Stanisław BAŃKA, Paweł DWORAK
SZCZECIN UNIVERSITY OF TECHNOLOGY, FACULTY OF ELECTRICAL ENGINEERING

On decoupling of LTI MIMO systems with guaranteed stability

D.Sc., Ph.D. Stanisław BAŃKA

He was born 1944, 24 September in Brudzowice. Ph.D. degree he had achieve in 1976 at Technical University of Wroclaw and habilitation degree in System Research Institute of Polish Academy of Sciences in Warsaw at 1992. Since 1969 hi is joined with Electrical Engineering Faculty of Szczecin University of Technology. Now hi is engaged as Professor of Szczecin University of Technology and Szczecin University. He is specialist in automation and information sciences.

e-mail: banka@ps.pl

Ph.D. Paweł DWORAK

Born in 17.09.1974r. in Gryfino, Poland. He received the PhD degree in engineering sciences from the Faculty of Electrical Engineering, Szczecin University of Technology, in 2005. Science March 2006 he has been Assistant Professor at the Institute of Control Engineering, Szczecin University of Technology. His interests are focused on analysis and synthesis of multidimensional control systems.

e-mail: Pawel.Dworak@ps.pl

Abstract

In the paper methods for decoupling of dynamic multivariable systems are presented. There is discussed both static and dynamic decoupling of systems for open-loop control. The designed elements of the system provides elimination of interactions between specific plants inputs and outputs in a steady and transition state respectively. Results of considerations are illustrated by a numerical example.

Keywords: multivariable systems, static decoupling, dynamic decoupling, stability.

O odsprzęganiu liniowych układów dynamicznych MIMO z zapewnieniem stabilności

Streszczenie

W artykule omawia się sposoby odsprzęgania dynamicznych wielowymiarowych układów liniowych do celów sterowania nimi w otwartej pętli. Rozważa się zarówno odsprzęganie statyczne eliminujące interakcje między poszczególnymi wyjściami obiektu w stanach ustalonych jak i odsprzęganie dynamiczne pozwalające na eliminację tych oddziaływań również w stanach przejściowych. Rozważania zilustrowano przykładem obliczeniowym.

Słowa kluczowe: układy wielowymiarowe MIMO, odsprzęganie statyczne, odsprzęganie dynamiczne, stabilność.

1. Introduction

A characteristic feature of multi-input multi-output (MIMO) dynamical systems, which differs them from single-input single-output (SISO) systems is coupling of their inputs and outputs. It means that each input (control signal) may affect many of their outputs (controlled signals) at the same time. Such interactions can be inconvenient in designing multivariable control system, as well as may cause serious difficulties during the control of such systems, especially, in open-loop control systems because apart from the expected (desirable) influence of the chosen input signals to the specified output system signals, an additional undesirable interactions may occur. Thus elimination or at least reduction of such undesirable interactions in MIMO system is a problem of a great practical and theoretical importance.

For dynamic systems the above mentioned requirements may concern the interactions between signals both in a steady state and transition states. Elimination of such undesirable interactions only for steady states of the system is called *static decoupling*, whereas elimination of these interactions in transition states is called *dynamic decoupling*.

2. The plant descriptions

We consider a controllable and observable linear multivariable dynamic LTI systems (continuous or discrete-time) described by the state and output equations

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{u}(t)\end{aligned} \quad \text{or} \quad \begin{aligned}\boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}\boldsymbol{u}(k)\\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k)+\boldsymbol{D}\boldsymbol{u}(k)\end{aligned} \tag{1}$$

where $\boldsymbol{x}(\cdot)\in R^n$, $\boldsymbol{u}(\cdot)\in R^m$ and $\boldsymbol{y}(\cdot)\in R^l$ are the state, input and output vectors respectively, with the matrices $\boldsymbol{A}\in\mathbb{R}^{n\times n}$, $\boldsymbol{B}\in\mathbb{R}^{n\times m}$, $\boldsymbol{C}\in\mathbb{R}^{l\times n}$ and $\boldsymbol{D}\in\mathbb{R}^{l\times m}$ where $m\geq l$. In a polynomial approach the systems are described by $l\times m$ rational (proper) transfer matrices $\boldsymbol{T}(\cdot)$ of full rank l:

$$\boldsymbol{T}(s)=\boldsymbol{C}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}=\boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s)$$

or

$$\boldsymbol{T}(z)=\boldsymbol{C}(z\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}=\boldsymbol{B}_1(z)\boldsymbol{A}_1^{-1}(z). \tag{2}$$

We assume that polynomial matrices $\boldsymbol{A}_1(\cdot)\in\mathbb{R}[\cdot]^{m\times m}$ and $\boldsymbol{B}_1(\cdot)\in\mathbb{R}[\cdot]^{l\times m}$ are relatively right prime (*r.r.p.)* with $\boldsymbol{A}_1(\cdot)$ column-reduced and they satisfy the conditions $\deg_{ci}\boldsymbol{B}_1(\cdot)\leq\deg_{ci}\boldsymbol{A}_1(\cdot)=d_i$, $i=1,2,...,m$.

Following the theorem given by [1] and results given in numerous afterward papers, we have assumed (for dynamic decoupling) that the system to be decoupled should be described by invertible or right-invertible transfer matrices $\boldsymbol{T}(\cdot)$ with $m\geq l$, (*i.e.* of full normal $rank\boldsymbol{T}(\cdot)=l$). Then it is possible to find a proper transfer matrix $\boldsymbol{T}_c(\cdot)$ of dimension $m\times l$, such that $\boldsymbol{T}(\cdot)\boldsymbol{T}_c(\cdot)=\boldsymbol{T}_d(\cdot)$ where $\boldsymbol{T}_d(\cdot)$ is nonsingular, diagonal (or block diagonal) rational transfer matrix. For static decoupling, systems to be decoupled may have any number of inputs m and outputs l, but should be described by the transfer matrices $\boldsymbol{T}(\cdot)$ of full rank being equal to m or l (*i.e.* they can be either invertible or right-invertible as well as left-invertible).

3. Methods of decoupling of MIMO systems

Depending on requirements imposed on a system, one may design the system (statically or dynamically decoupled) for manual control or as a dynamically decoupled part of the automatic control system (regulation system).

In the first case, the system being dynamically (block or diagonal) decoupled standalone should also be internally stable and internally proper (physically realizable). In the second situation, when the dynamically decoupled system will be a part of a composed (multiloop) control system only – it may be, for example, a part of a multipurpose control systems MCS – it does not have to be stable. It will suffice, after decoupling, the system will be controllable and observable, *i.e.* do not consist any unstable uncontrollable and/or unobservable "hidden" modes.

Since static decoupling of MIMO dynamic systems concern their steady-state properties, static decoupled systems must be stable (internally stable) in any case. Additional requirements may concern dynamical properties of such decoupled system in a transition phase of its behavior. It can be noted that any multivariable control system which guarantees zero steady-state errors is a static decoupled one.

3.1. Static decoupling

For static decoupling of stable MIMO systems it is enough to use a static feedforward compensator (*static precompensator*) $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ included in the input $\boldsymbol{u}(\cdot)$ of a system, such that

$$\boldsymbol{K}_P \boldsymbol{G} = \boldsymbol{I}_l \tag{3}$$

for a case when $m \geq l$. If $m < l$ a *static postcompensator* $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ adjoined to the output $\boldsymbol{y}(\cdot)$ of a system such that

$$\boldsymbol{G}\boldsymbol{K}_P = \boldsymbol{I}_m, \tag{4}$$

may be used. In these equations $\boldsymbol{K}_P \in \mathbb{R}^{l \times m}$ is the "gain" matrix of a system (1) defined by

$$\boldsymbol{K}_P = [\boldsymbol{C}(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}]_{|s=0} = \boldsymbol{C}(-\boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{5}$$

for continuous systems and

$$\boldsymbol{K}_P = [\boldsymbol{C}(z\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}]_{|z=1} = \boldsymbol{C}(\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{6}$$

for discrete-time systems, respectively. If the MFD forms (2) are used as a plant description these gain matrices can be calculated from relationships

$$\boldsymbol{K}_P = \left[\boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s)\right]_{|s=0} \text{ or } \boldsymbol{K}_P = [\boldsymbol{A}_2^{-1}(s)\boldsymbol{B}_2(s)]_{|s=0} \tag{7}$$

for continuous and

$$\boldsymbol{K}_P = \left[\boldsymbol{B}_1(z)\boldsymbol{A}_1^{-1}(z)\right]_{|z=1} \text{ or } \boldsymbol{K}_P = [\boldsymbol{A}_2^{-1}(z)\boldsymbol{B}_2(z)]_{|z=1} \tag{8}$$

for discrete-time systems, respectively [2].

To obtain the matrix $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ for *static precompensator* the eq. (3) should be solved. For systems with the same number of outputs and inputs ($m = l$), it is defined simply by inversion $\boldsymbol{K}_P^{-1}$ (if nonsingular), *i.e.* $\boldsymbol{G} = \boldsymbol{K}_P^{-1}$. If $m > l$ pseudoinverse operations of matrices in terms of (7) and (8) must be used to obtain

$$\boldsymbol{G} = [\boldsymbol{A}_1(0)\boldsymbol{B}_1^{\#}(0)] \text{ or } \boldsymbol{G} = [\boldsymbol{B}_2^{\#}(0)\boldsymbol{A}_2(0)] \tag{9}$$

and

$$\boldsymbol{G} = [\boldsymbol{A}_1(1)\boldsymbol{B}_1^{\#}(1)] \text{ or } \boldsymbol{G} = [\boldsymbol{B}_2^{\#}(1)\boldsymbol{A}_2(1)] \tag{10}$$

respectively, where $\boldsymbol{B}_1^{\#}(\cdot)$ and $\boldsymbol{B}_2^{\#}(\cdot)$ denote pseudoinversion of matrices $\boldsymbol{B}_1(\cdot)$ and $\boldsymbol{B}_2(\cdot)$. The same relationships are also true for *static postcompensators* $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ appended to the system output $\boldsymbol{y}(\cdot)$ to produce an "external" signal vector $\boldsymbol{z}(\cdot) = \boldsymbol{G}\boldsymbol{y}(\cdot)$, if the system to be decoupled has less inputs m than outputs l ($m < l$). They follow directly from solution of the eq. (4).

When a decoupled MIMO system is unstable it should be stabilized before its decoupling. If the state vector $\boldsymbol{x}(\cdot)$ of a decoupled system is accessible for measurement, one can apply linear state variable feedback defined (*l.s.v.f.*) by (static) matrix $\boldsymbol{F} \in \mathbb{R}^{m \times n}$ to obtain (stable) transfer matrices

$$\boldsymbol{T}_F(s) = \left(\boldsymbol{C} - \boldsymbol{D}\boldsymbol{F}\right)\left(s\boldsymbol{I}_n - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{F}\right)^{-1}\boldsymbol{B} + \boldsymbol{D} = \boldsymbol{B}_1(s)\boldsymbol{C}_1^{-1}(s), \tag{11}$$

for continuous-time and

$$\boldsymbol{T}_F(z) = \left(\boldsymbol{C} - \boldsymbol{D}\boldsymbol{F}\right)\left(z\boldsymbol{I}_n - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{F}\right)^{-1}\boldsymbol{B} + \boldsymbol{D} = \boldsymbol{B}_1(z)\boldsymbol{C}_1^{-1}(z) \tag{12}$$

for discrete-time systems, respectively, where $\boldsymbol{C}_1(\cdot) \in \mathbb{R}[\cdot]^{m \times m}$ is stable (Hurwitz or Schur) polynomial matrix. Then using *static precompensators* $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ given by

$$\boldsymbol{G} = [\boldsymbol{C}_1(0)\boldsymbol{B}_1^{-1}(0)] \tag{13}$$

for continuous-time and

$$\boldsymbol{G} = [\boldsymbol{C}_1(1)\boldsymbol{B}_1^{-1}(1)] \tag{14}$$

for discrete-time systems with the same number of inputs and outputs $m = l$, as well as by

$$\boldsymbol{G} = [\boldsymbol{C}_1(0)\boldsymbol{B}_1^{\#}(0)] \tag{15}$$

and

$$\boldsymbol{G} = [\boldsymbol{C}_1(1)\boldsymbol{B}_1^{\#}(1)] \tag{16}$$

for systems with $m > l$, we get static decoupled systems which satisfy the conditions

$$[\boldsymbol{T}_F(s)\boldsymbol{G}]_{|s=0} = \boldsymbol{I}_l \ \left([\boldsymbol{T}_F(z)\boldsymbol{G}]_{|z=1} = \boldsymbol{I}_l, \text{ respectively}\right) \tag{17}$$

This is the most efficient method for static decoupling by using *l.s.v.f.* together with *static precompensators* when plant state vector is accessible for measurement. The same relationships are also true for *static postcompensators* $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ appended to the system output $\boldsymbol{y}(\cdot)$ after its stabilizing.

When a state vector of the system to be decoupled is not accessible (or contaminated by stochastic disturbances like white Gaussian noise), then either a Luenberger observer (full or reduced order) or Kalman filter is to be applied in (LQ/LQG or modal) compensators (controllers) designed. The feedback matrix $\boldsymbol{F} \in \mathbb{R}^{m \times n}$ is included then into the structure of that observer or Kalman filter as an output matrix of their "standard" state space realization (using a copy of the state space plant description) [3].

These (optimal or modal) compensators can also be designed in $s \in \mathbb{C}$ (or $z \in \mathbb{C}$) domain in the form of relatively left prime (*l.r.p.*) MFD of (proper) transfer matrices $\boldsymbol{M}_2^{-1}(\cdot)\boldsymbol{N}_2(\cdot)$ obtained from the minimal degree solution of the unilateral (left-side) polynomial matrix equation

$$\boldsymbol{N}_2(\cdot)\boldsymbol{B}_1(\cdot) + \boldsymbol{M}_2(\cdot)\boldsymbol{A}_1(\cdot) = \boldsymbol{Q}(\cdot)\boldsymbol{C}_1(\cdot) \tag{18}$$

where $\boldsymbol{Q}(\cdot) \in \mathbb{R}[\cdot]^{m \times m}$ is a stable (Hurwitz or Schur) denominator matrix of the observer or Kalman filter. After applying so obtained compensators as a controllers in a classic feedback structure presented in Fig. 1, we get the transfer matrix for (stable) closed-loop system between the signals $\boldsymbol{u}_o(\cdot)$ and $\boldsymbol{y}(\cdot)$ in the form

$$\begin{aligned}\boldsymbol{T}_{\boldsymbol{y}\boldsymbol{u}_o}(\cdot) &= \boldsymbol{B}_1(\cdot)\left[\boldsymbol{M}_2(\cdot)\boldsymbol{A}_1(\cdot) + \boldsymbol{N}_2(\cdot)\boldsymbol{B}_1(\cdot)\right]^{-1}\boldsymbol{M}_2(\cdot) = \\ &= \boldsymbol{B}_1(\cdot)\boldsymbol{C}_1^{-1}(\cdot)\boldsymbol{Q}^{-1}(\cdot)\boldsymbol{M}_2(\cdot)\end{aligned} \tag{19}$$

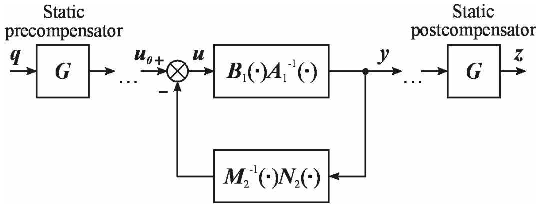

Fig. 1. Feedback control system structure with dynamic compensator in $s \in \mathbb{C}$ or $z \in \mathbb{C}$ domains
Rys. 1. Klasyczna struktura wielowymiarowego układu sterowania z dynamicznym sprzężeniem zwrotnym w dziedzinach operatorowych

For plants with the same number of inputs and outputs $m = l$, the *static precompensator* $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ is given by

$$\begin{aligned}\boldsymbol{G} &= [\boldsymbol{B}_1(s)\Delta^{-1}(s)\boldsymbol{M}_2(s)]^{-1}_{|s=0} = [\boldsymbol{M}_2^{-1}(s)\Delta(s)\boldsymbol{B}_1^{-1}(s)]_{|s=0} = \\ &= [\boldsymbol{A}_1(s)\boldsymbol{B}_1^{-1}(s)]_{|s=0} + [\boldsymbol{M}_2^{-1}(s)\boldsymbol{N}_2(s)]_{|s=0} = \boldsymbol{K}_P^{-1} + \boldsymbol{K}_R\end{aligned} \quad (20)$$

for continuous systems and

$$\begin{aligned}\boldsymbol{G} &= [\boldsymbol{B}_1(z)\Delta^{-1}(z)\boldsymbol{M}_2(z)]^{-1}_{|z=1} = [\boldsymbol{M}_2^{-1}(z)\Delta(z)\boldsymbol{B}_1^{-1}(z)]_{|z=1} = \\ &= [\boldsymbol{A}_1(z)\boldsymbol{B}_1^{-1}(z)]_{|z=1} + [\boldsymbol{M}_2^{-1}(z)\boldsymbol{N}_2(z)]_{|z=1} = \boldsymbol{K}_P^{-1} + \boldsymbol{K}_R\end{aligned} \quad (21)$$

for discrete-time systems respectively, can be used for static decoupling of them.

If the number of plant inputs m is greater than the number of its outputs l ($m > l$), then the right pseudoinversion $\boldsymbol{B}_1^{\#}(\cdot)$ of the numerator matrix $\boldsymbol{B}_1(\cdot)$ should be applied.

The $\boldsymbol{K}_R \in \mathbb{R}^{m \times l}$ and $\boldsymbol{K}_P \in \mathbb{R}^{l \times m}$ in above presented relationships are the „gain" matrices of the controller $\boldsymbol{M}_2^{-1}(\cdot)\boldsymbol{N}_2(\cdot)$ and the plant $\boldsymbol{B}_1(\cdot)\boldsymbol{A}_1^{-1}(\cdot)$, respectively.

The static *precompensators* $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ defined by the eqs. (20) and (21) decouple those (stabilized) systems, which in steady-state satisfy the conditions

$$[\boldsymbol{T}_{\boldsymbol{y}\boldsymbol{u}_o}(s)\boldsymbol{G}]_{|s=0} = \boldsymbol{I}_l \ \ ([\boldsymbol{T}_{\boldsymbol{y}\boldsymbol{u}_o}(z)\boldsymbol{G}]_{|z=1} = \boldsymbol{I}_l \text{, respectively}) \quad (22)$$

for a case when $m \geq l$. If the system to be decoupled has the number of inputs m less than outputs l ($m < l$) then static *postcompensators* $\boldsymbol{G} \in \mathbb{R}^{m \times l}$ appended to the system output $\boldsymbol{y}(\cdot)$ and defined by the same relationships (20) – (21) can be used to satisfy the condition

$$[\boldsymbol{G}\boldsymbol{T}_{\boldsymbol{y}\boldsymbol{u}_o}(s)]_{|s=0} = \boldsymbol{I}_m \ \ ([\boldsymbol{G}\boldsymbol{T}_{\boldsymbol{y}\boldsymbol{u}_o}(z)]_{|z=1} = \boldsymbol{I}_m \text{, respectively}) \quad (23)$$

These *postcompensators* decouple left-invertible systems in steady-state between the input vector signal $\boldsymbol{u}(\cdot) \in R^m$ and "external" signal vector $z(\cdot) \in R^m$.

3.2. Dynamic decoupling

A general requirement for the dynamic decoupling of a multivariable system with m inputs and l outputs is to lead the system to the situation when some specific group of inputs affects only a specific group of outputs and all other interactions between these input and output groups are eliminated. It is called *dynamic block decoupling*.

If the goal of the decoupling is to create separable pairs of signals with one input to one output then the decoupling is called *diagonal dynamic decoupling* or *row-by-row decoupling*. It is the most rigorous but the most typical goal for dynamic MIMO systems, where one input of the control system affects only one of its outputs, both in a transition and steady state of the system.

Let $l_1, l_2, ..., l_k$ and $p_1, p_2, ..., p_k$ be the sets of positive integers which satisfy the condition

$$\sum_{i=1}^{k} l_i = l \quad \text{and} \quad \sum_{i=1}^{k} p_i = p, \quad (24)$$

for which the output vector $\boldsymbol{y}(\cdot) \in R^l$ of the decoupled system and the vector of exogenous signals $\boldsymbol{q}(\cdot) \in R^p$ have the form

$$\boldsymbol{y}(\cdot) = \left[\boldsymbol{y}_1^{\mathrm{T}}(\cdot) \ \dots \ \boldsymbol{y}_{\mathrm{i}}^{\mathrm{T}}(\cdot) \ \dots \ \boldsymbol{y}_{\mathrm{k}}^{\mathrm{T}}(\cdot)\right]^{\mathrm{T}}, \ \boldsymbol{y}_i(\cdot) \in R^{l_i}$$

and

$$\boldsymbol{q}(\cdot) = \left[\boldsymbol{q}_1^{\mathrm{T}}(\cdot) \ \dots \ \boldsymbol{q}_{\mathrm{i}}^{\mathrm{T}}(\cdot) \ \dots \ \boldsymbol{q}_{\mathrm{k}}^{\mathrm{T}}(\cdot)\right]^{\mathrm{T}}, \ \boldsymbol{q}_i(\cdot) \in R^{p_i}. \quad (25)$$

The goal of a (block) dynamic decoupling of a linear dynamic MIMO system with inputs $\boldsymbol{u} \in R^m$ and outputs $\boldsymbol{y} \in R^l$ in $s \in \mathbb{C}$ (or $z \in \mathbb{C}$, respectively) by a rational transfer matrix $\boldsymbol{T}(\cdot)$ $l \times m$, is to create a system with inputs $\boldsymbol{q} \in R^p$ and outputs $\boldsymbol{y} \in R^l$, described by diagonal transfer matrix $\boldsymbol{T}_d(\cdot) = diag[\boldsymbol{T}_{ii}(\cdot)]$, where $\boldsymbol{T}_{ii}(\cdot)$, $i = 1, 2, ..., k$ are nonsingular transfer matrices of dimensions $l_i \times p_i$; k is a number of groups (blocks) into which signals $\boldsymbol{q}$ and $\boldsymbol{y}$ are partitioned. In a block decoupling it is possible to take an exogenous vector $\boldsymbol{q} \in R^p$ either as $p = m$ or $p = l$. In the case of diagonal decoupling $l_i = p_i = 1$, and $p = l$.

A special type of dynamic decoupling is a triangular decoupling where it is required that each i-th input controls i-th output not affecting any j-th output for $j > i$. Triangular decoupling is of minor practical significance and it is easier in realization, so it will not be considered in the paper.

Dynamic decoupling may be realized in many different ways, depending on the structure of the system to be decoupled, requirements imposed on a system after decoupling, the method of decoupling and the forms of describing systems in time or frequency domain [4-8].

3.2.1. Dynamic decoupling by output feedback

Due to possibility of measuring outputs of the system, one of the most popular method of dynamic decoupling is the use of the output feedback. It may be realized by the output feedback $\boldsymbol{H}(\cdot)$ together with *an input dynamic compensators* $\boldsymbol{G}(\cdot)$

$$\boldsymbol{u}(\cdot) = \boldsymbol{H}(\cdot)\boldsymbol{y}(\cdot) + \boldsymbol{G}(\cdot)\boldsymbol{q}(\cdot), \quad (1)$$

where $\boldsymbol{H}(\cdot)$ and $\boldsymbol{G}(\cdot)$ are proper transfer matrices in $s \in \mathbb{C}$ or $z \in \mathbb{C}$, respectively. Special cases of such method of decoupling are situations when one or both of the matrices $\boldsymbol{H}(\cdot)$ or $\boldsymbol{G}(\cdot)$ are static (of zero degree).

A special case of the decoupling by output feedback is a dynamic decoupling in an unity output feedback structure [9]. Such a system has the structure of a typical control system (with *unity output feedback*), where onto inputs to the plant dynamic compensators are placed. This allows one to achieve simultaneously decoupling and stabilization of the closed-loop control system.

From the analysis of many papers concerning dynamic decoupling it follows that the inner stability and property of the (parts of) decoupled systems by using the structure with output feedback, is not simple to ensure and in many cases is simply impossible.

The most of the proposed methods allows some fixed poles to exist in the decoupled system which can result in the system instability. Moreover, they are often confined to square plants with minimum-phase transmission zeros only.

3.2.2. Dynamic decoupling by l.s.v.f.

Linear state variable feedback (*l.s.v.f.*) usually together with an input dynamics (*dynamic feedforward compensator*) seems to be the most effective way of decoupling. The decoupling law takes here the following form:

$$\boldsymbol{u}(\cdot) = -\boldsymbol{F}(\cdot)\boldsymbol{x}(\cdot) + \boldsymbol{G}(\cdot)\boldsymbol{q}(\cdot). \tag{27}$$

One of the imposed requirements for the designed system is its possibly the lowest rank. From that point of view the best solution of the problem is the use of static feedback matrices $\boldsymbol{F} \in R^{m\times n}$ and $\boldsymbol{G} \in R^{m\times p}$ (zero degree).

However, decoupling by static matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ apart of block or diagonal decoupling of the system do not guarantee achievement of many other features important from the practical point of view. Namely, systems with only static feedbacks can be unstable, which makes them impractical for open-loop (manual) control. They can also be unobservable and/or uncontrollable (with unstable hidden parts) which means that such decoupled system cannot even be automatically controlled, *i.e.* it cannot be a part of a multipurpose control system [10, 11].

In the papers [12, 13] we have presented the algorithm (using a *l.s.v.f.* with *dynamic precompensator*) for dynamic decoupling MIMO systems, which provides a decoupled (block or diagonal) system without any unstable uncontrollable and unobservable parts. The decoupled right-invertible or invertible plant ($m \ge l$) can be unstable, non-minimum phase or both. The algorithm ensures that the decoupled system is always internally stable and internally proper (physically realizable).

The idea of the method is as follows. By using the linear state variable feedback along with the dynamic feedforward $\boldsymbol{G}^{-1}(s)\boldsymbol{L}(s)$ presented in Fig. 2, we decouple the system between the signals $\boldsymbol{q}$ and $\boldsymbol{y}$ to obtain a block diagonal transfer matrix

$$\boldsymbol{T}_{\boldsymbol{yq}}(s) = \boldsymbol{B}_1(s)\left[\boldsymbol{G}(s)\boldsymbol{A}_1(s) - \boldsymbol{F}(s)\right]^{-1}\boldsymbol{L}(s) = \boldsymbol{N}(s)\boldsymbol{D}^{-1}(s) \tag{28}$$

with

$$\boldsymbol{N}(s) = \text{block diag}[\boldsymbol{N}_{ii}(s), i = 1,2,...,k] \in R[s]^{l\times l} \tag{29}$$

and

$$\boldsymbol{D}(s) = \text{block diag}[\boldsymbol{D}_{ii}(s), i = 1,2,...,k] \in R[s]^{l\times l}. \tag{30}$$

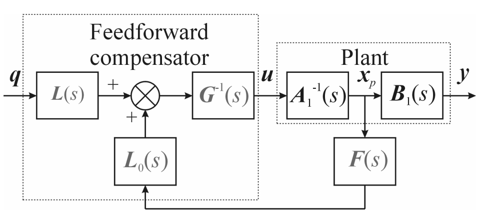

Fig. 2. Structure of the dynamically decoupled system with accessible state vector of the plant

Rys. 2. Struktura dynamicznie odsprzężonego układu z dostępnym wektorem stanu obiektu

The algorithm starts with determination of the numerator matrix $\boldsymbol{N}(s)$ of the decoupled system which is taken as a block diagonal matrix $\boldsymbol{N}(s) = \text{block diag}[\boldsymbol{N}_{ii}(s), i = 1,2,...,k]$, where particular blocks $\boldsymbol{N}_{ii}(s)$ are great common left divisors of columns of *i*-th row-block of $\boldsymbol{B}_1(s)$ caused by the partition (25)

$$\boldsymbol{B}_1(s) = \begin{bmatrix} \boldsymbol{B}_{11}(s) \\ \vdots \\ \boldsymbol{B}_{1i}(s) \\ \vdots \\ \boldsymbol{B}_{1k}(s) \end{bmatrix}. \tag{31}$$

Then $\boldsymbol{B}_1(s)$ takes the form

$$\boldsymbol{B}_1(s) = \boldsymbol{N}(s)\boldsymbol{B}(s)\,. \tag{32}$$

Next steps of the algorithm allows us to calculate a feedforward $\boldsymbol{G}^{-1}(s)\boldsymbol{L}(s)$ and the state vector feedback matrix $\boldsymbol{F}$. The denominator matrix $\boldsymbol{D}(s)$ is arbitrary set which guarantees free location of all controllable and observable poles of the system, independently for each loop (block) of the system. Some unobservable and uncontrollable poles, if exist, are freely chosen in designing of the system. The algorithm provides also handling a situation when the plant has non-minimumphase interconnection transitions zeros. Then an additional dynamic element is added into inputs of the plant and finally moved to a *dynamic feedforward compensator* [12, 13].

The state vector $\boldsymbol{x}(\cdot)$ of the decoupled system is usually not accessible and/or contaminated by stochastic disturbances. Then either a Luenberger observer (full or reduced order) or Kalman filter has to be applied. The feedback matrix $\boldsymbol{F}$ (designed as if the state vector was accessible) is then included in the structure of an observer or filter as an output matrix of its state space description [12].

4. A numerical example

In order to illustrate the above considerations the design procedure of systems to be manually controlled for a multivariable dynamical system of rank $n = 5$ with $m = 3$ inputs and $l = 2$ outputs described by the state and output equations (1) where appropriate matrices take the form

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ -2 & -1 & 1 & 5 & 1 \\ -4 & 0 & 2 & 1 & -1 \\ 1 & -1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

will be presented below.

In a polynomial approach its transfer matrix $\boldsymbol{T}(\cdot)$ is described by *r.r.p.* polynomial fraction

$$\boldsymbol{T}(s) = \boldsymbol{B}_1(s)\boldsymbol{A}_1^{-1}(s) = \begin{bmatrix} -4 & 0 & 4 \\ s-2 & 0 & 3s-8 \end{bmatrix} \begin{bmatrix} s^2-1.25s-5.5 & -3 & -10.75s-4 \\ 2 & s & 8 \\ 1.75s-2.5 & 0 & s^2+0.25s-13 \end{bmatrix}^{-1}$$

The system is unstable and has a non-minimumphase transmission zero $s_1^o = 2.5$, and its gain matrix (7) has the form

$$\boldsymbol{K}_P = \begin{bmatrix} 0 & -10.33(3) & -6.66(6) \\ 0 & -1 & 0 \end{bmatrix}$$

Since the system is unstable, it has to be stabilized by the feedback to obtain (11) and statically decoupled afterwards.

Taking values for poles of the closed-loop system as

$$s_{1,2} = -2.12180 \pm j0.53925\,, \quad s_3 = -3.11453$$

and

$$s_{4,5} = -2.58765 \pm j3.20271$$

we have set the matrix

$$C_1(s) = \begin{bmatrix} s^2+4.24361s+4.79285 & 0 & 0 \\ 0 & s+3.11453 & 0 \\ 0 & 0 & s^2+5.17530s+16.95326 \end{bmatrix}$$

which yields the feedback matrix

$$\boldsymbol{F} = \begin{bmatrix} 11.237216 & 4.060902 & -9.140032 & -1.432706 & 6.070016 \\ -2.778632 & 0 & 2.557265 & 0 & 0.278632 \\ 3.988315 & 0.793825 & 0.500000 & 2.543825 & -0.250000 \end{bmatrix}.$$

Then the *static precompensator* $\boldsymbol{G}$, which decouples the system, calculated from the eq. (15) is as follows

$$\boldsymbol{G} = [\boldsymbol{C}_1(0)\boldsymbol{B}_1^{\#}(0)] = \begin{bmatrix} -0.958569 & -0.479285 \\ 0 & 0 \\ 0.847663 & -1.695326 \end{bmatrix}.$$

Results of simulation of such statically decoupled system are shown in Fig. 3. The simulation was done with the assumption of non zero initial conditions of the state vector $\boldsymbol{x}(0) = [1,-2,3,-4,5]^T$. The reference signal was taken as $\boldsymbol{q}(t) = \boldsymbol{q}_0 \boldsymbol{1}(t)$ for $q_{1o}(t) = 10$ and $q_{2o}(t) = -10$ changed at the time $t_1 = 10s$ and $t_1 = 20s$, respectively.

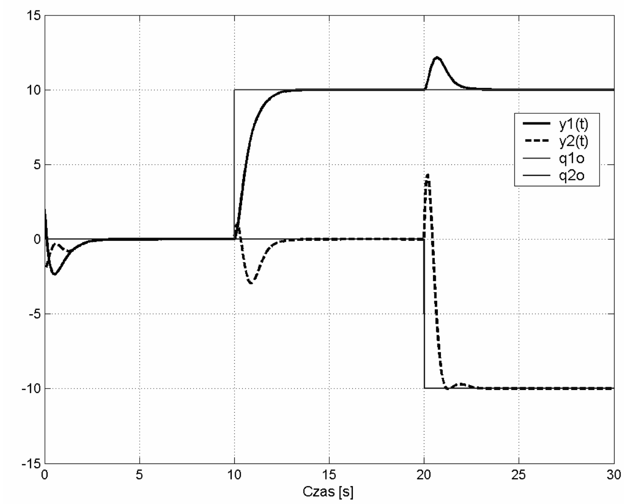

Fig. 3. Step responses of the statically decoupled system with accessible plant state vector

Rys. 3. Odpowiedzi skokowe statycznie odsprzężonego układu z dostępnym wektorem stanu

Assuming the plant state vector is inaccessible, the modal compensator $\boldsymbol{M}_2^{-1}(s)\boldsymbol{N}_2(s)$ based on a Luenberger observer has been calculated. Taking poles for the observer as

$$s_1 = -90.4579,\ s_2 = -5.3220,\ s_3 = -2.50874$$

and

$$s_{4,5} = -5.08145 \pm j5.82653$$

(including the above presented matrix $\boldsymbol{F}$) we have obtained an unstable compensator with the "gain" matrix

$$\boldsymbol{K}_R = [\boldsymbol{M}_2^{-1}(0)\boldsymbol{N}_2(0)] = \begin{bmatrix} -4.70133 & 2.38186 \\ -1.93082 & 2.17558 \\ -0.05748 & -1.39459 \end{bmatrix}$$

So the static precompensator for this system has been given as follows

$$\boldsymbol{G} = [\boldsymbol{A}_1(0)\boldsymbol{B}_1^{\#}(0)] + \boldsymbol{K}_R = \begin{bmatrix} -3.801325 & 3.331855 \\ -1.930824 & 1.175577 \\ -0.207483 & 0.155413 \end{bmatrix}.$$

Results of simulation of such decoupled system are shown in Fig. 4. The simulation was done with the same (non-zero) initial conditions $\boldsymbol{x}(0) = [1,-2,3,-4,5]^T$ for plant state vector and (zero) initial conditions for state vector of the observer.

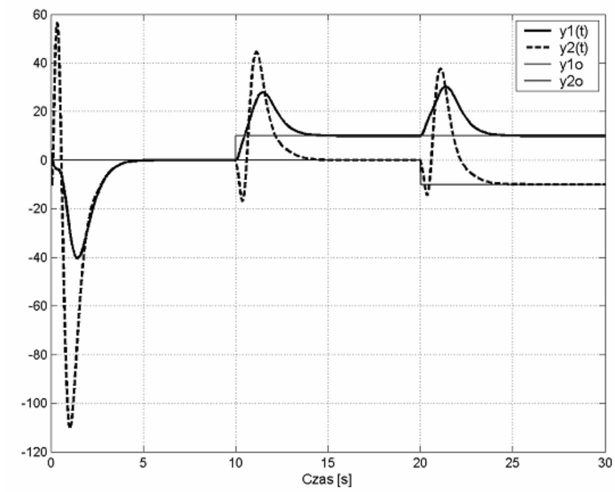

Fig. 4. Step responses of the statically decoupled system with an inaccessible plant state vector

Rys. 4. Odpowiedzi skokowe statycznie odsprzężonego układu z niedostępnym wektorem stanu

To compare the features of a statically and dynamically decoupled systems the same plant has been assumed to be dynamically decoupled by using *l.s.v.f.* together with a dynamic feedforward compensator. All calculations have been performed using the algorithm given in details in [12].

Taking the controllable and observable poles

$$s_{1,2} = -2.12180 \pm j0.53925,\ s_3 = -3.11453$$

and

$$s_{4,5} = -2.58765 \pm j3.20271$$

and one uncontrollable pole

$$s_6 = -5$$

for the dynamic decoupled system we obtain a feedforward compensator $\boldsymbol{G}^{-1}(s)\left[\boldsymbol{L}(s) \quad \boldsymbol{L}_0(s)\right]$ with matrices

$$\boldsymbol{G}(s) = \begin{bmatrix} s-25.8218 & -0.06096s+1.5741 & -0.27039s+6.9818 \\ 5.8629 & 1 & 21.4581 \\ -0.76448 & -16.9761 & 1 \end{bmatrix},$$

$$\boldsymbol{L}(s) = \begin{bmatrix} -0.20440s-0.23235 & 0.18240s-15.7306 \\ 0.24184 & 6.8303 \\ 0.20584 & 0.05888 \end{bmatrix}$$

and

$$\boldsymbol{L}_0(s) = \begin{bmatrix} -0.20440s-0.23235 & 0.18240s-15.7306 & -0.09483s+23.0204 \\ 0.24184 & 6.8303 & 1.555(5) \\ 0.20584 & 0.05888 & -26.4072 \end{bmatrix}$$

and the state vector feedback matrix $\boldsymbol{F}$

$$\boldsymbol{F}=\begin{bmatrix} -181.9180 & -38.0063 & 37.4788 & -141.4452 & 18.4644 \\ -16.5539 & -4.7426 & 10.5962 & -1.1346 & -7.9577 \\ 0.1896 & -0.4011 & -1.1564 & -0.8021 & -0.7883 \end{bmatrix}.$$

Transfer matrix of the system $\boldsymbol{T}_{yq}(\cdot)=\boldsymbol{T}_d(s)=\boldsymbol{N}(s)\boldsymbol{D}^{-1}(s)$ is finally described by the matrices

$$\boldsymbol{N}(s)=\begin{bmatrix} s-2.5 & 0 \\ 0 & s-2.5 \end{bmatrix}$$

and

$$\boldsymbol{D}(s)=\begin{bmatrix} s^3+7.3581s^2+18.010s+14.927 & 0 \\ 0 & s^2+5.1753s+16.953 \end{bmatrix}.$$

Its gain matrix has the form

$$\boldsymbol{K}_d=\left[\boldsymbol{N}(s)\boldsymbol{D}^{-1}(s)\right]_{s=0}=\begin{bmatrix} -0.1675 & 0 \\ 0 & -0.1475 \end{bmatrix}.$$

In order to obtain $\boldsymbol{K}_d=\boldsymbol{I}_2$ the above mentioned $\boldsymbol{L}(s)$ has been postmultiplied by

$$\left[\boldsymbol{D}(s)\boldsymbol{N}^{-1}(s)\right]_{s=0}=\begin{bmatrix} -5.9710 & 0 \\ 0 & -6.7813 \end{bmatrix}$$

which finally gives

$$\boldsymbol{L}(s)=\begin{bmatrix} 1.2205s+1.3874 & -1.2369s+106.674 \\ -1.4440 & -46.3182 \\ -1.2291 & -0.39929 \end{bmatrix}$$

and

$$\boldsymbol{N}(s)=\begin{bmatrix} -5.9710(s-2.5) & 0 \\ 0 & -6.7813(s-2.5) \end{bmatrix}.$$

Results of simulation of such designed system are shown in Fig. 5.

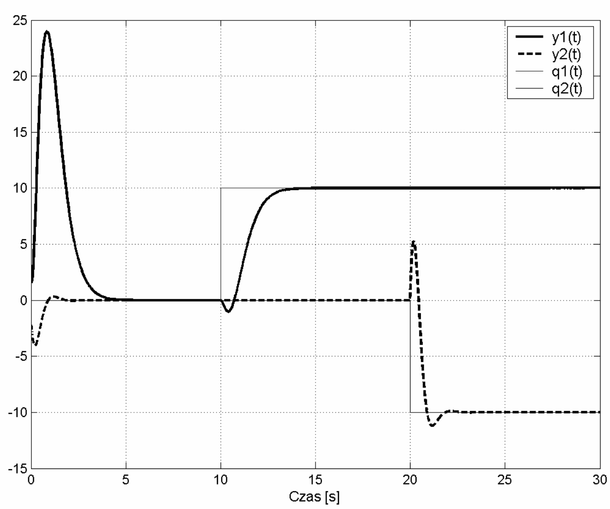

Fig. 5. Step responses of the dynamically decoupled system with accessible plant state vector
Rys. 5. Odpowiedzi skokowe dynamicznie odsprzężonego układu z dostępnym wektorem stanu

Simulations were done with the same (non-zero) initial conditions $\boldsymbol{x}(0)=\left[1,-2,3,-4,5\right]^T$ for plant state vector and (zero) initial conditions for state vector of the dynamic feedforward compensator. Similar results may have been obtained by using an observer or a Kalman filter in the case when the state vector $\boldsymbol{x}(t)$ is inaccessible and/or noised.

5. Conclusion

In the paper problems of static and dynamic decoupling of linear MIMO dynamic systems have been presented. The algorithms that may be used to design a system for open-loop control ensure stability and free assignment of all poles of decoupled systems. In such a system we achieve desired values for outputs $\boldsymbol{y}(\cdot)\to\boldsymbol{y}_0$ when $\boldsymbol{q}(\cdot)=\boldsymbol{y}_0$ is given on external inputs of this system. The proposed method for static decoupling may be applied to systems with any number of inputs m and outputs l by using a *static precompensator* for $m\geq l$ or *static postcompensator* for $m<l$, after the systems have been stabilized. All presented methods provide internal stability and internal property for both unstable and non-minimum phase proper plants. Results of considerations are illustrated by a numerical example.

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Artykuł recenzowany