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Spectral analysis of deck structures excited by sea waves

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Abstract

Two computational formulations for spectral analysis of linear superstructures subject to deck motion excitations are presented in the paper. Finite element equations of motion are decoupled by using the modal transform from the generalized nodal displacements to a set of the normal coordinates. Time and maximum responses of the system are estimated by using alternatively the root-mean-square technique or spectral density method. Computer implementations of the two approaches are discussed. Numerical algorithms worked out can be incorporated to fit into existing finite element codes with no difficulty. Illustrative results show that the approaches appear to be effectively employed in ship engineering.

Keywords: spectral analysis, deck structures, sea waving modeling

Analiza widmowa konstrukcji pokładowych wymuszonych falami morskimi

Streszczenie

W pracy przedstawiono dwa komputerowe sformułowania dla liniowej analizy widmowej konstrukcji pokładowych, wymuszonych ruchem pokładu. Układ równań ruchu, opisany w kontekście elementów skończonych, rozprzężono poprzez transformację modalną z uogólnionych przemieszczeń węzłowych do zbioru współrzędnych głównych. Odpowiedzi: czasowa oraz maksymalna zostały oszacowane alternatywnie drogą średniej kwadratowej lub metodą gęstości widmowej. Przedyskutowano implementacje komputerową obydwóch sformułowań. Opracowane algorytmy numeryczne mogą być wygodnie zaimplementowane w istniejących pakietach programów elementów skończonych. Wyniki ilustrujące wykazują, że podejścia te mogą być skutecznie zastosowane w inżynierii okrętowej.

Słowa kluczowe: analiza widmowa, konstrukcje pokładowe, fale morskie

1. Introduction

Time and spectral methodologies for ship and offshore structures such as superstructures, deck cranes, etc., have been extensively discussed in the literature. In almost all the formulations, however, stress-displacement behaviour or response maxima are treated in the framework of static systems or dynamic systems under structural loadings, described de-terministically as well as stochastically, cf. [1-4]. In contrast to the massive literature on earthquake problems (onshore systems), cf. [5], theoretical and practical applications to super-structural structures subject to deck motion have had little attention. Much essential work remains to be done, and state-of-the-art software for such a computational option is rather scarce.

In the context of the finite element setting this study is a numerical attempt to spectral analysis of superstructures excited by their rigid-base motion. After a brief description of the decoupling technique, Sections 2 and 3 deal with the spectral response and the first two probabilistic moments. Analysis of maxima is discussed in Section 4. This is followed by two illustrative examples and concluding remarks, Section 5.

2. Spectrum response

In accordance with the finite element formalism response of a *N-DOF* linear structural system subject to base accelerations can be described by a coupled system of linear ODEs

$$M_{\alpha\beta}\ddot{q}_\beta(\tau)+C_{\alpha\beta}\dot{q}_\beta(\tau)+K_{\alpha\beta}q_\beta(\tau)=-M_{\alpha\beta}\ddot{q}^{\mathrm{b}}_\beta(\tau) \quad (1)$$

with the two initial conditions prescribed. In this system q_α, $\alpha = 1, 2,..., N$, is the vector of nodal displacements from the undeformed configuration, q^{b}_α the vector of nodal reference (to the rigid-base) displacements, while the dot and double-dot denote first and second time derivatives. The symbols $M_{\alpha\beta}$, $C_{\alpha\beta}$ and $K_{\alpha\beta}$; $\alpha, \beta = 1, 2,..., N$, indicate the system mass, damping and stiffness matrix, respectively. The summation convention is applied throughout in the text.

Equation (1) can be numerically integrated over the time and frequency domains by using various algorithms, cf. [1-6], out of which the mode superposition technique is essential in spectral analysis. Let q_α be approximated via the vector of normal (modal) coordinates x_ρ, $\rho = 1,2,...,V$, $V \ll N$, as $q_\alpha = \varphi_{\alpha\rho}x_\rho(\tau)$, where the time-independent matrix $\varphi_{\alpha\rho}$ contains V mode shapes, each with ρ fixed, are solved for from a generalized eigenproblem. Further, by assuming in the obtained system that the mass-orthonormality and stiffness-orthogonality conditions are satisfied and the damping effect are of the Rayleigh type, i.e. it can be expressed as a linear combination of the system mass stiffness, $C_{\alpha\beta} \cong aM_{\alpha\beta} + bK_{\alpha\beta}$, a and b being constants, we arrive at the decoupled system

$$\ddot{x}_\rho + 2\xi_\rho\omega_\rho\dot{x}_\rho + \omega_\rho^2 x_\rho = \ddot{x}^{\mathrm{b}}_\rho, \quad \rho = 1,2,\ldots,V \quad (2)$$

where ω_ρ^2, $\rho = 1, 2,\ldots,V$, is the ρ-th system eigenvalue, being squares of the natural frequency ω_ρ, while the ρ-th modal damping coefficient reads

$$\xi_\rho = \frac{1}{2}\left(\frac{a}{\omega_\rho} + b\omega_\rho\right) \quad (3)$$

and the ρ-th modal acceleration of the rigid base is expressed as

$$\ddot{x}^{\mathrm{b}}_\rho = \varphi_{\rho\alpha}M_{\rho\alpha}\ddot{q}^{\mathrm{b}}_\beta, \quad \alpha, \beta = 1,2,...,N; \ \rho = 1,2,...,V \quad (4)$$

The modal response $x_\rho(\tau)$ to the problem (2) can be written at any time $\tau = t \in [0, \infty)$ as

$$x_\rho(t) = \int_0^\infty h_\rho(t-\tau)\ddot{x}^{\mathrm{b}}_\rho(\tau)\,\mathrm{d}\tau \quad \text{(no sum on } \rho) \quad (5)$$

where the unit-impulse-response function $h(\tau)$ is defined as

$$\ddot{h}_\rho(\tau) + 2\xi_\rho\omega_\rho\dot{h}_\rho(\tau) + \omega_\rho^2 h_\rho(\tau) = \delta(\tau) \quad (6)$$

with $\delta(\tau)$ being the Dirac-delta distribution. The Fourier transform of Eq. (5) reads

$$\int_0^\infty x_\rho(t)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t = \int_0^\infty\left[\int_0^\infty h_\rho(t-\tau)\ddot{x}_\rho^{\mathrm{b}}(\tau)\mathrm{d}\tau\right]\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}t =$$
$$= \int_0^\infty\int_0^\infty h_\rho(t-\tau)\ddot{x}_\rho^{\mathrm{b}}(\tau)e^{-\mathrm{i}\omega(t-\tau+\tau)}\overbrace{\mathrm{d}(t-\tau)}^{=\mathrm{d}t}\mathrm{d}\tau = \quad (7)$$
$$= \int_0^\infty h_\rho(v)\,\mathrm{e}^{-i\omega v}\mathrm{d}v\int_0^\infty \ddot{x}_\rho^{\mathrm{b}}(\tau)\,\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau$$

(no sum on ρ) with 'i' being the imaginary unit. We rewrite Eq. (7) as

$$X_\rho(\omega) = H_\rho(\omega)\ddot{X}_\rho^{\mathrm{b}}(\omega) \quad \text{(no sum on } \rho) \tag{8}$$

where

$$X_\rho(\omega) = \int_0^\infty x_\rho(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau\,,$$
$$\ddot{X}_\rho^{\mathrm{b}}(\omega) = \int_0^\infty \ddot{x}_\rho^{\mathrm{b}}(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau \tag{9}$$

stand for the spectral nodal displacements and reference accelerations, respectively, and

$$H_\rho(\omega) = \int_0^\infty h_\rho(\tau)\mathrm{e}^{-i\omega\tau}\mathrm{d}\tau \tag{10}$$

is the complex-frequency-response function, which is time-independent. (For nonlinear or non-stationary systems H is a function of both frequency and time and Eqs. (2), (5) and (8) do not hold true. This aspect, however, goes beyond the scope of the text.) The inverse transforms of Eqs. (9) and (10) give

$$x_\rho(\tau) = \frac{1}{2\pi}\int_0^\infty X_\rho(\omega)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\omega\,,$$
$$\ddot{x}_\rho^{\mathrm{b}}(\tau) = \frac{1}{2\pi}\int_0^\infty \ddot{X}_\rho^{\mathrm{b}}(\omega)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\omega \tag{11}$$

and

$$h_\rho(\tau) = \frac{1}{2\pi}\int_0^\infty H_\rho(\omega)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\omega \tag{12}$$

To obtain the explicit expression for the complex-frequency-response function H_ρ we transform both sides of Eq. (6) as

$$\int_0^\infty\left[\ddot{h}_\rho(\tau)+2\xi_\rho\omega_\rho\dot{h}_\rho(\tau)+\omega_\rho^2h_\rho(\tau)\right]\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = \int_0^\infty\delta(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = 1 \tag{13}$$

By using Eq. (12) yields

$$\dot{h}_\rho(\tau) = \mathrm{i}\omega\frac{1}{2\pi}\int_0^\infty H_\rho(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega = \mathrm{i}\omega h_\rho(\tau)$$
$$\ddot{h}_\rho(\tau) = -\omega^2\frac{1}{2\pi}\int_0^\infty H_\rho(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega = -\omega^2h_\rho(\tau) \tag{14}$$

This implies, cf. Eq.(l0)

$$\dot{H}_\rho(\omega) = \int_0^\infty \dot{h}_\rho(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = \mathrm{i}(\omega)\int_0^\infty h_\rho(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = \mathrm{i}\omega H_\rho(\omega)$$
$$\ddot{H}_\rho(\omega) = \int_0^\infty \ddot{h}_\rho(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = -\omega^2\int_0^\infty h_\rho(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = -\omega^2H_\rho(\omega) \tag{15}$$

Substituting Eqs (10) and (15) into Eq. (13) we obtain

$$H_\rho(\omega) = \left\{\omega_\rho^2\left[1-\left(\frac{\omega}{\omega_\rho}\right)^2+\mathrm{i}2\xi_\rho\frac{\omega}{\omega_\rho}\right]\right\}^{-1} \tag{16}$$

which is the complex-frequency-response function for the ρ-th mode, and

$$\left|H_\rho(\omega)\right| = \sqrt{H_\rho(\mathrm{i}\omega)H_\rho^*(\mathrm{i}\omega)} = \left(\omega_\rho^2\left\{\left[1-\left(\frac{\omega}{\omega_\rho}\right)^2\right]^2+\left(2\xi_\rho\frac{\omega}{\omega_\rho}\right)^2\right\}^{\frac{1}{2}}\right)^{-1} \tag{17}$$

Following the same line as for Eq. (15) the relationships between spectral displacements and spectral velocities and accelerations for the ρ-th mode take the form

$$\dot{X}_\rho(\omega) = \mathrm{i}\omega X_\rho(\omega),\quad \ddot{X}_\rho(\omega) = -\omega^2X_\rho(\omega) \tag{18}$$

Having solved Eq. (8) for the all the mode shapes the total spectral displacements of the system can be computed straightforward. According to the definition

$$Q_\alpha(\omega) = \int_0^\infty q_\alpha(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau \quad \alpha = 1,2,\ldots,N \tag{19}$$

we have, from Eqs. (8) and $(9)_1$

$$Q_\alpha(\omega) = \int_0^\infty x_\rho(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = \varphi_{\alpha\rho}\int_0^\infty x_\rho(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau =$$
$$= \varphi_{\alpha\rho}X_\rho(\omega) = \sum_{\rho=1}^{V}\varphi_{\alpha\rho}H_\rho(\omega)\ddot{X}_\rho^{\mathrm{b}}(\omega) \tag{20}$$

and, consequently

$$q_\alpha(\tau) = \frac{1}{2\pi}\int_0^\infty Q_\alpha(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega\,,\quad \alpha = 1,2,\ldots,N \tag{21}$$

Computationally, to estimate response maxima Eq. (5) has to be integrated step-by-step over time and the extremum values are then selected from each time signal sequence of all the dominating modes. The computations may turn out high-costly. The approximate method based on [5] and discussed below appears to be more efficient. We begin with Eq.$(ll)_1$,cf.Eq.(8)

$$x_\rho(\tau) = \frac{1}{2\pi}\int_0^\infty X_\rho(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega = \frac{1}{2\pi}\int_0^\infty H_\rho(\omega)\ddot{X}_\rho^{\mathrm{b}}(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega \tag{22}$$

(no sum on ρ) that describes the modal time response obtained by integration over the frequency domain. It is observed that the only difference between Eq. (22) and Eq. $(11)_2$ is the function $H_\rho(\omega)$ involved in Eq. (22). Thus, this function can be interpreted as the influence coefficient of the reaction $x_\rho(\tau)$ to harmonic excitation

$$\ddot{x}_\rho^{\mathrm{b}}(\tau) = \ddot{X}_\rho^{\mathrm{b}}(\omega)\mathrm{e}^{\mathrm{i}\omega\tau} \tag{23}$$

i.e.

$$x_\rho(\tau) = X_\rho(\omega)\mathrm{e}^{\mathrm{i}\omega\tau} = H_\rho(\omega)\ddot{X}_\rho^{\mathrm{b}}(\omega)\mathrm{e}^{\mathrm{i}\omega\tau} \quad \text{(no sum on } \rho) \tag{24}$$

In other words, the coefficient $X_\rho(\omega)$ being the amplitude of the ρ-th harmonic component is a measure of the maximum displacement for the ρ-th mode. Therefore, for a specific mode the maximum response can be obtained directly as

$$x_\rho^{\max} \cong X_\rho\left(\omega_\rho\right) = H_\rho\left(\omega_\rho\right)\ddot{X}_\rho^{\mathrm{b}}\left(\omega_\rho\right) \tag{25}$$

The maximum total response $q_\alpha(\tau)$ cannot be evaluated, however, by merely adding the modal maxima according to the typical superposition scheme, since these maxima in general do not occur at the same time. Thus, although the spectral mode superposition provides information of an upper limit to the total response, it frequently over-estimates this maximum by a significant amount.

A number of approaches have been suggested to obtain a more reasonable estimate of the maximum response from the spectral responses. The simplest and most popular is that the α-th displacement and β-th stress maxima are treated as the root-mean-squares of all the modes of the α-th displacements and β-th stresses considered in the system. That is, respectively

$$q_\alpha^{\max} \cong \left[\sum_{\rho=1}^{V}\left(x_\rho^{\max}\right)^2\right]^{\frac{1}{2}}, \quad f_\beta^{\max} \cong \left[\sum_{\rho=1}^{V}\left(\omega_\rho^2 x_\rho^{\max}\right)^2\right]^{\frac{1}{2}} \tag{26}$$

3. Response of the first two probabilistic moments

By the assumption that the base modal excitations $\ddot{x}_\rho^{\mathrm{b}}(\tau)$ are stationary random variables, the modal reactions $x_\rho(\tau)$ of the linear system considered will also be stationary. The expectations for $x_\rho(\tau)$ at time $\tau = t$ can be expressed as, cf. Eq. (5)

$$\begin{aligned} E\left[x_\rho(t)\right] &= E\int_0^\infty\left[h_\rho(t-\tau)\ddot{x}_\rho^{\mathrm{b}}(\tau)\mathrm{d}\tau\right] = E\left[\int_0^\infty h_\rho(\tau)\ddot{x}_\rho^{\mathrm{b}}(t-\tau)\mathrm{d}\tau\right] = \\ &= \int_0^\infty h_\rho(\tau)E\left[\ddot{x}_\rho^{\mathrm{b}}(t-\tau)\right]\mathrm{d}\tau = \int_0^\infty h_\rho(\tau)E\left[\ddot{x}_\rho^{\mathrm{b}}(t)\right]\mathrm{d}\tau = \\ &= E\left[\ddot{x}_\rho^{\mathrm{b}}(t)\right]\int_0^\infty h_\rho(\tau)\mathrm{d}\tau \qquad \text{(no sum on } \rho) \end{aligned} \tag{27}$$

since the convolution integral is symmetric in h_ρ and $\ddot{x}_\rho^{\mathrm{b}}$ and the expectations of stationary random variables are constant. Further, by setting $\omega = 0$ Eq. (10) becomes, cf. Eq. (16)

$$H_\rho(0) = \int_0^\infty h_\rho(\tau)\mathrm{d}\tau = \frac{1}{\omega_\rho^2} \tag{28}$$

which, substituted into Eq. (27), gives

$$E\left[x_\rho\right] = \frac{1}{\omega_\rho^2}E\left[\ddot{x}_\rho^{\mathrm{b}}\right] \tag{29}$$

This equation describes the relation of the ρ-th spectral acceleration of the structure base and the ρ-th spectral displacement (ρ-th normal coordinate) of the system. We also note that the expectations $E[x_\rho]$ ate independent of the damping effects and may be interpreted as the solution to the corresponding static problem; excitations typical of inertia forces due to the base accelerations are treated as static loads according to the well-known d'Alembert principle.

To compute the second probabilistic moments we write the Fourier transform pair

$$\begin{aligned} S_{x_\rho x_\sigma}(\omega) &= \int_0^\infty R_{x_\rho x_\sigma}(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau \\ R_{x_\rho x_\sigma}(\tau) &= \frac{1}{2\pi}\int_0^\infty S_{x_\rho x_\sigma}(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega \end{aligned} \qquad \rho,\ \sigma = 1,2,\ldots,V \tag{30}$$

where $S_{x_\rho x_\sigma}(\omega)$ and $R_{x_\rho x_\sigma}(\tau)$ are respectively the spectral density functions and correlation functions for two stationary random variables $x_\rho(t)$ and $x_\sigma(t)$, which are in this case two spectral reactions. The correlation functions, by definition, read

$$R_{x_\rho x_\sigma}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_0^\infty x_\rho(t)x_\sigma(t+\tau)\mathrm{d}t \tag{31}$$

the symbol τ denotes from now on the time shift. Because the Dirac distribution is even we rewrite Eq. (5) for $x_\rho(t)$ and $x_\sigma(t)$ as

$$\begin{aligned} x_\rho(t) &= \int_0^\infty h_\rho(v)\ddot{x}_\rho^{\mathrm{b}}(t-v)\mathrm{d}v \\ x_\sigma(t) &= \int_0^\infty h_\sigma(\eta)\ddot{x}_\sigma^{\mathrm{b}}(t-\eta)\mathrm{d}\eta \end{aligned} \qquad \text{(no sum on } \rho,\ \sigma) \tag{32}$$

which, introduced in Eq. (31), result in

$$\begin{aligned} R_{x_\rho x_\sigma}(\tau) &= \lim_{T\to\infty}\int_0^T\left[\int_0^\infty h_\rho(v)\ddot{x}_\rho^{\mathrm{b}}(t-v)\mathrm{d}v\right]\left[\int_0^\infty h_\sigma(\eta)\ddot{x}_\sigma^{\mathrm{b}}(t+\tau-\eta)\mathrm{d}\eta\right]\mathrm{d}t = \\ &= \int_0^\infty\int_0^\infty h_\rho(v)h_\sigma(\eta)\left[\lim_{T\to\infty}\int_0^T\ddot{x}_\rho^{\mathrm{b}}(t-v)\ddot{x}_\sigma^{\mathrm{b}}(t+\tau-\eta)\mathrm{d}t\right]\mathrm{d}v\,\mathrm{d}\eta \end{aligned} \tag{33}$$

(no sum on ρ, σ) or

$$R_{x_\rho x_\sigma}(\tau) = \int_0^\infty\int_0^\infty h_\rho(v)h_\sigma(\eta)R_{\ddot{x}_\rho^{\mathrm{b}}\ddot{x}_\sigma^{\mathrm{b}}}(\tau+v-\eta)\mathrm{d}v\,\mathrm{d}\eta \tag{34}$$

From Eq. $(30)_1$ it follows that

$$\begin{aligned} S_{x_\rho x_\sigma}(\omega) &= \int_0^\infty\left[\int_0^\infty\int_0^\infty h_\rho(v)h_\sigma(\eta)R_{\ddot{x}_\rho^{\mathrm{b}}\ddot{x}_\sigma^{\mathrm{b}}}(t+v-\eta)\mathrm{d}v\,\mathrm{d}\eta\right]\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = \\ &= \left[\int_0^\infty h_\rho(v)\mathrm{e}^{-\mathrm{i}\omega v}\mathrm{d}v\right]\left[\int_0^\infty h_\sigma(\eta)\mathrm{e}^{-\mathrm{i}\omega\eta}\mathrm{d}\eta\right]\times \\ &\times\left[\int_0^\infty R_{\ddot{x}_\rho^{\mathrm{b}}\ddot{x}_\sigma^{\mathrm{b}}}(\tau+v-\eta)\,\mathrm{e}^{-\mathrm{i}\omega(\tau+v-\eta)}\overbrace{\mathrm{d}(\tau+v-\eta)}^{=\mathrm{d}\tau}\right] \end{aligned} \tag{35}$$

which, on account of Eqs. (10) and $(30)_1$, becomes

$$S_{x_\rho x_\sigma}(\omega) = H_\rho^*(\omega)H_\sigma(\omega)S_{\ddot{x}_\rho^{\mathrm{b}}\ddot{x}_\sigma^{\mathrm{b}}}(\omega) \quad \text{(no sum on } \rho,\ \sigma) \tag{36}$$

Specifically, when $\rho \equiv \sigma$ we get

$$S_{x_\rho x_\rho}(\omega) = \left|H_\rho(\omega)\right|^2 S_{\ddot{x}_\rho^{\mathrm{b}}\ddot{x}_\rho^{\mathrm{b}}}(\omega) \tag{37}$$

The total second probabilistic moments can be evaluated by superposition of the obtained above modal quantities. The correlation function for two displacement components can be written as, cf. Eq. (31)

$$
\begin{aligned}
R_{q_\alpha q_\beta}(\tau) &= \lim_{T\to\infty}\frac{1}{T}\int_0^T q_\alpha(t)q_\beta(t+\tau)\,\mathrm{d}t = \\
&= \varphi_{\alpha\rho}\varphi_{\beta\sigma}\lim_{T\to\infty}\frac{1}{T}\int_0^T x_\rho(t)x_\sigma(t+\tau)\,\mathrm{d}t = \\
&= \varphi_{\alpha\rho}\varphi_{\beta\sigma}R_{x_\rho x_\sigma}(\tau)
\end{aligned} \qquad (38)
$$

and, consequently, cf Eq. $(30)_1$

$$
S_{q_\alpha q_\beta}(\omega) = \int_0^\infty \varphi_{\alpha\rho}\varphi_{\beta\sigma}R_{x_\rho x_\sigma}(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau = \varphi_{\alpha\rho}\varphi_{\beta\sigma}S_{x_\rho x_\sigma}(\omega) \qquad (39)
$$

4. Analysis of maxima

To this topic we define a non-dimensional random variable

$$
r_\alpha = \frac{\tilde{q}_\alpha}{\sqrt{\mu_0}} = \frac{q_\alpha(t) - E\left[q_\alpha(t)\right]}{\sqrt{\mu_0}} \qquad (40)
$$

with (cf. Eq. $(30)_2$ written for $\tilde{q}_\alpha$ at $\tau = 0$)

$$
\mu_0 = R_{\tilde{q}_\alpha\tilde{q}_\beta}(0) = \frac{1}{2\pi}\int_0^\infty S_{\tilde{q}_\alpha\tilde{q}_\beta}(\omega)\mathrm{d}\omega \qquad (41)
$$

Observing that $R_{\tilde{q}_\alpha\tilde{q}_\beta}(0) = R_{q_\alpha q_\beta}(0) - E\left[q_\alpha\right]E\left[q_\beta\right]$ the relationship between $S_{q_\alpha q_\beta}(\omega)$ and $S_{\tilde{q}_\alpha\tilde{q}_\beta}(\omega)$ takes the form

$$
S_{\tilde{q}_\alpha\tilde{q}_\beta}(\omega) = S_{q_\alpha q_\beta}(\omega) - E\left[q_\alpha\right]E\left[q_\beta\right]\delta(\omega) \qquad (42)
$$

since $E[q_\alpha]$ and $E[q_\beta]$ are constant and $\int_0^\infty \mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\omega = \delta(\omega)$ with $\delta(\omega)$ defined here for the finite time interval $[0, T]$ as, [5]

$$
\delta(\omega) = \begin{cases} T/(2\pi) & \text{for} \quad \omega \le \pi/\mathrm{T} \\ 0 & \text{for} \quad \omega > \pi/\mathrm{T} \end{cases} \qquad (43)
$$

From M independent maxima with the probability density function $p(r_\alpha)$ the cumulative distribution function is defined for the maxima as

$$
P_{\mathrm{extr}}(r_\alpha) = \text{Prob (all } M \text{ maxima} < r_\alpha) = \mathrm{e}^{\vartheta} \qquad (44)
$$

where

$$
\vartheta = -\frac{T}{2\pi}\left(\frac{\mu_2}{\mu_0}\right)^{1//2} e^{-r_\alpha^2//2} \quad \text{with} \quad \mu_2 = \frac{1}{2\pi}\int_0^\infty \omega^2 S_{\tilde{q}_\alpha\tilde{q}_\beta}(\omega)\mathrm{d}\omega \qquad (45)
$$

The probability density functions $p_{\mathrm{extr}}(r_\alpha)$ for maxima takes the form

$$
p_{\mathrm{extr}}(r_\alpha) = \frac{\mathrm{d}P_{\mathrm{extr}}(r_\alpha)}{\mathrm{d}r_\alpha} = \frac{Tr_\alpha}{2\pi}\left(\frac{\mu_2}{\mu_0}\right)^{1//2}\mathrm{e}^{\vartheta - r_\alpha^2//2} \qquad (46)
$$

The expectations and standard deviations for the maxima r_α^{extr} can be approximately calculated from Eq. (44) as

$$
\begin{aligned}
E\left[r_\alpha^{\mathrm{extr}}\right] &\cong \sqrt{\kappa} + \frac{\gamma}{\sqrt{\kappa}} \\
\sigma_{r_\alpha^{\mathrm{extr}}} &\cong \frac{\pi}{\sqrt{6\kappa}}
\end{aligned}
\quad \text{with} \quad \kappa = \ln\frac{T^2\mu_2}{4\pi^2\mu_0} \qquad (47)
$$

where $\gamma = 0.5772$ is the Euler constant.

5. Illustrative examples and concluding remarks

Two deck cranes, produced by TOWIMOR for the ships B-570 and B-577, modelled as superstructures subject to deck motion are considered. Input data are defined by the spectra of acceleration amplitudes and the spectral density functions of ship decks. The data on input are processed by using the design sea's state method supplied from CTO. The finite element model for the first crane consists of 4942 thin-shell elements and 332 3-D beam elements (22325 DOFs). For the second crane, the finite element setting includes 2122 thin shell elements and 20 cable elements (16088 DOFs). For both the two systems 30 dominated modal shapes are used in the spectral analysis and numerical integrations. To evaluate the relative nodal displacements and accelerations, element stress response and estimate response maxima both the approaches based on the root-mean-squares and the spectral density functions are employed.

It is observed from the obtained numerical results that in accordance with the design sea's state method the displacement-acceleration spectra and spectral densities of the crane structures and those of the ‘design seas’ are respectively different in a considerable extent. Geometrically interpreting, the common domain of their spectral density functions is relatively small, being about 2-4 percent. In view of this, the currently existing supports of the cranes can safely be removed during the ship's cruise, since the vibration amplitudes of the deck cranes are small and can be neglected. For the cases of ship’s swinging and/or swaying with smaller periods, 1-2 seconds for instance, when comparedwith the so-called design sea’s state swinging and swaying, i.e. ’shorter’ sea’s waves and consequently ‘faster’ vibrations of the ship's desks, the time and spectral responses of the deck cranes seem to be more dangerous and the deck supports may turn out necessarily needed.

It should be pointed out in ending that the spectral approach, extensively applied in onshore civil engineering for earthquake problems, may stand for an alternative methodology in analysis of offshore travelling units. By the significant difference in the nature of sea’s waves and earthquake, the problem should be put towards more thoroughly, though.

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