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### Jacek PISKOROWSKI, Roman KASZYNSKI

POLITECHNIKA SZCZECIŃSKA, INSTYTUT AUTOMATYKI PRZEMYSŁOWEJ

# Effective Filtering in Signal Multiplexing Systems

#### Ph.D. Jacek PISKOROWSKI

Was born in Pila Poland in 1977. He received the M.Sc. degree in electronic engineering from Szczecin University of Technology, Poland, in 2002, and the Ph.D. degree from the same university in 2006. Since 2002 he has been with the Institute of Control Engineering, Szczecin University of Technology. His research activity is mainly focused on signal processing and measurement with particular interest in analysis, synthesis and design of systems and circuits with time-varying parameters.



 $e\hbox{-}mail\hbox{:} jacek.piskorowski@ps.pl$ 

#### Abstract

In this article, we propose a novel concept of a continuous-time timevarying filter of constant component that exhibits a very short response in the time domain without significant distortions in the frequency domain. Such a designed filter is then applied in the system containing analog multiplexer, where it is able to reveal its advantageous properties. Results verifying the effectiveness of the proposed approach are presented and compared to the traditional filters.

Keywords: signal processing, data aquisition, analog multiplexer, timevarying systems, filter of constant component.

# Efektywna filtracja w systemach multipleksowania sygnałów

#### Streszczenie

Artykuł przedstawia nową koncepcję filtru składowej stałej o zmiennych parametrach, który charakteryzuje się bardzo krótką odpowiedzią w dziedzinie czasu, nie wykazując przy tym istotnych zaburzeń filtrowanych sygnałów w dziedzinie częstotliwości. Zaprojektowany filtr wykorzystano w procesie multipleksowania silnie zaszumionych sygnałów, co pozwoliło na zaprezentowanie zalet zaproponowanego układu. W celu przedstawienia korzyści z wprowadzenia do struktury filtru zmiennych w czasie parametrów porównano jego właściwości z klasycznym układem o stałych współczynnikach.

Słowa kluczowe: przetwarzanie sygnałów, akwizycja danych, multiplekser analogowy, układy o zmiennych parametrach, filtr składowej stałej.

### 1. Introduction

Although increasingly many filtering applications are now handled with digital signal processing techniques and digital filters, generally there remains the question of whether to choose an analog or a digital filter for a particular application. In practice, there are a number of situations in which analog continuous-time filters are either a necessity or provide a more economical solution. The design methods of traditional continuous-time filters are described in detail in the rich literature [1-3]

In this article, the authors will address the question of how to design a continuous-time filter of constant component that exhibits a very short response in the time domain without significant distortions in the frequency domain. Such a filter will be applied in the system containing analog multiplexer.

For time-invariant filters there are only small possibilities of shortening the transient state, since the filter parameters are calculated on the basis of the assumed approximation method of the frequency characteristics. This guarantees that the frequency requirements are satisfied without taking into consideration the character of the transient state. If the requirements on the frequency characteristics are imposed, we can slightly influence on the reduction of the transient state of the n-th order filter

#### D.Sc., Ph.D. Roman KASZYŃSKI

Was born in Uniejow, Poland, in 1950. He received the M.Sc. degree from the Szczecin University of Technology, Poland, in 1973, and the Ph.D. degree from the same university in 1978. He received the D.Sc. degree from Silesian Technical University in Gliwice in 2002. Since 1973 he has been with the Institute of Control Engineering, Szczecin University of Technology. His research activity focuses on signal processing and measurement with particular interest in analysis and synthesis of systems with time-varying parameters.



e-mail: roman.kaszynski@ps.pl

by choosing different methods of the approximation. The indeterminacy principle says that it is not possible to achieve a shorter rise time of the low-pass filter output signal when the filter passband is constant. One can obtain significant changes of the duration of the transient state by the variation of the filter

passband [4-6]. This procedure is connected with the change of the value of filter coefficients. The theory of linear time-varying continuous-time systems is well established and was widely described [7-10].

Time-varying filtering has been applied in many fields of signal processing, but mainly in digital systems where it can be more easily implemented. Very good results were achieved e.g. in seismic data processing [11, 12] and medical measurements [13].

This paper describes a novel theoretical approach to the filters of constant component by using the time-varying parameters technique. Such a designed filter is then applied in the signals multiplexing system. The outline of the paper is as follows. Section 2 discusses the stationarity of solutions of time-varying systems. In Section 3, the analysis of the filter of constant component both in the time domain and the frequency domain is presented. The concept and main assumptions of the time-varying filters of constant component are discussed in Section 4. Section 5 then presents the results of simulations of the time-varying filtering in the signals multiplexing system, carried out with the aid of Matlab-Simulink software. The conclusions are presented in Section 6.

# 2. Stationarity of solutions of parametric systems for $t\rightarrow \infty$

In order to analyze spectral properties of filters with timevarying parameters one can use methods applicable to timeinvariant systems under the conditions that the filter parameters stabilize (with  $\alpha$ -accuracy) after passing of the transient state. To show this the theorem on spectral density of the output signal after passing of the transient state in linear systems was used. One proved that this result held also for systems with time-varying parameters if their values stabilize when  $t \to \infty$ .

In the relation (1), (2) and (3) one introduced the spectral transmittance  $K(j\omega)$  of a system with constant parameters which corresponds to a system with time-varying parameters at  $t \to \infty$ ,

denoting it by denoting it by  $K_{t\to\infty}^{var}(j\omega)$  if the condition  $\lim a_i(t) = a_i = \text{const.}$ 

Taking into account

$$\int_{-\infty}^{+\infty} k(\tau_1) e^{j\omega \tau_1} d\tau_1 = K(-j\omega) \triangleq K_{t\to\infty}^{\text{var}} (-j\omega) \Big|_{\substack{\lim a_i(t) = const \\ t\to\infty}} (1)$$

$$\int_{-\infty}^{+\infty} k(\tau_2) e^{-j\omega \tau_2} d\tau_2 = K(j\omega) \triangleq K_{t\to\infty}^{\text{var}} (j\omega) \Big|_{\substack{\lim a_i(t) = const \\ t\to\infty}} (2)$$

$$\int_{-\infty}^{+\infty} k(\tau_2) e^{-j\omega\tau_2} d\tau_2 = K(j\omega) \hat{=} K_{t\to\infty} (j\omega) \Big|_{\substack{\lim a_i(t)=const}} (2)$$

one can as follows

$$S_{y}(\omega) = K(-j\omega)K(j\omega)S_{x}(\omega) = |K(j\omega)|^{2} S_{x}(\omega) = |K_{t\to\infty}(j\omega)|^{2} \cdot S_{x}(\omega)$$
(3)

The above presented proof allows to apply the spectral relations which hold in the steady state for linear time-invariant systems to systems with time-varying parameters if these parameters stabilize their values with time.

#### 3. Filter of constant component

A filter of constant component can be proposed as an effective tool enabling determination of a constant component signal. The constant component filter can be roughly treated as a low-pass filter with the passband narrowed to the single frequency  $\omega=0$ . The stopband of the filter of constant component is determined by the cutoff frequency  $\Omega$  and assumed, admissible value  $\alpha$  limiting the amplitudes of the frequency response  $|K(j\omega)|$  for  $\Omega=\omega$ . Therefore, the magnitude response of a constant component filter can be written as

$$|K(j\omega)| \begin{cases} = 1 & \text{for } \omega = 0 \\ < 1 & \text{for } 0 < \omega < \Omega. \\ \le \alpha & \text{for } \Omega \le \omega \end{cases}$$
 (4)

The general filtration requirements given by (4) enable the synthesis of constant component filters. Of course, using relation (4) one can not explicitly determine the filter structure and its parameters. Thus, the additional filter quality criteria must be taken into consideration.

The filter settling time  $t_{s1}$ , i.e. time, that for each  $t = t_{s1}$  the filter step response is never more than  $\alpha$  different from its final value, can be considered as one among possible quality coefficients for constant component filters design. The evaluation based on a value of the settling time can not be treated as sufficient for the filters belonging to a class of systems with time-variable parameters. The mentioned quality coefficient does not take into account the altering parameters of the spectral characteristic. Using the formula determining the power spectral density of the filter output signal  $S_y(\omega) = S_x(\omega) \cdot \left| K(j\omega) \right|^2$ , the new, spectral "measure" of the quality for constant component filters fulfilling relation (4) has been proposed in the form of the following coefficient:

$$g_{\eta} = \sqrt{\int_{0}^{\infty} |K(j\eta)|^{2} d\eta}$$
 (5)

where  $K(j\eta)$  is the filter transfer function for  $t \to \infty$ , and  $\eta = \omega/\Omega$  is the normalized frequency.

The so-called "time of operation" can be calculated either considering the settling time  $t_{s1}$  of the step response h(t) fulfilling the following relation:

$$\begin{cases} \left|1 - h(t_{s1})\right| = \alpha \\ \left|1 - h(t)\right|_{t > t_{s1}} \le \alpha \end{cases}$$
(6)

or considering the time  $t_{52}$  determined by the filter response y(t) to the sine input signal  $x_1(t)$ , as it follows:

$$\begin{cases} |y(t_{s2})| = 2\alpha \\ |y(t)|_{t>t_{s2}} \le 2\alpha \end{cases}$$
 (7)

functions as well as the input sine signals are identical and equal to the amplitude range of the filter input. The longer among the times  $t_{s1}$ ,  $t_{s2}$  has been established as the operation time  $t_s$ , deciding about the filter quality. The results of calculations and simulations can be easily compared, when the relative operation time will be introduced in the following form:  $t_s = \frac{t_s}{t_s}$  where  $T_{cs}$  denotes the

It has been assumed that the amplitudes of the input step

introduced in the following form:  $t_r = \frac{t_s}{T_\Omega}$  where  $T_\Omega$  denotes the

period corresponding to the angular frequency  $\Omega$ . All the calculations and comparisons have been carried out for  $\alpha = 0.05$ .

The filter quality coefficient k has been assumed in the form:

$$k = t_r \cdot g_n. \tag{8}$$

The product (8) is sensitive for filter improvements created by the introduction of time-varying parameters. The idea of the choice of the quality coefficient in the form (8) can be supported by the following rezoning: the first factor, i.e. the operation time, expresses the duration of the transient state under spectral assumptions (4), the second one determines additional damping of the harmonic components of the signal (comparing to the damping resulting from assumptions (4)). It can be easily perceived that minimizing the coefficient k one improves the quality of the filter [14]. Of course, the coefficient k can be successfully applied to comparative evaluations of constant component filters of both types, i.e. filters with constant and time-varying parameters.

### 4. Time-varying approach

Examinations were carried out for the constant component filter of the second order with varying parameters, which is described by the following differential equation:

$$\frac{1}{\omega_0^2(t)} \cdot y''(t) + \frac{2\beta(t)}{\omega_0(t)} \cdot y'(t) + y(t) = x(t)$$
 (9)

where x(t) and y(t) are the filter input and output, respectively. Moreover,  $\omega_0$  is the characteristic frequency of the filter, and  $\beta$  is referred as the damping factor.

The functions of filter parameters were formulated as follows:

$$\omega_0(t) = \left[1 - c_1 \cdot h_1(t)\right] \cdot \overline{\omega}_0 \tag{10}$$

$$\beta(t) = [b_1 + b_2 \cdot h_2(t)] \cdot \overline{\beta}$$
 (11)

where  $\overline{\omega}_0$  and  $\overline{\beta}$  are the characteristic frequency and the damping factor, respectively, following from the approximation, and  $c_1$ ,  $b_1$ , and  $b_2$  are the functions coefficients. The functions  $h_1(t)$  and  $h_2(t)$  are the step responses of the second order supportive system. These functions have the following forms:

$$h_{1}(t) = L^{-1} \left[ \frac{1}{s} \cdot \frac{1}{(\omega_{01})^{-2} s^{2} + 2\beta_{1}(\omega_{01})^{-1} s + 1} \right]$$

$$= \left[ 1 - \left( 1 + \omega_{01} t \right) \exp(-\omega_{01} t) \right] \cdot 1(t) \quad \text{for} \quad \beta_{1} = 1$$
(12)

and

$$h_{2}(t) = L^{-1} \left[ \frac{1}{s} \cdot \frac{1}{(\omega_{02})^{-2} s^{2} + 2\beta_{2}(\omega_{02})^{-1} s + 1} \right]$$

$$= I(t) - \left[ \cos \left( \omega_{02} t \sqrt{1 - \beta_{2}^{2}} \right) + \frac{\sin \left( \omega_{02} t \sqrt{1 - \beta_{2}^{2}} \right)}{\sqrt{1 - \beta_{2}^{2}}} \right]$$

$$\cdot \exp \left( -\beta_{2} \omega_{02} t \right) \cdot I(t) \quad \text{for} \quad \beta_{2} < 1.$$
(13)

where  $x_1(t)=1(t)\sin(\Omega t+\varphi)$ .

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The parameters  $\omega_{01}$  and  $\omega_{02}$  determine the variation rate of the functions  $\omega_0(t)$  and  $\beta(t)$ , respectively. The range of changes of the parameters has been chosen in the following way:

$$\frac{\omega_0(0)}{\overline{\omega}_0} = 10 \quad \text{and} \quad \frac{\beta(0)}{\overline{\beta}} = 0.5 \tag{14}$$

which means that in the initial phase of the filter work the characteristic frequency is 10-times greater and the damping factor 2-times smaller than the ones following from the approximation, i.e. when the parameters are settled.

### 5. Signals multiplexing system

The time-varying filters can be useful in many signal processing applications. However, these filters are useful if we know the moments in which the filter parameters should be changed. Such a situation appears in multiplexing systems. A block diagram of the signals multiplexing system which cooperates with the time-varying filter of constant component is shown in Fig. 1. Each of N signals from multiplexer inputs  $x_N(t)$  consists of the useful constant component, undesired harmonic components, and noise.

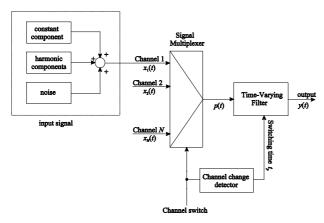


Fig. 1. Block diagram of the signals multiplexing system Rys. 1. Schemat blokowy systemu multipleksowania sygnałów

The input signals are switched according to the control signal which is fed to the channel switch input. As a result of the switching process, the highly noised rectangular signal denoted by p(t) is transferred on the multiplexer output. Such a signal is presented in Fig. 2.

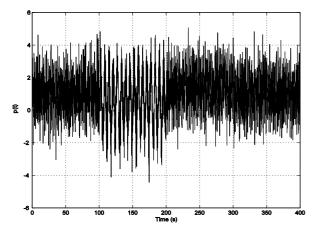


Fig. 2. Highly noised rectangular signal from the output of the analog multiplexer

Rys. 2. Silnie zaszumiony sygnał prostokątny na wyjściu multipleksera analogowego

The functions  $\beta(t)$  and  $\omega_0(t)$  from (9) which vary the parameters of the filter are presented in Figs. 3 and 4, respectively. The multiplexer is switched every 100 s, so the functions  $\beta(t)$  and  $\omega_0(t)$  are generated periodically.

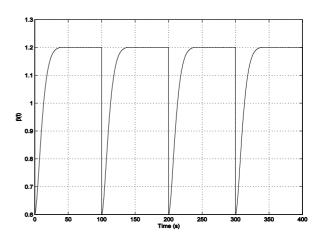


Fig. 3. Function  $\beta(t)$ Rys. 3. Funkcja  $\beta(t)$ 

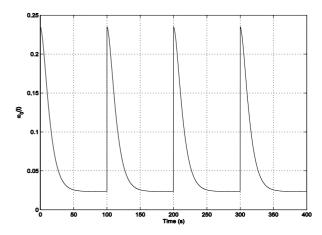


Fig. 4. Function  $\omega_0(t)$ Rys. 4. Funkcja  $\omega_0(t)$ 

Fig. 5 show the results of the process of the filtration in the signals multiplexing system by using the time-invariant filter and its time-varying equivalent.

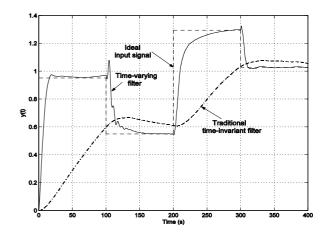


Fig. 5. Filtering graphs in the multiplex system by using the 2-nd order filter of constant component with time-varying and time-invariant parameters

Rys. 5. Przebiegi filtracji w systemie multipieksowania sygnałów przy użyciu filtru składowej stałej 2-go rzędu o zmiennych oraz stałych parametrach

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How one can see, the introduction of the time-varying filter to the signals multiplexing system gives good results. While the time-invariant filter is not able to work out the useful signal in the form of the constant component, the time-varying filter is considerably faster, and is able to follow the shape of the ideal rectangular signal.

#### 6. Conclusions

As it has been proven, the introduction of time-varying coefficients to the filter structure of constant component brings good results. By using the described filtering approach it is possible to obtain an efficient filter that ensures a very fast response if compared with the traditional filter. Such a designed filter was used to the signals multiplexing system, where it confirmed its good properties. It seems that further examinations of time-varying filters of constant component are needed.

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Artykuł recenzowany

### **INFORMACJE**

# Krótkie podsumowanie konferencji IMTC 2007



W dniach od 1 do 3 maja br., w warszawskim hotelu *Marriott*, odbyła się międzynarodowa konferencja 2007 *IEEE Instrumentation and Measurement Technology Conference (IMTC 2007)*. Wyłącznym sponsorrem tej konferencji było *IEEE Instrumentation and Measurement Society* (USA). Komitetowi organizacyjnemu przewodniczył prof. dr hab. Roman Z. Morawski z Wydziału Elektroniki i Technik Informacyjnych Politechniki Warszawskiej. Pełny

skład komitetu organizacyjnego, komitetu doradczego i komitetu programowego IMTC 2007 znaleźć można na stronie internetowej: http://ewh.ieee.org/soc/im/imtc/imtc2007/. Tam też dostępne są szczegółowe informacje dotyczące organizacji konferencji i jej programu merytorycznego.

Odbywająca się corocznie, poczynając od roku 1984, konferencja *IMTC* uważana jest za najważniejsze międzynarodowe wydarzenia naukowo-techniczne w dziedzinie metrologii i aparatury pomiarowej. *IMTC* jest forum wymiany myśli naukowej i technicznej – wymiany zorientowanej nie tylko na merytoryczny rozwój przedmiotowej dziedziny, ale także na rozwój międzynarodowej współpracy o charakterze akademickim i akademicko-przemysłowym. Wszystkie referaty prezentowane podczas konferencji drukowane są w materiałach

konferencyjnych; najlepsze z nich, w liczbie ok. 60, publikowane są następnie – w rozszerzonej i uaktualnionej wersji – w specjalnym wydaniu dwumiesięcznika *IEEE Transactions on Instrumentation and Measurement*.

Spośród 23 konferencji *IMTC*, zorganizowanych w latach 1984-2006, 15 odbyło się w USA; trzy we Włoszech; dwie w Kanadzie; po jednej w Belgii, Japonii i na Węgrzech. W wyniku selekcji ofert, trwającej ponad dwa lata (2003-2004), *IMTC Board of Directors* powierzył organizację *IMTC 2007* polskiemu środowisku metrologicznemu. A oto wybrane dane statystyczne charakteryzujące zakończoną właśnie konferencję w Warszawie:

- o liczba zgłoszonych 4-stronicowych streszczeń 694;
- o liczba zaakceptowanych streszczeń 505;
- liczba referatów w programie konferencji 477 (w tym 236 referatów prezentowanych w formie plakatów);
- o liczba zarejestrowanych uczestników konferencji 406;
- o liczba reprezentowanych krajów 44;
- o najliczniej reprezentowane kraje Włochy (70 zarejestrowanych uczestników), USA (48), Polska (38), Hiszpania (23), Kanada (19).

Przewodniczący IMTC 2007 prof. dr hab. Roman Z. MORAWSKI Politechnika Warszawska Wydział Elektroniki i Technik Informacyjnych Instytut Radioelektroniki