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Method of Evaluation of Roundness Profiles of Machine Parts Using The Wavelet Analysis

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Jest adiunktem w Katedrze Technologii Mechanicznej i Metrologii Politechniki Świętokrzyskiej w Kielcach. W swoich pracach zajmuje się zagadnieniami inżynierii jakości oraz podstaw metrologii i pomiarów wielkości geometrycznych, a w szczególności analizą amplitudowo-częstotliwościową zarysów chropowatości, prostoliniowości i okrągłości. Posiada ponad 20-letnią praktykę pracy w przemyśle w zapleczu naukowym branży armatury przemysłowej.

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Abstract

In hitherto practice, roundness and cylindricity profiles have been evaluated using the Fourier transform. In numerous metrological branches, however, concerned with the evaluation of variable signals, the wavelet analysis is applied. Efforts have been undertaken to apply this method to the evaluation of geometrical surface structure. This paper describes the application of the Haar expansion to the approximation of roundness profiles and to the presentation of the concept of comparison of two roundness profiles decomposed by the wavelet method. The examples presented in the paper were used for preliminary evaluation of the proposed concept.

Keywords: Haar wavelet, Fourier transform, wavelet transform, geometrical surface structure, roundness, cylindricity.

Zastosowanie analizy falkowej do oceny zarysów okrągłości części maszyn

Streszczenie

W dotychczasowej praktyce w ocenie zarysów struktury geometrycznej powierzchni szeroko wykorzystuje się zasadę transformacji Fouriera. Znalazło to szczególne zastosowanie w analizie pomiarów okrągłości i walcowości. Z tego względu, że w wielu dziedzinach metrologii do oceny zarysów zmiennych stosuje się z dużym powodzeniem transformację falkową, podjęto prace nad aplikacją tej metody do oceny zarysów struktury geometrycznej powierzchni. Niniejszy referat poświęcony jest zastosowaniu rozwinięcia Haara do aproksymacji sygnałów zarysu nierówności powierzchni części maszyn oraz prezentacji koncepcji porównywania zarysów chropowatości powierzchni, które zostały poddane dekompozycji falkowej. Przedstawione w referacie przykłady posłużyły do wstępnej oceny zaproponowanej metody.

Słowa kluczowe: falka Haara, transformata Fouriera, transformata falkowa, struktura geometryczna powierzchni, okrągłość, walcowość.

1. Introduction

The geometrical surface structure has usually been evaluated by means of the discrete Fourier transform [1, 2]. This method requires conducting the harmonic analysis of the experimentally obtained surface profiles. Such an analysis usually allows establishing the following parameters of expansion of the profiles into a trigonometric Fourier series: A_n and B_n coefficients of individual harmonic component of roundness, waviness and roughness profile, amplitudes C_n of each harmonic component, phase shift φn of individual harmonic components [1, 2].

Algorithms of the above mentioned methods allow performing a frequency analysis that makes it possible to investigate the properties of the signal changing in time in the domain of frequency [3]. These methods enable analysis of stationary signals. However, most physical phenomena are non-stationary and they can be regarded as stationary only after applying specific approximation techniques [4]. Because traditional frequency analysis does not allow accurate observation of properties of non-stationary signals, therefore in such cases joint time-frequency representations of signals should be applied. These representations can be divided into two groups: time-frequency representations and time-scale representations. In order to improve quality of Fourier transform analysis, the method of wavelet analysis is getting more common. Wavelet analysis is applied in numerous areas of science and technology. It seems that this method should be adopted to analysis of signals representing irregularities of surfaces of machine parts. In the first stage of the investigation developing of the methodology of evaluation of roundness profiles and comparison of its efficiency to the Fourier analysis efficiency is planned. The aim of this paper is description of the method of decomposition of the signal recorded during roundness measurement by the Haar wavelets and presentation of the concept of comparison of decomposed profiles.

2. Decomposition of measured signal with use of Haar wavelets

2.1. Definition of Haar wavelet

The basis of the wavelet theory is Haar wavelet, which is defined as follows [5, 6]:

$$\begin{aligned} \psi(t) &= 1 \quad \text{for } 0 \le t < 0{,}5 \\ \psi(t) &= -1 \quad \text{for } 0{,}5 \le t < 1 \\ \psi(t) &= 0 \quad \text{for other t} \end{aligned} \tag{1}$$

This function generates the set of wavelets containing following elements:

$$\psi_{mn}(t) = 2^{-m/2}\psi(2^{-m} \cdot t - n) \quad \text{for m,n} = ...,-2,-1,0,1,2\ldots \tag{2}$$

Time intervals, in which elementary signals are not equal to zero are called support. On the basis of the equation (2) we can say that width of such interval for Haar wavelets is equal to 2m, so we can conclude that support of the function $\psi_{mn}(t)$ equals 2m, where integer value *m* is called a scale coefficient. Accordingly, n is called a shift coefficient, although complete shift of the basis wavelet Ψ depends also on the scale coefficient and it is equal to $2^m n$ [6].

2.2. Wavelet transform

Continuous wavelet transform (CWT) of the signal x(t) for given wavelet Ψ(t) is defined by the formula [3, 4]:

$$W(\tau,\sigma)=\int x(t)\psi^{*}_{t,\sigma}(t)dt,\quad \psi_{t,\sigma}(t)=\frac{1}{\sqrt{\sigma}}\psi\left(\frac{t-\tau}{\sigma}\right) \tag{3}$$

where:

τ – time shift
σ – scale (frequency).

Discrete wavelet transform (DWT) can be developed assuming discretization of the signal x(t) and following conditions:

$$\sigma=2^{-s},\qquad \tau=2^{-s}\cdot I \tag{4}$$

where:

I – shift coefficient.

Taking to account relationships (4), one obtains:

$$\begin{aligned}W(\tau,\sigma)&=W(I\cdot 2^{-s},2^{-s})=W(I,s)=\\&=2^{s/2}\int_{-\infty}^{\infty}\psi(2^{s}\cdot t-I)dt=\\&=2^{s/2}\sum_{n=0}^{N-1}x(n)\psi(2^{s}\cdot n-I)\end{aligned} \tag{5}$$

After developing above mentioned definitions one can formulate expansion of the function into wavelet series:

$$x(t)=\sum_{s}\sum_{I}W_{I,s}\cdot\psi_{I,s}(t),\quad \psi_{I,s}(t)=2^{s/2}\cdot\psi(2^{s}\cdot t-I) \tag{6}$$

2.3. Approximation of the signal using Haar expansion

Equation of the function $f^{(-m_0)}(t)$, which is assumed to be a measured signal is described by formula 7

$$f^{(-m_0)}(t)=\sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1}f_n^{(-m_0+1)}\varphi_{(-m_0+1)n}(t)+\sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1}d_n^{(-m_0+1)}\psi_{(-m_0+1)n}(t). \tag{7}$$

The scheme of the algorithm presented above, which is based on averaging of the function values for adjacent determination intervals and on respecting the difference of details in each interval is presented in the Fig. 1.

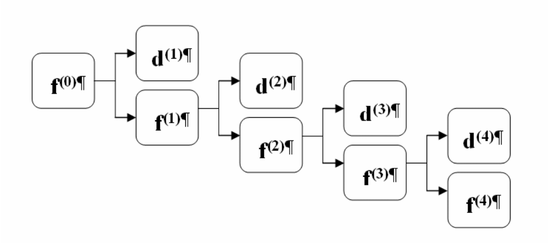


Fig. 1. The tree of the wavelet decomposition: $f^{(0)}$ - recorded signal, $f^{(i)}$ – i-step approximation, $d^{(i)}$ – detail in the step i

Rys. 1. Drzewo dekompozycji falkowej: $f^{(0)}$ - sygnał zarejestrowany, $f^{(i)}$ - aproksymacja w i – tym kroku, $d^{(i)}$ - element w i – tym kroku.

2.4. Example of the model decomposition

In order to investigate of the process of the approximation of the measured signal (for example in measurement of the surface irregularities) with Haar wavelets practically, simulation calculations of the non-real signals assuming low values of parameters m_0 and m_1 were performed.

The assumptions are:
Output function of the profile $f^{(0)}$ is defined in the second row of the Table 1, Calculation parameters: $m_0=0$ and $m_1=3$.

Tab. 1. Values of the output data and calculation of the simulation results of the approximation of the function $f^{(0)}$ for parameters $m_0=0$ and $m_1=3$

Tab. 1. Zestawienie danych wyjściowych i wyników obliczeń symulacyjnych aproksymacji funkcji $f^{(0)}$ dla parametrów $m_0=0$ i $m_1=3$

t	$f^{(0)}$	$f^{(1)}$	$d^{(1)}$	$f^{(2)}$	$d^{(2)}$	$f^{(3)}$
-8	-2	-2	0	-0.25	-1.75	0.375
-7	-2	-2	0	-0.25	-1.75	0.375
-6	1	1.5	-0.5	-0.25	1.75	0.375
-5	2	1.5	0.5	-0.25	1.75	0.375
-4	0	0.5	-0.5	1	-0.5	0.375
-3	1	0.5	0.5	1	-0.5	0.375
-2	1	1.5	-0.5	1	0.5	0.375
-1	2	1.5	0.5	1	0.5	0.375
0	1	2	-1	2.75	-0.75	1.75
1	3	2	1	2.75	-0.75	1.75
2	3	3.5	-0.5	2.75	0.75	1.75
3	4	3.5	0.5	2.75	0.75	1.75
4	4	3.5	0.5	0.75	2.75	1.75
5	3	3.5	-0.5	0.75	2.75	1.75
6	-2	-2	0	0.75	-2.75	1.75
7	-2	-2	0	0.75	-2.75	1.75

3. The principle of profiles comparison using wavelets

In order to compare profiles transformed by the wavelet analysis, it is necessary to present obtained profile in a discrete form in the variability interval (measurement distance), which is divided by the number of intervals, being the multiplicity of natural power of 2.

The comparison should be started from the expanding of two profiles: datum one (f_w) and compared one (f_p) according to following equations:

$$f_w^{(-m_0)}(t)=f_w^{m_1}(t)+\sum_{m=m_0+1}^{m_1}\sum_{n=-2^{m_1-m}}^{2^{m_1-m-1}}d_n^{m}\psi_{mn}(t) \tag{8}$$

$$f_p^{(-m_0)}(t)=f_p^{m_1}(t)+\sum_{m=m_0+1}^{m_1}\sum_{n=-2^{m_1-m}}^{2^{m_1-m-1}}d_n^{m}\psi_{mn}(t) \tag{9}$$

The comparative analysis of individual profiles should be preceded by assuming values of indicators „m_0". defining the length of the quantization interval „m_1" and measurement distance (function domain). whose length is equal to $2*2^{m_1}$ in the range $< 2^{-m_1}; 2^{m_1} >$. Then profiles are compared by the indicator of partial fitting D_p and the indicator of full fitting. which are calculated by the three different values of the parameter w_j for given value of the parameter $k = 2^{m_1}$:

$$D_{pi} = (f_p^{m_1} - f_w^{m_1})^2 + \sum_{j=1}^{m_1} w_j (d_{jp} - d_{jw})^2 \tag{10}$$

$$D_p = \sum_{i=-k}^{k-1} D_{pi} \tag{11}$$

where:

$$k = 2^{m_1+m_0} \tag{12}$$

Application of three versions of the fitting indicator is the result of the fact that presented method is still in the stage of tests and therefore different criteria allowing determination of the dependence between profiles should be investigated.

Values of w_j used in the calculations are following:

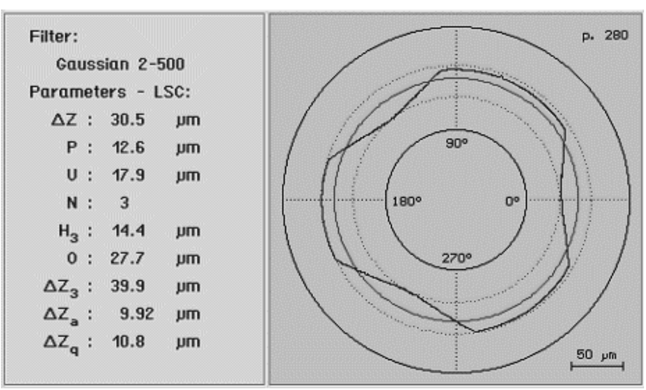
- $w_j = 1$ - contribution of the main component is equal to the contribution of other subsequent corrections.
- $w_j = 2^{-j}$ - contribution of corrections in relation to the contribution of the main component is getting smaller as the number „j" of the corrections grows
- $w_j = 2^{-m_1}$ - all the corrections have the same significance in relation to the main component. but their contribution into the value of the fitting indicator is much smaller.

4. Verification of the method for the case of roundness measurement

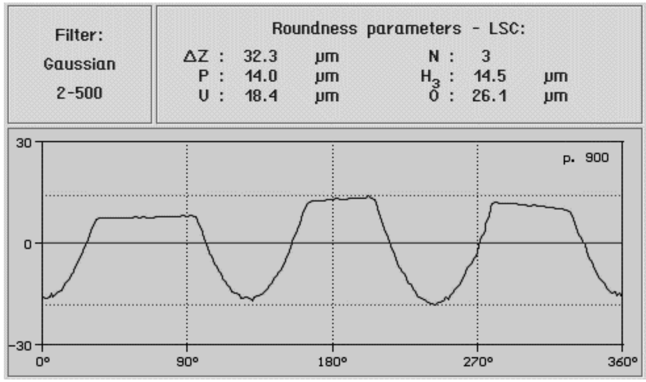
In Fig. 2, 3 and 4 real roundness profiles obtained through measurement of the cylindrical workpiece in three cross-sections with Talyrond 200 measuring device and the profiles obtained through the discretization (Figures c) are presented. Discretization was performed assuming that the domain of the profile should be divided into 16 intervals. The number of intervals is the result of assumed approximation parameters. i.e. $m_0 = 0$ and $m_1 = 3$. Real profiles are presented in polar (figures a) and Cartesian (Figures b) coordinates. Each profile in discrete form had been processed through approximation by the wavelet analysis and then selected couples of profiles were compared according to proposed method.

Results of the calculations for different methods of comparison of profiles are presented in Table 2.

a)



b)



c)

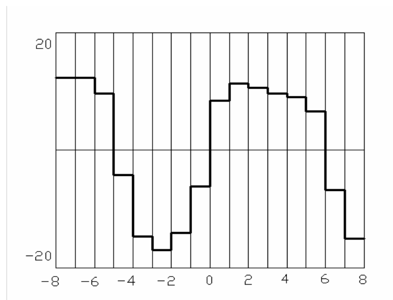
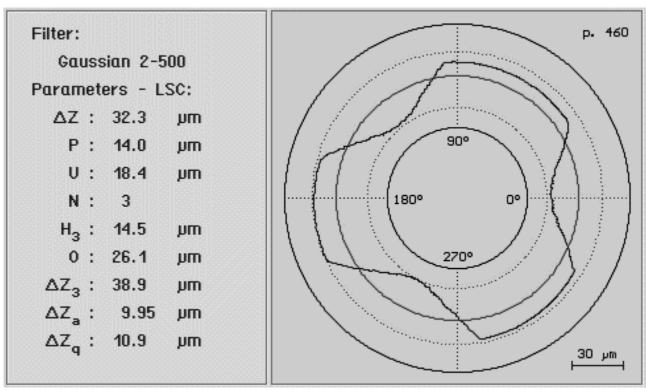


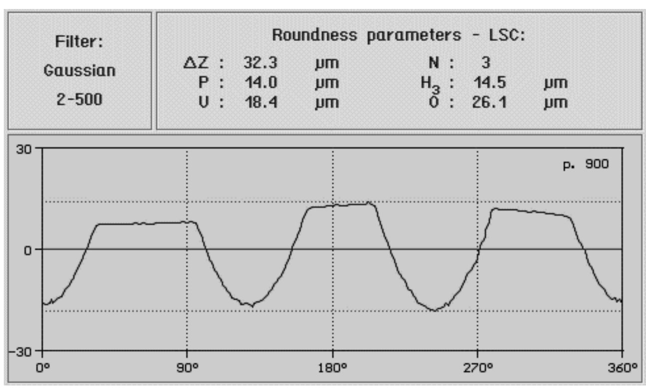
Fig. 2. Roundness profile of the cylindrical workpiece – cross-section no. 1
a) real profile measured with Talyrond 200 (polar coordinates)
b) real profile measured with Talyrond 200 (Cartesian coordinates)
c) the fragment of the profile after discretization selected for the wavelet analysis

Rys. 2. Zarys okrągłości mierzonej części – przekrój poprzeczny nr 1:
a) zarys zmierzony za pomocą przyrządu Talyrond 200 (współrzędne biegunowe),
b) zarys zmierzony za pomocą przyrządu Talyrond 200 (współrzędne prostokątne),
c) odcinek zarysu po dyskretyzacji, wybrany do analizy falkowej

a)



b)



c)

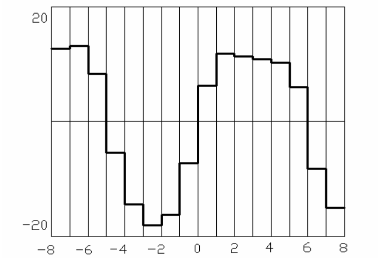
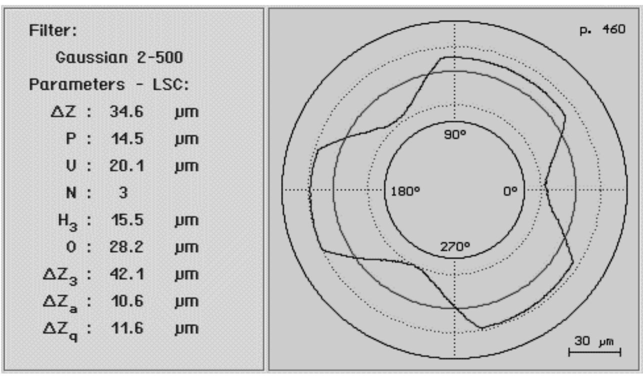


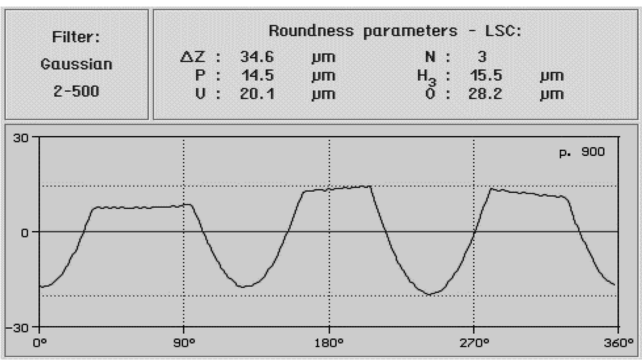
Fig. 3. Roundness profile of the cylindrical workpiece – cross-section no. 2
a) real profile measured with Talyrond 200 (polar coordinates)
b) real profile measured with Talyrond 200 (Cartesian coordinates)
c) the fragment of the profile after discretization selected for the wavelet analysis

Rys. 3. Zarys okrągłości mierzonej części – przekrój poprzeczny nr 2:
a) zarys zmierzony za pomocą przyrządu Talyrond 200 (współrzędne biegunowe),
b) zarys zmierzony za pomocą przyrządu Talyrond 200 (współrzędne prostokątne),
c) odcinek zarysu po dyskretyzacji, wybrany do analizy falkowej

a)



b)



c)

Fig. 4. Roundness profile of the cylindrical workpiece – cross-section no. 3
a) real profile measured with Talyrond 200 (polar coordinates)
b) real profile measured with Talyrond 200 (Cartesian coordinates)
c) the fragment of the profile after discretization selected for the wavelet analysis

Rys. 4. Zarys okrągłości mierzonej części – przekrój poprzeczny nr 3:
a) zarys zmierzony za pomocą przyrządu Talyrond 200 (współrzędne biegunowe),
b) zarys zmierzony za pomocą przyrządu Talyrond 200 (współrzędne prostokątne),
c) odcinek zarysu po dyskretyzacji, wybrany do analizy falkowej

Tab. 2. Results of the calculations of profiles comparison
Tab. 2. Wyniki obliczeń porównywania zarysów

Compared profiles	Variant of the calculation	Value of the fitting indicator D_p [μm²]
1 - 2	$w_j = 1$	20.49
	$w_j = 2^{-j}$	9.99
	$w_j = 2^{-m_1}$	6.16
1 - 3	$w_j = 1$	45.92
	$w_j = 2^{-j}$	16.59
	$w_j = 2^{-m_1}$	8.92
2-3	$w_j = 1$	17.00
	$w_j = 2^{-j}$	6.38
	$w_j = 2^{-m_1}$	4.12

5. Summary and conclusions

The algorithm described in the paper has been presented in a model way for very small values of the signal decomposition parameters. i.e. $m_0 = 0$ and $m_1 = 3$. In practice these parameters should be much larger, but then the algorithm presented in the paper would require special computer software and it would not be clear for purposes of presentation of the method. Among the investigated calculation variants, the variant of equal contribution of the approximated profile and subsequent corrections ($w_j = 1$) seems to be the most appropriate. Further research work on application of the wavelet theory to evaluation of surface irregularities of machine parts should be carried out in following research areas:

Development of the computer software performing the process of the approximation and expansion of the measured signal using wavelets for any value of parameters m_0 and m_1.

Performing practical verification of the software through its application to evaluation of real signals of roundness profiles and comparison of its results to the results of Fourier analysis.

Development of the concept of comparison of profiles obtained with different measuring instruments or with the same instrument for different workpieces with use of the wavelet analysis.

Conducting optimization investigations aiming at defining criteria allowing correct selection of calculation parameters and form of basic wavelet and determination if their influence on the quality of performed calculations.

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