles was verified by computer simulations. The results confirmed the correctness of the relationships derived. It was also shown that accurate results can be obtained after the second iteration is completed.

The paper was presented at the 3rd International Conference on "Metrology in Manufacturing Technologies" at Białystok University of Technology in September 2007.

## 6. References

[1] H. Bosse, F. Lüdicke and H. Reimann : An intercomparison on roundness and form measurement. Measurement. Vol. 13, No 2, 1994, pp. 107-117.
[2] Henke R. P. et al., Methods for evaluation of systematic geometric deviations in machine parts and their relationships to process variables, Precision Engineering, Vol. 23., Elsevier, 1999, pp. 273-292.
[3] Żebrowska-Łucyk S.: Non-reference method of measurement of macrogeometry of rotary elements. Warsaw University of Technology. Science Reports. Mechanics, no. 187, Warsaw University of Technology Publ., Warsaw 2000.
[4] Sacconi A., Pasin W.: An intercomparison of roundness measurements between ten European national standards laboratories. Measurement, Vol. 13, Nr 2, 1994, s. 119-128.
[5] Spiegel M. R.: Theory and Problems of Advanced Mathematics for Engineers and Scientists. Schaum's outline series, McGraw-Hill Book Company, New York,1971.

*Artykuł recenzowany*