APPLICATION OF THE CORRELATION FUNCTION FOR DETERMINATION OF DAMPING DURING IN-FLIGHT FLUTTER TESTS

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The paper presents a method for determination of damping during in-flight flutter tests. The method is based on a new estimate of the correlation function. Actually, the necessity has emerged of a new estimator of the autocorrelation function which, when applied to finite unsteady signals of short duration, does not change the values and signs of damping coefficients. The paper provides theoretical backgrounds of the proposed method. The new correlation function was applied to the ANDAT procedure [4,5,6] for flutter analysis. The procedure was verified based on both simulation data and in-flight vibration measurements.
Keywords: Flight tests, vibration measurement, damping coefficients, autocorrelation function.

1. INTRODUCTION

The flight flutter tests are performed on the prototype versions of new or significantly modified commercial and military aircraft in order to demonstrate freedom from flutter over the entire flight envelope.

Traditionally, the most widely used indicators of the stability in aeroelastic systems are the values of modal damping coefficients and their airspeed-variations. In the typical flight flutter tests, the damping coefficients for all significant modes are evaluated at a number of sub critical speeds using the system identification methods. Classical methods for flight flutter investigations require a special excitation of vibrations. These vibrations can be forced in a harmonic, impulse or stochastic way.

Up till now the ANDAT system has been used at the Institute of Aviation to analyze these vibrations.

The ANDAT software package is based on the least-square-technique in time domain, employing the time-frequency method for the analysis of impulse responses. The modes of importance are determined using a statistical method in terms of the rest sum of squares procedure (F-Snedecor distribution). The frequency and damping coefficients are determined versus the airspeed. The ANDAT system employs the real and imaginary parts of the Fourier transform for the determination of vibration phase.

One of the aims of the FliTE project consists in the development of flight flutter test techniques under natural excitation conditions [8], with no additional control surface excitation applied. To make the flight flutter tests under natural excitation conditions (in normal flight), the impulse response can be replaced with the autocorrelation function of the measured vibration signals. This function represents the same frequency and damping factors as the infinite response function.

The estimators of autocorrelation function being used now [2,7] were developed for stationary signals and when applied to the analysis of signals of finite duration they can produce errors in the determination of damping coefficients.

Therefore, the necessity has emerged for determination of a new estimator of the autocorrelation function, which when applied to finite unsteady signals of short duration does not change the values and signs of damping coefficients.

The paper provides theoretical backgrounds of the proposed method. The new estimator of the correlation function was applied to the ANDAT procedure for flutter analysis. The procedure was verified using both the model data and the real data from in-flight vibration measurements.

2. DETERMINATION OF THE AUTOCORRELATION FUNCTION ESTIMATOR FOR NON STATIONARY PROCESSES

In the literature on the possibilities of replacement of the impulse response signal with the autocorrelation function of this signal when calculating the damping coefficients, in the course of proving their equivalence, the following formula was obtained:

$$R_{ijk}(\tau) = \sum_{r=1}^{N}\sum_{s=1}^{N} \alpha_k \Psi_{ir}\Psi_{kr}\Psi_{js}\Psi_{ks} \int_{0}^{+\infty} g_r(t+\tau) g_s(t) dt \qquad (1)$$

where

$$g_s(t) = \frac{1}{m_s \omega_{sd}} \exp(-\zeta_s \omega_{s0} t) \sin(\omega_{sd} t) \qquad (2)$$

R_{ijk} is the cross-correlation function defined for two response signals at points i and j, that result from the white noise excitation applied at point k,
α_k is the correlation function of the white noise at point k,
Ψ_{ir} is the i-th component of r-th vibration form,
Ψ_{kr} is the k-th component of r-th vibration form,
Ψ_{js} is the j-th component of s-th vibration form,
Ψ_{ks} is the k-th component of s-th vibration form.
The formula for $g_r(t)$ has a similar form.

The integral appearing in Eq. *(1)* stands therefore for the cross-correlation function between the signals $g_s(t)$ and $g_r(t)$. For the objects with positive damping coefficients of vibrations these signals reveal the nature of free decaying signals (nonstationary).

The estimator of autocorrelation function commonly applied to stationary processes of finite duration cannot be used in this case. For example, the MATLAB package provides the following unbiased estimator of autocorrelation function of a discrete stationary signal of finite duration (for the sake of simplicity hereinafter we consider only the autocorrelation function):

$$R_{yy}(k) = \frac{1}{N-k}\sum_{n=1}^{N-k} y(n)y(n+k) \quad k = 0,1,2,...,N-1 \quad (3)$$

This estimator, however, changes the values of damping coefficients of non-stationary signals, the detailed reasons behind that will be presented below.

Let us now search for some hints about the determination of the autocorrelation function estimator for damped , exponential increasing or stationary signals.

It is well-known that for the decaying infinite (from 0 to $+\infty$) signal, the integral appearing in Eq. (1) can be determined and damping coefficient of autocorrelation function is equal to the damping coefficients of the signal under consideration.

It should be noted, however, that the real signals we deal with are of finite duration and the integral is calculated over that very finite duration (being approximated in terms of a sum) not up to infinity.

Thus, we have arrived at the first conclusion that a definite integral over a constant range should appear in the formula for the sought estimator of autocorrelation function , i.e. the sum should have constant limits, independent of the shift τ: despite of the value of k. Therefore the estimator given in Eq. *(3)* does not satisfy this condition since the calculated sum depends on k.

To simplify the process of finding the required properties of the estimator that does not change the values of damping coefficient let us deal with the integral for a sample signal:

$$y(t) = e^{-\lambda t} \quad t \in \langle 0, +\infty \rangle \quad (4)$$

For this signal we have:

$$R_{yy}(\tau) = \int_0^{+\infty} e^{-\lambda t} e^{-\lambda(t+\tau)} dt = e^{-\lambda\tau}\int_0^{+\infty} e^{-2\lambda t} dt =$$
$$= e^{-\lambda\tau}[-\frac{1}{2\lambda}(e^{-2\lambda t}\Big|_0^{+\infty})] = e^{-\lambda\tau}[-\frac{1}{2\lambda}(0-1)] = \quad (5)$$
$$= \frac{1}{2\lambda}e^{-\lambda\tau}$$

It can be clearly seen that the autocorrelation function for the exponential curve has not changed the value of the exponent λ (the damping, e. q. in the impulse response signal $\lambda = \zeta\omega_0$) replacing however the initial value *1* with $1/2\lambda$.

Let us ask the question if there exists the estimator of the considered exponential function over a finite interval that does not change the value of damping coefficient λ.

We can answer the question by means of calculation of the integral given above from *0 to T* :

$$R_{yy}(\tau) = \int_o^{T} e^{-\lambda t} e^{-\lambda(t+\tau)} dt = e^{-\lambda\tau}\int_0^{T} e^{-2\lambda t} dt =$$
$$= e^{-\lambda\tau}[-\frac{1}{2\lambda}(e^{-2\lambda t}\Big|_0^{T})] = e^{-\lambda\tau}[-\frac{1}{2\lambda}(e^{-2\lambda T}-1)] = \quad (6)$$
$$= \frac{1}{2\lambda}(1-e^{-2\lambda T})e^{-\lambda\tau}$$

It can be found that for a finite duration for the considered signal the integral can be calculated and the resulting autocorrelation function does not change the value and sign of the coefficient λ. Only the initial value of autocorrelation signal has been changed.

It should be noted that before assuming the upper limit of integration as T, one should know the segment of the considered function $exp[-\lambda(t+\tau)]$ within the interval from *0* to $(T+\tau_{max})$.

Let us analyse in detail what the above formula for transformation really means, using the following diagram (see Fig. 1).

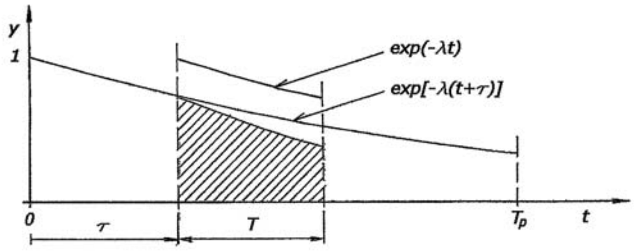

Fig. 1

The integral is calculated in terms of determination of the marked area. The value of autocorrelation function for the shift τ is represented by the marked area. Therefore, the calculation process of autocorrelation function consists in calculating of the area (for t within the {0-T} interval) under a segment of the signal shifted along the considered signal. For the initial location ($\tau = 0$), one obtains the maximal value of autocorrelation function, while the minimal one is observed for τ_{max} location, when the end of the shifted signal coincides with the end of the considered one. Generally, when one deals with a signal of, e.g. 2s duration, then if a signal of 1 s duration is shifted, the maximum shift τ_{max} can be only 1 s. Generally, the formula for maximum shift τ_{max} for the segment duration T and the considered signal duration Tp , respectively, can be written as:

$$\tau_{max} = T_p - T \quad (7)$$

Therefore we can assume, e.g., that $T = T_p/2$ and then:

$$\tau_{max} = Tp/2.$$

For a discrete signal the aforementioned calculation rules take the following estimator form:

$$R_{yy}(k) = \frac{1}{N/2}\sum_{i=1}^{N/2} y(i)\ y(i+k)$$
$$k = 0, 1, 2, 3, ..., N/2-1 \quad (8)$$

where: $k\Delta t = \tau$ $k = 0,1,2,..., N/2$

$n\Delta t = t$

$(N/2)\Delta t = T$

N – represents the number of samples in the signal: $N\Delta t = T_p$.

To reveal clearly the differences as compared to that obtained from Eq. 3 the result of calculations is divided by $N/2$ (the division by a constant number does not affect the value of λ). In practice, dividing the result by $N/2$ makes the codomain of autocorrelation function and the values of the considered signal closer to each other.

Graphical rules for the determination of autocorrelation function as shown in Fig. 1. for the proposed estimator (see Eq. 8) can be also formulated for the unbiased estimator (see Eq. 3) for stationary signals, applicable also to visibly non-stationary signals (see Fig. 2).

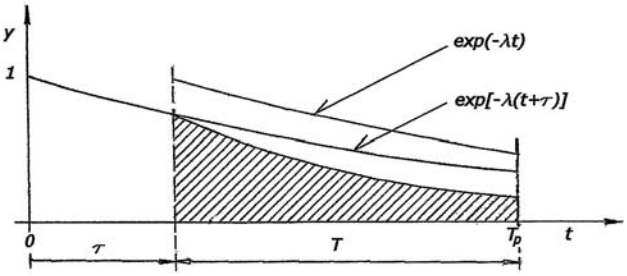

Fig. 2

It can be clearly seen from the picture that the shifted segment changes its duration depending on the value of shift τ. The segment duration reads:

$$T(k) = T_P - \tau$$

Roughly speaking in terms of changing the segment duration we change the calculation rules. At the same time, the fact that the calculation results are divided by the segment duration ($N - k$) affects the average segment duration growth as the shift τ rises for decaying signals. As a result we obtain higher calculation results and lower values of the coefficient λ of the calculated autocorrelation function as compared to the coefficient of the considered signal.

It should be emphasised, however, that the fact that the results were divided by the segment duration allowed for obtaining of the unbiased estimator of autocorrelation function for steady signals.

The proposed estimator of autocorrelation function for exponent decaying , increasing or stationary signals can be considered as a tool for time-frequency analysis of non-stationary signals. It can be therefore applied to the analysis of momentary dynamical properties of non-stationary processes.

3. VERIFICATION OF THE PROCEDURE

To verify that the new estimator can be used for the time-frequency analysis of non-stationary signals, a number of analyses have been carried out at the Institute of Aviation in Warsaw, Poland.

The following three methods were taken into account:

- **ANDAT1** procedure, in which the impulse responses is analysed with no autocorrelation function introduced;
- **ANDAT1acp** procedure, which uses the proposed estimator of autocorrelation function;
- **ANDAT1unb** procedure, which employs the estimator of autocorrelation function obtained from the MATLAB package.

The following types of vibrations representative of the problem were analyzed:

- Damped model vibrations - the results obtained are shown in Table 1
- Damped real vibrations measured on the aircraft wing after buffeting – the results obtained are shown in Table 2
- Model vibrations with increasing amplitude - the results obtained are shown in Table 3
- Stationary model vibrations - the results obtained are shown in Table 4

Tab.1

Mode number	Model damping coefficient [%]	Damping coefficient resulting from ANDAT1 [%]	Damping coefficient resulting from ANDAT1acp [%]	Damping coefficient resulting from ANDAT1unb [%]
1	1.5	1.5	1.5	0.5
2	2.0	2.1	2.3	1.2
3	2.0	2.0	2.1	1.4
4	1.5	1.6	1.4	1.0

Tab.2

Mode number	Damping coefficient resulting from ANDAT1 [%]	Damping coefficient resulting from ANDAT1acp [%]	Damping coefficient resulting from ANDAT1unb [%]
1	4.6	4.8	3.3
2	2.3	3.9	3.1
3	-	7.6	9.7

Tab.3

Mode number	Model damping coefficient [%]	Damping coefficient resulting from ANDAT1 [%]	Damping coefficient resulting from ANDAT1acp [%]	Damping coefficient resulting from ANDAT1unb [%]
1	-1.5	-1.5	-1.5	+0.6 (!!!)

Tab.4

Mode number	Model damping coefficient [%]	Damping coefficient resulting from ANDAT1 [%]	Damping coefficient resulting from ANDAT1acp [%]	Damping coefficient resulting from ANDAT1unb [%]
1	0.0	0.0	0.0	0.0

The obtained results prove that the proposed estimator does not change the damping coefficients (both the value and sign remain unchanged). The autocorrelation function of exponent decaying signal is a signal of the same frequency components and the same damping coefficients; i.e., a decaying signal. The autocorrelation function of an increasing signal is an increasing signal. The proposed estimation of autocorrelation function for a stationary signal is a stationary signal.

4. FINAL REMARS

Finally, let us present a concise calculation algorithm of the proposed estimator of autocorrelation function.

A new estimator of the autocorrelation function:

The data from measurements: $y(i)$ $i = 1, 2, 3, \ldots, N$

$$R_{yy}(k) = \frac{1}{N/2} \sum_{i=1}^{N/2} y(i)\, y(i+k)$$

$$k = 0, 1, 2, 3, \ldots, N/2 - 1$$

Computation procedure for the FFT [1] autocorrelation:

The data from measurements: $y(i)$ $i = 1, 2, 3, \ldots, N$

Define $yp(i)$ data:

$$yp(i) = y(i) \quad i = 1, 2, \ldots, N/2$$

$$yp(i) = 0 \quad i = N/2+1, \ldots, N$$

Compute the discrete transform of $y(i)$ and $yp(i)$:

$$Y(j) = \mathrm{FFT}[y(i)]$$

$$YP(j) = \mathrm{FFT}[yp(i)]$$

Compute the product:

$$Y_{yy}(j) = Y(j)\, YP^*(j) \qquad j=1,2,3,\ldots,N$$

Averaging:

$$\overline{Y_{yy}(j)} = \overline{Y(j)\ YP^*(j)}$$

Compute the inverse transform:

$$R_{yy}(k) = (2/N) \bullet \mathrm{FFT}^{-1}[Y_{yy}(j)] \qquad k = 0, 1, 2, 3, \ldots, N/2-1$$

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ZASTOSOWANIE FUNKCJI KORELACJI DO WYZNACZANIA WSPÓŁCZYNNIKA TŁUMIENIA W CZASIE PRÓB FLATTEROWYCH W LOCIE

Artykuł prezentuje metodę wyznaczania tłumienia w czasie prób flatterowych w locie. Metoda oparta jest o nową estymację funkcji korelacji. Prezentowany estymator funkcji autokorelacji, zastosowany do skończonych, niestacjonarnych, krótkich sygnałów nie zmienia wartości i znaków wyznaczonych współczynników tłumienia.

Artykuł przedstawia teoretyczne podstawy proponowanej metody. Metoda ta zastosowana została w procedurze analizy flatterowej ANDAT. Testowano ją stosując sygnały symulowane oraz zarejestrowane w czasie prób w locie.

Praca zrealizowana w ramach programu EUREKA E!2419 „FLITE”.

Ф. Ленорт, А. Непокульчицки

ПРИМЕНЕНИЕ ФУНКЦИЙ КОРРЕЛЯЦИИ ДЛЯ ОПРЕДЕЛЕНИЯ КОЭФФИЦЕНТА ДЕМПФИРОВАНИЯ ВО ВРЕМЯ ФЛАТТЕРНЫХ ЛЕТНЫХ ИСПЫТАНИЙ

В стате представлен метод определения демпфирования во время флаттерных летных испытаний. Метод основан на новой эстимации функции корреляции. Представляемый эстиматор функции автокорреляции примененный к конечным, нестационарным, коротким сигналам не меняет величины и знаков определенных коэффициентов демпфирования. В статье обсуждаются теоретические основы предлагаемого метода. Метод был применен процедуре флаттерного анализа ANDAT. Тестировали его применяя имитационные и зарегистрованные сигналы во время летных испытаний.

Работа была осуществлена в рамках программа EUREKA E! 2419 „FLITE”.