HYPOTHESIS OF LOADS CYCLE REDUCTION TO EQUIVALENT FATIGUE CYCLE

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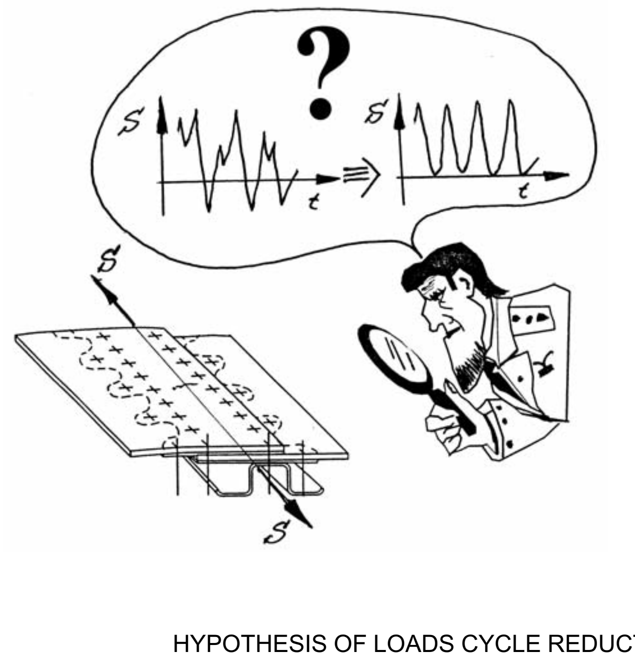
Paper presets a new scientific hypothesis for reduction of any given fatigue cycle to the equivalent cycle, in sense of the fatigue wear. This hypothesis should allow designers to enhance quality of the computational fatigue analyses, particularly analyses of the riveted structures. In effect, implementation of the presented hypothesis should allow designers create lighter, more stressed and – hopefully – equally safe riveted structures.



NOMENCLATURE:

l - crack length,
a - one-half of the crack length,
C1 - Forman constant,
C2 - Erdogan constant,
k - exponent in cycle reduction relationship,
$K = \sigma \sqrt{\pi a}$ - Stress Intensity Factor (SIF),
K_C - critical value of K,
K_r - K value in equivalent load cycle,
$K_{\max}$ - maximum K value in load cycle,
$K_{\min}$ - minimum K value in load cycle,
$\Delta K = K_{\max} - K_{\min}$,
m, n - exponents in Forman or Erdogan formulas,
N -number of load cycle,
R - stress ratio: $K_{\min}/K_{\max} = \sigma_{\min}/\sigma_{\max} = P_{\min}/P_{\max}$,
P_r - equivalent load cycle,
$P_{\max}$ - maximum load value,
$P_{\min}$ - minimum load value,
$\Delta P = P_{\max} - P_{\min}$,
$\sigma_{\max}$ - maximum stress,
$\sigma_{\min}$ - minimum stress,
$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$,
$\sigma_a = \Delta\sigma/2$,
$\sigma_m = (\sigma_{\max} + \sigma_{\min})/2$,
$\sigma_{r\,\text{sym}}$ - stress in equivalent symmetric stress cycle,
$\sigma_c = \dfrac{K_c}{\sqrt{\pi \cdot a}}$ - critical stress,
ψ - cycle asymmetry susceptibility coefficient,
σ_r - stress in equivalent stress cycle,
$\sigma_0(T)$ – strength parameter, dependent on temperature (e.g. Rm(T) material tensile strength),
$P_0(T)$ - strength parameter, dependent on temperature (e.g. element tensile strength),
T - temperature,
$\sigma_{r\,\text{comp}}$ - equivalent stress for complex stress state.

Development of computational methods and tools, allows us to realize computational analysis of complex and more complex models. It is always a better or worse trial of complex phenomena description by simplified phenomena models. Dependent on experience and knowledge level, those models describe the phenomena with not always sufficient approximation. In papers [1 ... 10], one can find foundations for issues, described here.

Creation of the models, computational models included, is an asymptotic aspiration to the perfect model, therefore, to the such model, which fully describes given phenomenon. But model is always an simplification. Such complex physical process is a phenomenon of material and construction fatigue under random or given in time loads. Computational fatigue analyses, realized on the basis of accepted process run computational models, allow us to:

- Optimisation and prognosis of constructions load structure critical points from fatigue strength point of view,
- Prognosis of the expected fatigue durability,
- Prognosis of fatigue failures initiation places and propagation velocities (for *Damage Tolerance* type structures),
- Selection of diagnostic methods.

Alongside summation model of fatigue failures, usually based on Palmgren - Miner cumulative linear damage hypothesis, reduced load cycle model is a principal element of every fatigue computational analysis. It results from the fact, that fatigue characteristics (Wholer curves) for materials and constructions are usually calculated for symmetric or pulsating cycles. One can provide examples of hypotheses on which models of load (strain) cycle reduction to equivalent fatigue cycle were build.

ODING HYPOTHESIS:

$$P_r = \sqrt{P_{\max} \cdot \Delta P} \tag{1}$$

Supposing linear dependence between *K* and loading *P* we obtain:

$$K_r = \sqrt{K_{\max} \cdot \Delta K} \tag{2}$$

and supposing linear dependence between loads and stresses:

$$\sigma_r = \sqrt{\sigma_{\max} \cdot \Delta\sigma} \tag{3}$$

CYCLE ASYMMETRY SUSCEPTIBILITY COEFFICIENT HYPOTHESIS:

$$\sigma_{r\,sym} = \sigma_a + \psi \cdot \sigma_m \tag{4}$$

D-G-D LOAD REDUCTION'S HYPOTHESIS [12]:

$$K_r = K_{\max}^{\frac{1}{3}} \cdot (\Delta K)^{\frac{2}{3}}$$

or as a function of cycle ratio **R**:

$$K_r = K_{\max} \cdot (1-R)^{\frac{2}{3}} \tag{5}$$

or

$$K_r = \frac{\Delta K}{(1-R)^{\frac{1}{3}}} \tag{6}$$

Supposing linear interdependence of load, stress, and Stress Intensity Factor we obtain identical relationships for loads:

$$P_r = P_{\max}^{\frac{1}{3}} \cdot (\Delta P)^{\frac{2}{3}} \tag{7}$$

or as a function of cycle ratio **R**:

$$P_r = P_{\max} \cdot (1-R)^{\frac{2}{3}} \tag{8}$$

or

$$P_r = \frac{\Delta P}{(1-R)^{\frac{1}{3}}} \tag{9}$$

and for stresses:

$$\sigma_r = \sigma_{\max}^{\frac{1}{3}} \cdot (\Delta\sigma)^{\frac{2}{3}} \tag{10}$$

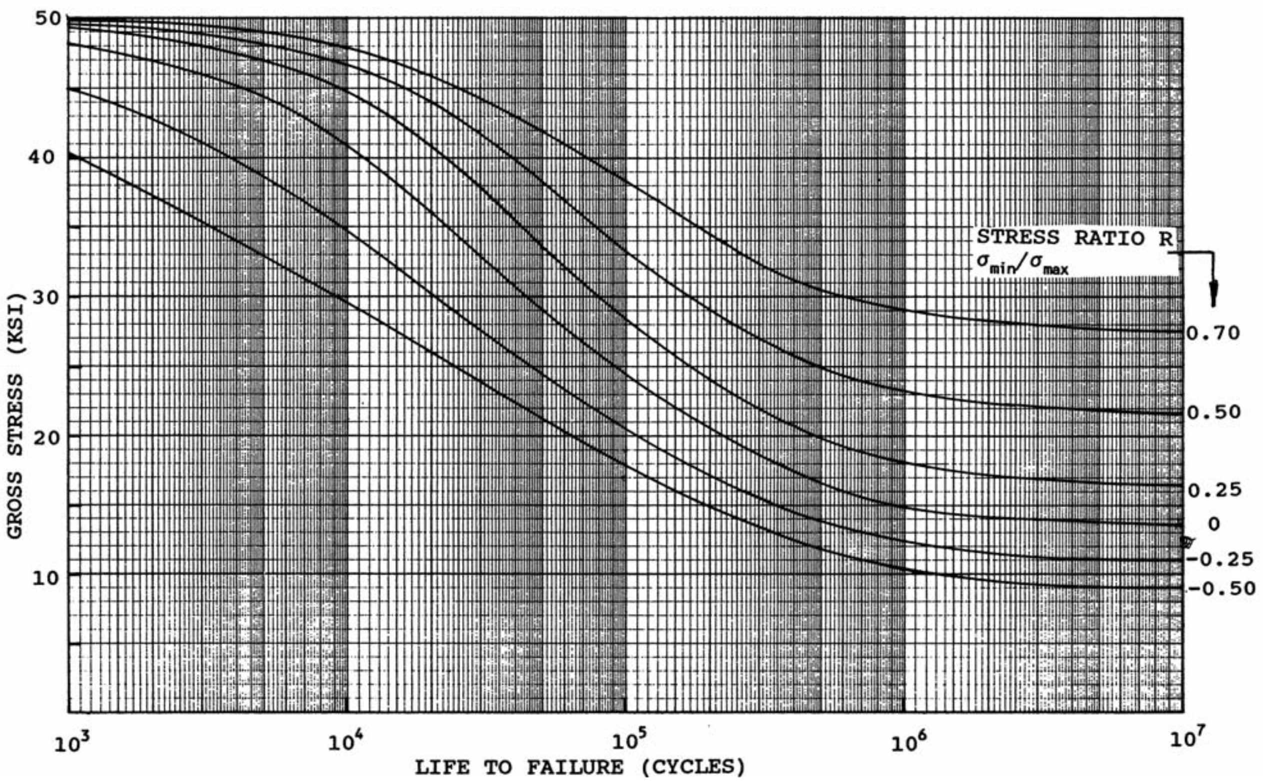


Fig. 1: Fatigue Sn data for good quality 3/16 inch diameter rivets in 2024-T3 Material [9]

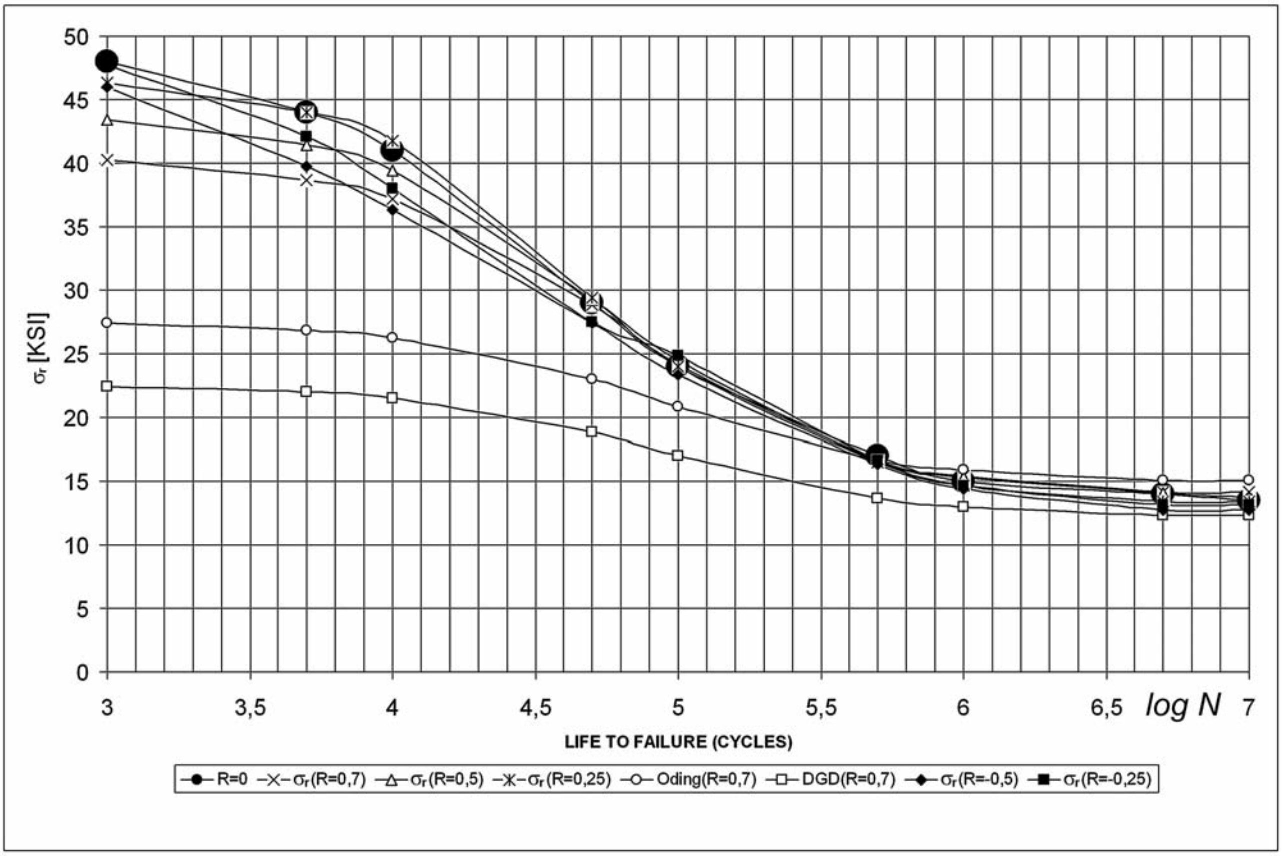


Fig. 2. Strain cycle reduction according to dependence (13) (σ_0 (T=20° C) = Rm = 61 KSI (420 MPa)) and, for the comparison, according to the Oding and DGD hypotheses.

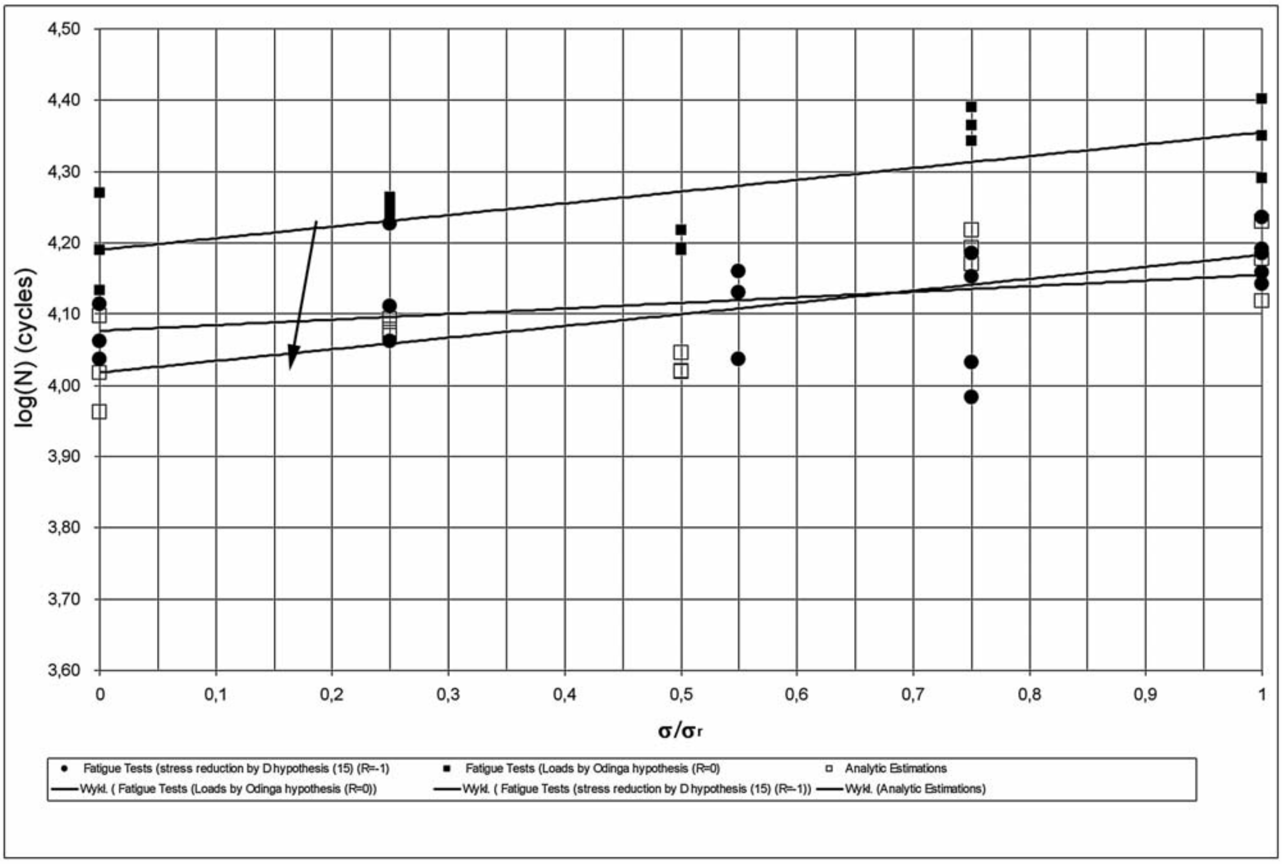


Fig. 3. Computational – experimental estimation of load cycle reduction in complex load state

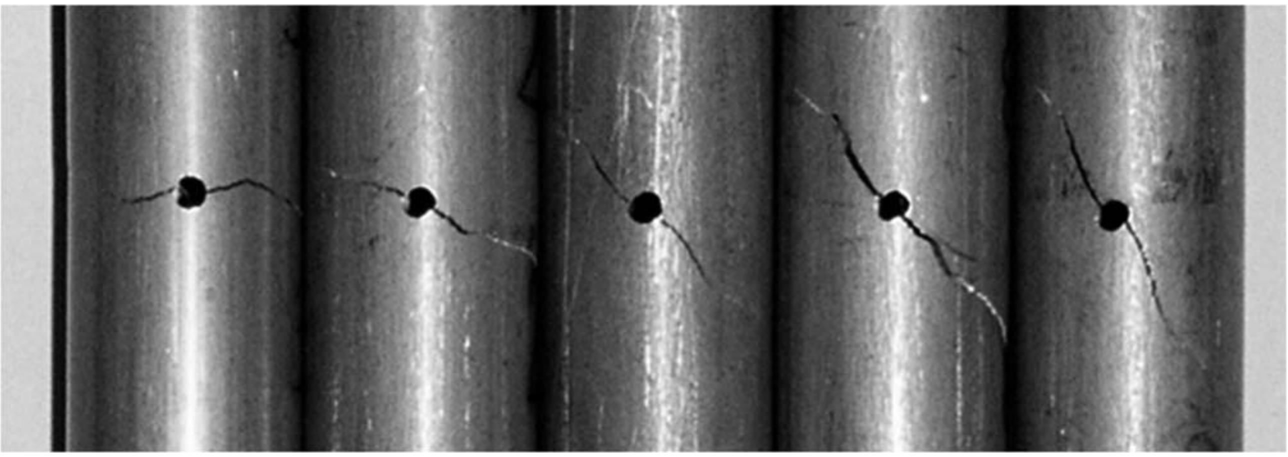


Fig. 4: Examples of fatigue crack in equivalent complex loads state by Huber-Mises hypothesis [11]

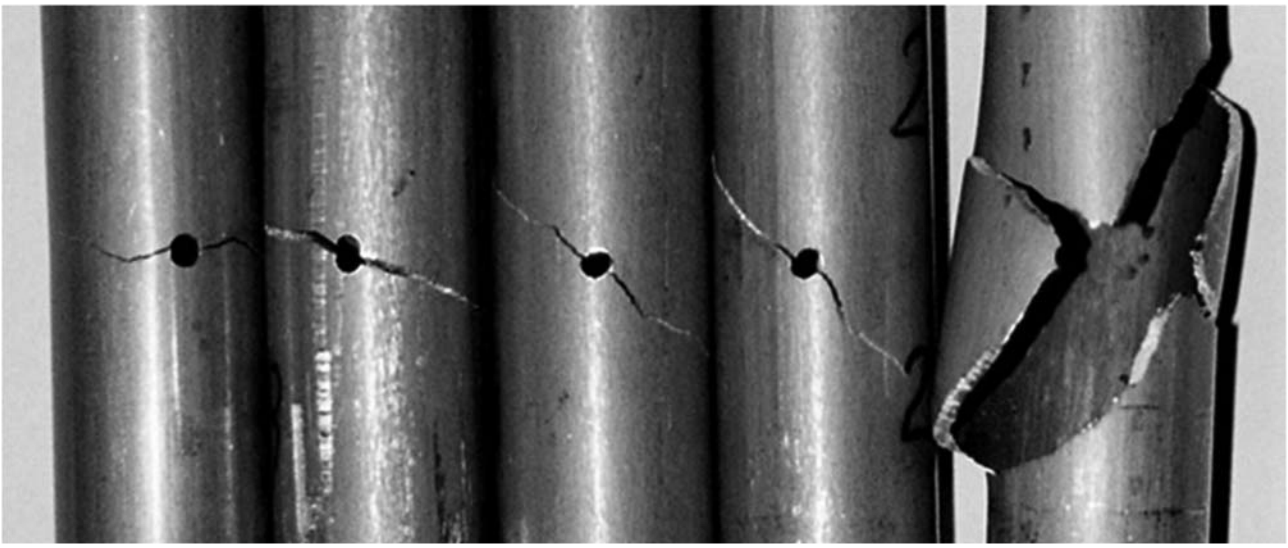


Fig. 5: Examples of fatigue crack in equivalent complex loads state by D hypothesis [11]

or as a function of cycle ratio **R**:

$$\sigma_r = \sigma_{\max} \cdot (1-R)^{\frac{2}{3}} \tag{11}$$

or

$$\sigma_r = \frac{\Delta\sigma}{(1-R)^{\frac{1}{3}}} \tag{12}$$

PROPOSED HYPOTHESIS OF DISCRETIONARY LOAD CYCLE REDUCTION TO THE EQUIVALENT FATIGUE LOAD CYCLE

Figure 1 shows fatigue curves Sn for riveted duralumin assembly. It is a typical construction assembly. It can be easily shown, that a good reduction dependency for discretionary load cycle with cycle asymmetry coefficient R (stress ratio) to the equivalent fatigue cycle with coefficient R=0 is dependence:

$$\boxed{\sigma_r = \sigma_{\max}\left(1-R\right)^{1-\frac{\sigma_{\max}}{\sigma_0(T)}}} \tag{13}$$

accordingly, for loads it will be:

$$\boxed{P_r = P_{\max}\left(1-R\right)^{1-\frac{P_{\max}}{P_0(T)}}} \tag{14}$$

or

$$\boxed{\sigma_r = \sigma_{\max}^{\frac{\sigma_{\max}}{\sigma_0(T)}} \cdot \left(\sigma_{\max} - \sigma_{\min}\right)^{1-\frac{\sigma_{\max}}{\sigma_0(T)}}} \tag{13A}$$

$$\boxed{P_r = P_{\max}^{\frac{P_{\max}}{P_0(T)}}\left(P_{\max} - P_{\min}\right)^{1-\frac{P_{\max}}{P_0(T)}}} \tag{14A}$$

Figure 2 presents runs of equivalent fatigue strain cycles, reduced according to proposed hypothesis (dependence 13).

For comparison, results of strain cycle reductions, according to Oding hypothesis and DGD hypothesis are also shown. Relatively good conformability for proposed hypothesis with fatigue tests results, shown on Figure 1, can be seen.

Proposed hypothesis projects results presented on Figure 1 much better, than Oding hyphothesis and DGD hypothesis. The truth is, that experimental results shown on Figure 1, does not include temperature influence.

In our opinion, this influence can be included through introduced here strength parameters $\sigma_0(T)$, $P_0(T)$.

Such parameter, accepted here for reduction shown on Figure 2, has value approaching the duralumin tensile strength σ_0 *(T*=20°C*)* = Rm = 61 KSI (420 MPa).

In case of loads reduction, loads in the form of tensile forces acting on given construction element, such parameter can be equal to the tensile strength value.

Best of all is, when selection of those parameters results from experimental data analysis, both own and taken from bibliography.

In paper [11], experimental comparison of complex stress state reduction hypotheses and estimation of material stress: $\sigma_{\max}$, $\tau_{\max}$, Huber – Mises, and presented in papers: [3, 4] hypothesis, called shortly D-hypothesis, is presented:

$$\sigma_{r\,\mathrm{comp}} = \frac{\sigma}{3} + \frac{2}{3}\sqrt{\sigma^2 + 4\tau^2} \qquad (15)$$

D-hypothesis appeared to be the best criterion.

Fatigue research tests were realized on constructional elements in form of cylindrical rods and thin walled tubing with diameter of 16 milimeters and with a notch in form of 2mm cylindrical opening. Elements were made from steel and duralumin material, as basic construction elements. Tests were performed in complex load (strain) state.

Axial force $P(\sigma)$ and torque $M_S(\tau)$ were applied simultaneously.

The basic assumption in those tests was the constant value of reduced strains σ_r , according to cited hypotheses.

Additionally, fatigue research tests for load cycles, reduced according to Oding hypothesis were made for cycles with coefficient R=0. Results of those tests are presented on Fig. 3. Those results were never published before. They will be published after future fatigue research tests for load cycles reduced according to hypothesis (13, 14).

Computational assumption was then made. Value of the fatigue curve Sn exponent was assumed as equal to 4. Result of these assumptions is presented on Fig. 3 (Analytic Estimation, Fatigue Tests, (stress reduction by D hypothesis)). Fatigue cracks in equivalent complex loads state by **Huber-Mises** and by **D** hypothesis are shown on Fig. 4, 5. The runs of the fatigue cracks are seemingly identical for both hypothesis, but numbers of cycles to fatigue crash are most different (pp 84, 85 [11]).

Small recapitulation:

It shows, that load cycles reduced according to hypothesis (13, 14) give better conformability than load cycles reduced, according to Oding hypothesis.

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HIPOTEZA REDUKCJI CYKLU OBCIĄŻENIA DO CYKLU ZMĘCZENIOWO EKWIWALENTNEGO

Streszczenie

W pracy zaproponowano nową hipotezę redukcji zadanego, dowolnego cyklu obciążenia zmęczeniowego do cyklu równoważnego w sensie zużycia zmęczeniowego. Hipoteza ta powinna pozwolić na na podniesienie jakości obliczeniowych analiz zmęczeniowych – zwłaszcza konstrukcji nitowanych.

W efekcie powinna też umożliwić projektowanie coraz lżejszych, bardziej wysilonych i co najmniej równie bezpiecznych struktur nośnych konstrukcji

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ГИПОТЕЗА ПРИВЕДЕНИЯ ЦИКЛА НАГРУЗКИ К ЭКВИВАЛЕНТНОМУ УСТАЛОСТНОМУ ЦИКЛУ

Резюме

В работе предложена новая гипотеза приведения, заданного произвольного усталостного цикла нагрузки к равноценному циклу в смысле усталостного износа. Эта гипотеза должна дать возможность повысить качество расчетных усталостных анализов – особенно клёпаных конструкций.

В результате далжна также сделать возможным проектирование более легких, более напряженных и по крайней мере одинаково безопасных структур несущих конструкуий.