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# Robust fuzzy predictive control structure

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#### Abstract

An effective method is proposed for robust predictive control of nonlinear processes that is easily implementable on commonly used equipment, such as PLC and PAC. The method is based on a two-loop model following control (MFC) system containing a nominal model of the controlled plant and two controllers: the nonlinear, suboptimal fuzzy predictive one as the main controller and the proposed robust controller as an auxiliary one. In the paper ways of employing Takagi-Sugeno fuzzy models to synthesize Model Predictive Control with State equations (MPCS) for nonlinear processes and basic features exhibited by the MFC structure are presented. The resulting controller has been incorporated into the MFC structure, and then a method for synthesizing the auxiliary controller has been given. The proposed control structure has been tested for its performance on control plants with perturbed parameters. Results of tests lend support to the view that the proposed control method may find wide application to robust control of nonlinear plants with time-varying parameters.

**Keywords**: Robustness, Model Predictive Control, Model Following Control, Fuzzy control.

# Odporny układ rozmytej regulacji predykcyjnej

### Streszczenie

W artykule zaproponowano efektywną metodę odpornej regulacji predykcyjnej procesów nieliniowych, łatwą do implementacji w powszechnie stosowanym komputerowym sprzęcie automatyki takim jak sterowniki PLC oraz PAC. Metoda wykorzystuje dwupętlowy układ sterowania ze śledzeniem modelu (ang. Model Following Control – MFC) zawierający model nominalnego nieliniowego obiektu oraz dwa regulatory: główny – nieliniowy, suboptymalny predykcyjny regulator rozmyty oraz proponowany odporny regulator pomocniczy. Pokazano sposób wykorzystania do budowy regulatora głównego obiektów nieliniowych rozmytych modeli Takagi-Sugeno w przestrzeni stanu oraz podstawowe zalety struktury MFC. Przedstawiono sposób syntezy regulatora pomocniczego. Proponowana struktura została przetestowana ze względu na jakość regulacji obiektów perturbowanych. Wyniki badań pokazują, że może ona znaleźć zastosowanie do odpornej regulacji obiektów nieliniowych o parametrach zmiennych w czasie.

**Słowa kluczowe**: odporność, regulacja predykcyjna, regulacja ze śledzeniem modelu, regulacja rozmyta.

## 1. Introduction

Model Predictive Control (MPC) belongs to the category of the so-called advanced control techniques, and has been employed with advantage for over 20 years to control complicated manufacturing processes that are difficult to govern by classic controllers [1, 2, 3, 4]. I recent years numerous attempts, both theoretical and practical, to develop predictive control techniques that would be effective for nonlinear processes [5, 6, 7, 8]. Unfortunately, synthesis of Nonlinear Model Predictive Control (NMPC) algorithms, leads in general to a very complicated problem of nonlinear programming, which is unconvex in most

cases. In order to find the current control here, the problem is to be solved at each sampling step. The numerical complexity of this approach is particularly inconvenient if we have to do with fast processes and those that are multi-input and multi-output. Effective methods that would ensure obtaining optimal solutions in a finite time are still lacking [1, 9]. Also, the choice of adequate nonlinear process models and their parameterization in industrial conditions in terms of needs of an NMPC controller presents a great problem. This is true for both phenomenological models that are mostly very expensive and lead to overly complicated analytical relationships, and empirical models that require determining their structure, testing signals and ways of their identification in real industrial conditions [4, 5].

Therefore, despite the great progress in analysis and synthesis of nonlinear predictive control made over the past years [6, 7], suboptimal methods still present the majority of proposed solutions [9], exemplified by approaches based on employing soft computing methods. Extensive use in NMPC algorithms is made of models based on artificial neural networks [1, 10] and nonlinear fuzzy models, among others Takagi-Sugeno models [11, 12, 13, 14].

Despite many remarkable theoretical achievements in analysis of nonlinear predictive control and first attempts to employ NMPC algorithms in industry, a number of problems still remain to be solved. Among them, realization of nonlinear predictive algorithms that would be robust to uncertainties and perturbations of the process structure and parameters unavoidable in industrial practice, and to the process-model mismatch seems to be the greatest.

In the paper a universal, robust and easy-to-realization predictive control method for nonlinear processes is proposed. Its idea consists in incorporating the suboptimal MPCS algorithm with Takagi-Sugeno controllers map into the robust MFC structure.

## 2. MPCS algorithm for nonlinear processes

First predictive control algorithms were developed for process models given in the form of discrete impulse and step responses or discrete transfer functions [2, 3, 9]. Later in the early 1990s, papers appeared where the process model in the form of discrete state and output equations has been used [15, 16, 17].

To employ the MPCS algorithm for nonlinear processes it is assumed in the paper that nonlinear time-invariant (NLTI) processes may be described by state and output equations, the structure of which is typical for linear systems with the difference that state, input and output matrices are dependent on the current value of the state vector and/or input vector. As it was shown in [5, 8], the approach where the NLTI model is replaced by a linear time-variant (LTV) one is possible for the majority of nonlinear time-invariant processes, notwithstanding the fact that the choice of the matrix variability is generally still an open problem. General remarks and recommendations on this point may be found, for example, in [19].

Among infinitely many possible transforms, one of the most computationally effective, it appears, is the method of Takagi-Sugeno fuzzy models presented originally in [9, 14]. Here, the following antecedent is adopted for the i-th from among p fuzzy rules

IF 
$$x_1(k) \subset S_{i,1} \text{ AND} \cdots \text{AND} x_n(k) \subset S_{i,n}$$
 (1)

where  $x_j(k) \subset S_{i,j}$  is the membership of the state variable  $x_j(k)$  in the fuzzy set  $S_{i,j}$  with the membership function  $\mu_{S_{i,j}}(x_j(k))$ , AND is the fuzzy logical product operator, n is the number of state

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variables, and i is the number of fuzzy sets for the variable in question.

The consequents have the so-called functional form and correspond to local linear models of the following structure

THEN 
$$\underline{x}_{i}(k+1) = \mathbf{A}_{i}\underline{x}(k) + \mathbf{B}_{i}\underline{u}(k)$$
  
 $\underline{y}_{i}(k) = \mathbf{C}_{i}\underline{x}(k)$  (2)

where  $\underline{x}(k) \in \mathcal{R}^{^{n \times l}}$ ,  $\underline{u}(k) \in \mathcal{R}^{^{l \times l}}$ ,  $\underline{y}(k) \in \mathcal{R}^{^{m \times l}}$  are state, input and output vectors, respectively. It can be shown that after inferencing and defuzzyficating by the gravity center method the following relationships for the state and output of the nonlinear model hold true [10, 13]

$$\underline{x}(k+1) = \frac{\sum_{i=1}^{p} w_i(k) \cdot \underline{x}_i(k+1)}{\sum_{i=1}^{p} w_i(k)} = \sum_{i=1}^{p} \widetilde{w}_i(k) \cdot \underline{x}_i(k+1) = \sum_{i=1}^{p} \widetilde{w}_i(k) \cdot \left[ \mathbf{A}_i \underline{x}(k) + \mathbf{B}_i \underline{u}(k) \right]$$
(3)

$$\underline{\underline{y}}(k+1) = \frac{\sum_{i=1}^{p} w_i(k) \cdot \underline{\underline{y}}_i(k+1)}{\sum_{i=1}^{p} w_i(k)} = \sum_{i=1}^{p} \widetilde{w}_i(k) \cdot \underline{\underline{y}}_i(k+1) = \sum_{i=1}^{p} \widetilde{w}_i(k) \cdot \mathbf{C}_i \underline{\underline{x}}(k) \quad (4)$$

where weighting coefficients that designate the so-called degrees of activation of individual rules are determined by the adopted definition of the fuzzy logical product operator [14], for example, that given by Zadeh

$$w_i(k) = \min \left\{ \mu_{S_{i,1}}(x_1(k)), \mu_{S_{i,2}}(x_2(k)), \cdots, \mu_{S_{i,n}}(x_n(k)) \right\}$$
 (5)

In such a case, the nonlinear time-invariant process may be described by a quasi-linear time-variant model with time-varying matrices "adjusted" in a fuzzy manner

$$\underline{x}(k+1) = \mathbf{A}_{k} \underline{x}(k) + \mathbf{B}_{k} \underline{u}(k)$$

$$y(k) = \mathbf{C}_{k} \underline{x}(k)$$
(6)

with

$$\mathbf{A}_{k} = \mathbf{A}(\underline{x}(k), \underline{u}(k)) = \sum_{i=1}^{p} \widetilde{w}_{i}(k) \cdot \mathbf{A}_{i}$$

$$\mathbf{B}_{k} = \mathbf{B}(\underline{x}(k), \underline{u}(k)) = \sum_{i=1}^{p} \widetilde{w}_{i}(k) \cdot \mathbf{B}_{i}$$

$$\mathbf{C}_{k} = \mathbf{C}(\underline{x}(k), \underline{u}(k)) = \sum_{i=1}^{p} \widetilde{w}_{i}(k) \cdot \mathbf{C}_{i}$$
(7)

following from (3) and (4).

Synthesis of the MPCS algorithm for a nonlinear process represented by the model (6) amounts to designing a linear MPCS controller defined by

$$\underline{\Delta U}_{k}(k) = \left(\mathbf{E}_{k}^{T} \mathbf{M} \mathbf{E}_{k} + \mathbf{L}\right)^{-1} \mathbf{E}_{k}^{T} \mathbf{M} \left[\underline{Y}^{r}(k) - \underline{Y}_{k}^{0}(k)\right]$$
(8)

for each k-th local submodel (2). Here  $\mathbf{M}$  and  $\mathbf{L}$  denote matrices of weighting coefficients,  $\underline{Y}_k^r(k)$  is the reference trajectory vector within the prediction horizon,  $\mathbf{E}_k$  is the so-called process dynamics matrix defined by

$$\mathbf{E}_{k} = \left[ \begin{cases} \mathbf{C}_{k+p} \left( \sum_{j=0}^{N_{1}+p-q-1} \prod_{i=1}^{j} \mathbf{A}_{k-1+i} \right) \mathbf{B}_{k-1+q} & \text{for} \quad N_{1}+p-q-1 \geq 0 \\ 0 & \text{for} \quad N_{1}+p-q-1 < 0 \end{cases} \right]_{pq} \right]_{\substack{p=1,2,\cdots,N_{2}-N_{1}+1\\ q=1,2,\cdots,N_{q}}} \tag{9}$$

and  $\underline{Y}_{k}^{0}(k)$  is the free component vector within the prediction horizon with prediction dependent on known future control signals

$$\underline{Y}_{k}^{0} = \begin{bmatrix} \mathbf{C}_{k+N_{1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{C}_{k+N_{2}} \end{bmatrix} \begin{bmatrix} \prod_{i=0}^{N_{1}-1} \mathbf{A}_{k+i} \\ \vdots \\ \prod_{i=0}^{N_{2}-1} \mathbf{A}_{k+i} \end{bmatrix} \underline{\underline{x}}(k) + \begin{bmatrix} \sum_{j=0}^{N_{1}-1} \prod_{i=1}^{j} \mathbf{A}_{k-1+i} \\ \vdots \\ \sum_{j=0}^{N_{2}-1} \prod_{i=1}^{j} \mathbf{A}_{k-1+i} \end{bmatrix} \mathbf{B}_{k-1} u(k-1)$$
(10)

The designed local controllers form the so-called controllers map. Employing them as functional consequents for antecedents (1) yields an analytical Fuzzy Model Predictive Controller FMPCS [9, 12]. Its structure is similar to that of the linear MPCS algorithm, and only the values of gain coefficients are LTV, being fuzzy weighted sums of local controllers gains. In this way a system stability analysis with such a model in the nominal case is made possible and is easy to carry out [9].

It should be noted that in the case of Takagi-Sugeno fuzzy models it presents a remaining problem to perform data validation, i.e. establishing a division of the whole operating domain of the process into a number of overlapping local subdomains through the appropriate choice of fuzzy sets and values of the parameters of the membership function of each set, and also formulating a linear process model for each subdomains. Furthermore, the state vector to be found in (1) and (10) need be fully measurable, which is most often impossible in industrial conditions. For these reasons, MPCS algorithms are used mainly to control those processes, description of which in the form of state equations is natural, e.g. in electromechanics [16, 17]. In other cases, the algorithm should be supplemented by a state observer or Kalman filter, which is generally not a trivial task, if nonlinear processes are concerned. Fortunately, it is possible to design a nonlinear observer, e.g. in the form of a local observer network [9], provided the process allows to be described by the fuzzy model (6).

## 3. Improving the robustness of the Fuzzy Model Predictive Controller

To improve the robustness of the proposed Fuzzy Model Predictive Controller (8) we suggest incorporating it into the Model Following Control (MFC) structure. In the MFC structure, described closer for the first time in [20, 21], the basic control task is performed by the main controller matched in a most optimal way to the process model. On the other hand, the task set for the auxiliary controller is to support the main controller by generating a corrective signal that depends on the difference between the outputs produced by the adopted model and the actual process. By this means the effect produced by the process-model mismatch (caused, e.g. by different structures) and by possible process perturbations can be neutralized. The system robustness to model inadequacy, as well as the control performance is thereby increased, and the effect produced by nonmeasurable disturbances is reduced. The principal virtues displayed by the MFC structure are its universality due to the feasibility of employing arbitrary control algorithms, and possibility to design controllers by familiar methods. To put this another way, the MFC structure permits achieving high control performance and robustness to disturbances and process perturbations (uncertainty) by simple means [5, 20].

Properties exhibited by the MFC structure are described most often for the linear case. Here we will generalize them by treating perturbations as a synthetic description of changes of all kinds experienced by the process in reference to the nominal model, thus indirectly as a description of the mismatch between the adopted nonlinear fuzzy model (6) and the actual nonlinear process [21].

We propose in this paper to employ the above-mentioned nonlinear predictive fuzzy algorithm (8)–(10) as the main controller in the MFC structure. To synthesize the auxiliary controller the nominal linear process model is adopted. It is

assumed that the model is stable and controllable with a full-order matrix  $\mathbf{B}_{\bullet}$  and is defined by

$$x_M(k+1) = \mathbf{A}x_M(k) + \mathbf{B}u_M(k)$$
 (11)

Next, it is assumed that the controlled process is subjected to unknown, yet bounded perturbations, and is acted upon by disturbances, not necessarily bounded, dependent in a known way on the state vector. Hence, the process is described by the following nonlinear state and output equations

$$\underline{x}(k+1) = \left[\mathbf{A} + \Delta \mathbf{A}(\underline{x}(k))\right] \underline{x}(k) + \left[\mathbf{B} + \Delta \mathbf{B}(\underline{x}(k), \underline{u}(k))\right] \underline{u}(k) + \mathbf{d}(\underline{x}(k))$$

$$y(k) = \left[\mathbf{C} + \Delta \mathbf{C}(\underline{x}(k))\right] \underline{x}(k)$$
(12)

with the perturbation matrices defined as

$$\Delta \mathbf{A}(\underline{x}(k)) = \mathbf{BF}_{0}(\underline{x}(k)) = \mathbf{BF}_{0k}$$

$$\Delta \mathbf{B}(\underline{x}(k),\underline{u}(k)) = \mathbf{BF}_{1}(\underline{x}(k),\underline{u}(k)) = \mathbf{BF}_{1k}$$

$$\Delta \mathbf{C}(x(k)) = \Delta \mathbf{C}_{k}$$
(13)

and the disturbance vector as

$$\mathbf{d}(\underline{x}(k)) = \mathbf{BF}_{2}(\underline{x}(k))\mathbf{a}(\underline{x}(k)) = \mathbf{BF}_{2k}\mathbf{a}_{k}$$
 (14)

where **a** is a known function of the state vector, and matrices  $\mathbf{F}_0$ ,  $\mathbf{F}_1$ ,  $\mathbf{F}_2$ ,  $\Delta \mathbf{C}$  are unknown yet bounded. Additionally, the vector (14) also may represent a nonlinear bounded process uncertainty. The presence of the **B** matrix in descriptions of perturbations (13) and disturbances follows from the need to compensate effects produced by perturbations and disturbances by means of the control signal [22].

Furthermore, it is assumed that the perturbed input matrix may be defined as

$$\mathbf{B} + \Delta \mathbf{B}(x(k), u(k)) = \mathbf{BH}(x(k), u(k)) = \mathbf{BH}_{k}$$
 (15)

and it may be found such  $\beta > 0$  that the following condition will be fulfilled by assumption for the **H** matrix for each v(k)

$$v^{T}(k)\mathbf{H}v(k) \ge \beta v^{T}(k)v(k) \tag{16}$$

With eqs. (12)–(15), the state equation for the perturbed process takes the form

$$x(k+1) = \mathbf{A}x(k) + \mathbf{B}\mathbf{H}_{k}u(k) + \mathbf{B}[\mathbf{F}_{0k}x(k) + \mathbf{F}_{2k}\mathbf{a}_{k}]$$
 (17)

The task set for the auxiliary controller is to generate an auxiliary control signal such that the model be well tracked by the process

$$||y(k) - y_{\mathcal{M}}(k)|| < \varepsilon, \quad \varepsilon > 0$$
 (18)

Following the approach proposed in [5, 22], we assume that the control signal is comprised of two components:  $u_N$  that has to linearize input nonlinearities, and  $u_R$  that has to compensate effects produced by bad estimation of nonlinear perturbations

$$u(k) = u_N(k) + u_R(k)$$
 (19)

assuming the former component is defined by

$$\underline{u}_{N}(k) = \hat{\mathbf{H}}^{-1} \left[ \underline{u}_{M}(k) - \hat{\mathbf{F}}_{0} \underline{x}(k) - \hat{\mathbf{F}}_{2} \mathbf{a}_{k} \right]$$
 (20)

From eqs. (17), (19) and (20) it follows that for perturbations accurately estimated ( $\hat{\mathbf{H}} = \mathbf{H}$ ,  $\hat{\mathbf{F}}_0 = \mathbf{F}_0$ ,  $\hat{\mathbf{F}}_2 = \mathbf{F}_2$ ) and  $\underline{u}_R(k) = 0$ , the process (17) shows identity with the model (11), hence, the model is tracked accurately by the process.

If perturbations are estimated inaccurately, then the tracking error may be found by subtracting side-by-side eq. (11) from eq. (17) with eq. (20) in mind:

$$\underline{x}(k+1) - \underline{x}_{M}(k+1) = \underline{e}(k+1) = \mathbf{A}\underline{e}(k) + \mathbf{B}(\mathbf{H}_{k}\underline{u}_{R}(k) + \mathbf{T}_{k}\mathbf{b}_{k})$$
(21)

$$\mathbf{T}_{k} = \left[ \mathbf{H}_{k} \hat{\mathbf{H}}^{-1} - \mathbf{I} \mid \mathbf{F}_{0k} - \mathbf{H}_{k} \hat{\mathbf{H}}^{-1} \hat{\mathbf{F}}_{0} \mid \mathbf{F}_{2k} - \mathbf{H}_{k} \hat{\mathbf{H}}^{-1} \hat{\mathbf{F}}_{2} \right]$$
(22)

$$\mathbf{b}_{k} = \begin{bmatrix} \underline{u}_{M}(k) \\ \underline{x}(k) \\ \mathbf{a}_{k} \end{bmatrix}$$
 (23)

Matrix  $\mathbf{T}_k$  determines the unknown, current (at the k instant) value of the perturbation estimation error with the proviso that

$$\|\mathbf{T}_k\| < T \,, \qquad \forall k > 0 \tag{24}$$

and components of the vector  $\mathbf{b}_k$  are known or available for being measured, according to the assumptions adopted.

For the proposed MFC approach to be effective, it is sufficient to prove that the model state is tracked by the process state, i.e. the state of the system (21) is bounded if excited by (20). Considering that the perturbation  $\Delta \mathbf{C}$  (13) is bounded, the condition (18) will be thereby met.

As suggested in [5, 22], we propose to employ the control component  $u_R$  as a signal with bounded amplitude

$$\underline{u}_{R}(k) = -\frac{\underline{v}(k)}{\|\underline{v}(k)\| + \delta} \cdot r_{0} \Big( r_{1} \|\underline{e}(k)\| + r_{2} \|\mathbf{b}_{k}\| \Big)$$
 (25)

where

$$\underline{v}(k) = (\mathbf{B}^T \mathbf{P} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A} \underline{e}(k)$$
 (26)

after the fashion of optimal control with a quadratic performance index

The matrix  $\mathbf{P}$  represents here a positively defined, easy to find solution of the Lyapunov equation

$$\mathbf{A}^T \mathbf{P} \mathbf{A} - \mathbf{P} = -\mathbf{Q} \tag{27}$$

with an arbitrary chosen matrix  $\mathbf{Q} > 0$ . The positive constant  $\delta$  prevents the control signal (25) from being discontinuous in the case that the model tracking error equals zero.

The parameters of the auxiliary controller (28) may be chosen accordingly to the rule

$$r_{0} > \frac{\gamma \mu \delta \gamma_{C} + \varepsilon}{\varepsilon \beta \|\mathbf{B}\|}$$

$$r_{1} \ge \|\mathbf{A}\|$$

$$r_{2} \ge \|\mathbf{B}\| \cdot T$$
(28)

where

$$\mu = \left[\operatorname{cond}\left(\mathbf{B}^{T}\mathbf{P}\mathbf{B}\right)\right]^{\frac{1}{2}}$$

$$\left\|\left(\mathbf{C} + \Delta\mathbf{C}_{k}\right)\right\| < \gamma_{C}, \quad \forall k > 0$$
(29)

and the parameter  $\gamma$  may be determined if only the matrices **A**, **B** (11) and **P** (27) are known [5].

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Hence, in order to determine the auxiliary controller (20), (25) the following steps are to be taken:

- adopting a process model (11),
- estimating perturbations (13) and (14), and thereby perturbations (24) and (29),
- adopting the coefficient (16) that defines the input nonlinearity matrix,
- adopting the maximum permissible model output tracking error (18),
- adopting the coefficient (25) that prevents from control discontinuity.

On this basis, the tunable controller parameters (28) are evaluated, and then the auxiliary matrix **P** is found by choosing the **Q** matrix and solving eq. (27).

It should be noted that physical interpretation of tunable controller parameters  $r_0$ ,  $r_1$ ,  $r_2$  is fairly understandable from (25), and enables their tuning also by experiment.

On the basis of relationships (19), (20) and (25), (26) the auxiliary controller may be presented in the form of a block diagram shown in Fig. 1. As mentioned above, this controller is incorporated into the MFC structure.

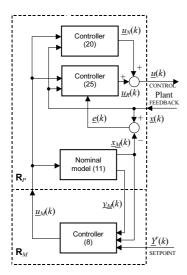


Fig. 1. The proposed MFC structure with the fuzzy predictive main controller  $\mathbf{R}_M$  and the robust auxiliary controller  $\mathbf{R}_P$ 

Rys. 1. Proponowany układ MFC z rozmytym predykcyjnym regulatorem głównym  $\mathbf{R}_M$  oraz odpomym regulatorem pomocniczym  $\mathbf{R}_P$ 

To determine permissible perturbations of the nonlinear control process, an appropriate nominal process model (11) should be adopted. This can be done on the basis of the set of Takagi-Sugeno local models given by eq. (2). If we adopt the central fuzzy submodel (submodel for the most frequently occurring process working point) as the nominal model (11), then the maximum perturbations experienced by the nominal model matrix can be easily estimated by comparing the relationship for fuzzy models (6) and that for the process model (12) and perturbation matrices (13):

$$\|\mathbf{A}_{k}\|_{\max} - \|\mathbf{A}\| < \|\Delta\mathbf{A}(\underline{x}(k))\|_{\max} = \mathbf{B}\hat{\mathbf{F}}_{0}$$

$$\|\mathbf{B}_{k}\|_{\max} - \|\mathbf{B}\| < \|\Delta\mathbf{B}(\underline{x}(k),\underline{u}(k))\|_{\max} = \mathbf{B}\hat{\mathbf{F}}_{1}$$

$$\|\mathbf{B}_{k}\|_{\max} < \mathbf{B}\hat{\mathbf{H}}$$

$$\|\mathbf{C}_{k}\|_{\max} - \|\mathbf{C}\| < \|\Delta\mathbf{C}(\underline{x}(k))\|_{\max}$$
(30)

# 4. Conclusions

The applicability of the proposed control method has been verified by simulation tests and by tests where a real electrothermal plant has been governed by a distributed

Programmable Automation Controller (PAC) system in which the proposed predictive control algorithm has been implemented. Tests have shown that the proposed solutions are easy to implement, and provide relatively high robustness and control performance [5].

## 5. References

- R. Bars, P. Colaneri, C. E. de Souza, F. Allgöwer, A. Kleimenov, C. Scherer: Theory, algorithms and technology in the design of control systems. In: Proc. 16th IFAC World Congress. Prague 2005, CD-ROM.
- [2] E. F. Camacho, C. Bordons: Model predictive control in the process industry. Advances in industrial control, Berlin, Springer-Verlag 1995.
- [3] D. W. Clarke, R. Scattolini: Constrained receding horizon predictive control. Proceedings IEE, Part D, 1991, Vol. 138, s. 347–354.
- [4] S. J. Qin, T. A. Badgwell: A survay of industrial model predictive control technology. Contr. Eng. Pract., 2003, Vol. 11(7), s. 733–764.
- [5] S. Domek: Odporna regulacja predykcyjna obiektów nieliniowych. Prace naukowe Politechniki Szczecińskiej, Zeszyt nr 593, Szczecin, Wydawnictwo Uczelniane PS 2006.
- [6] R. Findeisen, F. Allgöwer: An introduction to nonlinear model predictive control. In: Proc. 21st Benelux Meeting on System and Control. Veldhoven 2002, CD-ROM.
- [7] L. Magni, G. De Nicolao, R. Scattolini, F. Allgöwer: Robust model predictive control of nonlinear discrete-time systems. Int. J. Robust and Nonlinear Contr., 2003, Vol. 13, s. 229–246.
- [8] D. Q. Mayne: Nonlinear model predictive control: challenges and opportunities. In: Nonlinear Predictive Control, eds. F. Allgöwer, A. Zheng. Basel, Birkhäuser 2000, s. 23–44.
- [9] P. Tatjewski: Sterowanie zaawansowane obiektów przemysłowych. Struktury i algorytmy. Seria: Monografie Komitetu Automatyki i Robotyki PAN, Warszawa, Akademicka Oficyna Wydawnicza EXIT 2002.
- [10] P. Tatjewski, M. Ławryńczuk: Soft computing in model-based predictive control. Int. J. Appl. Math. Comput. Sci., 2006, Vol. 16(1), s. 7–26
- [11] R. Babuška, H. B. Verbrungen: An overwiew of fuzzy modelling for control. Control Eng. Practice, 1996, Vol. 4(11), s. 1593–1606.
- [12]Z. Huaguang, L. Cai: Multivariable fuzzy generalized predictive control. Cybernetics & Systems, 2002, Vol. 33(1), s. 66–99.
- [13] P. Marusak, P. Tatjewski: Stability analysis of nonlinear control systems with unconstrained fuzzy predictive controllers. Arch. Contr. Sci., 2002, Vol. 12, s. 267–288.
- [14]T. Takagi, M. Sugeno: Fuzzy identification of systems and its application to modeling and control. IEEE Trans. Systems, Man, and Cybernetics, 1985, Vol. 15, s. 116–132.
- [15] J. H. Lee, M. Morari, C. E. Garda: State space interpretation of model predictive control. Automatica, 1994, Vol. 30(4), s. 707–717.
- [16]S. Li, K. Lim, D. Fisher: A state-space formulation for model predictive control. AIChE Journal, 1989, Vol. 35, s. 241–249.
- [17] A. W. Ordys, D. W. Clarke: A state-space description for GPC controllers. Int. J. Syst. Sci., 1993, Vol. 24(9), s. 1727–1744.
- [18] P. M. Mäkilä, J. R. Partington: On linear models for nonlinear systems. Automatica, 2003, Vol. 39, s. 1–13.
- [19] P. Orłowski: Convergence of the optimal non-linear GPC method with iterative state-dependent, linear time-varying approximation. In: Prep. Int. Workshop on assessment and future directions of nonlinear model predictive control. Freudenstadt-Lauterbad 2005, s. 491–497.
- [20] S. Skoczowski, S. Domek: Robustness of a model following control system. In: Proc. Int. Conf. Mathematical Theory of Networks and Systems — MTNS. Perpignan 2000, CD-ROM.
- [21] S. Skoczowski, S. Domek, K. Pietrusewicz, B. Broel-Plater: A method for improving the robustness of PID control. IEEE Trans. Industrial Electronics, 2005, Vol. 52(6), s. 1669–1676.
- [22] T. Sugie, K. Osuka: Robust model following control with prescribed accuracy for uncertain nonlinear systems. Int. J. Contr., 1993, Vol. 58, s. 991–1009.

Artykuł recenzowany