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# Measure of self-similarity in network traffic – evaluation of selected estimation methods

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### Abstract

Fractal nature of computer network traffic involves a big impact on performance of queueing systems (routers, switches, etc.), increasing delay values and packet losses. In this article, methods of measurement of network traffic as well as estimation methods of the self-similarity degree are presented and discussed. The most popular time-domain estimation techniques are evaluated from the point of view of reliability: R/S, variance-time and IDC analysis.

**Keywords**: Network traffic, self-similarity, Hurst exponent, estimation.

## Miara samopodobieństwa natężenia ruchu sieciowego – ocena wybranych metod estymacji

### Streszczenie

Fraktalny charakter natężenia ruchu w sieciach komputerowych wywiera duży wpływ na wydajność systemów kolejkowych (rutery, przełączniki sieciowe, itd.), zwiększając wartości opóźnień oraz poziom strat pakietów. W niniejszym artykule zostanie podjęta dyskusja dotycząca zarówno metod pomiaru natężenia ruchu jak i metod późniejszej estymacji poziomu samopodobieństwa. Najbardziej popularne techniki estymacji w dziedzinie czasu (analiza R/S. variance-time oraz IDC) zostały ocenione z punktu widzenia wiarygodności dostarczanych przez nie wyników.

**Słowa kluczowe**: natężenie ruchu, samopodobieństwo, wykładnik Hursta, estymacja.

## 1. Network traffic measurements

Traffic measurements in packet computer networks show the presence of a significant phenomenon that can be seen at multiple time scales [1, 2]. This phenomenon is called Long Range Dependence (LRD) and is connected with self-similarity. To examine the self-similar behavior in real computer networks, measurements have been carried out at rush hours (about 11:00-15:00) both at Szczecin University of Technology (SUoT) as well as University of Szczecin (UoS). The network traffic was measured at the main gateways that links universities to the Internet network. Since the incoming traffic intensity is in general higher than outgoing, we focus on analyzing the first one.

In the case of SUoT, measurements was based on reading packet counters every 10 ms. The main task of the special program installed on the main gateway+firewall+router - *violet.tuniv.szczecin.pl* on March 2002 was to save differences in packet counters from pseudo-file /proc/net/dev which includes basic statistics for particular network interfaces. Since the operating system installed on a router was not a real-time operating system (Linux Debian with kernel 2.4.18), first the accuracy of 3 different delaying methods (10 ms periods) was tested:

a) reading current time in delay loop using *gettimeofday()* function:

```
#include <sys/time.h>
int gettimeofday(struct timeval *tv, struct timezone
*tz);
```

where particular structures are defined as follows:

```
struct timeval {
time_t      tv_sec;  /* seconds */
useconds_t  tv_usec; /* microseconds */
};
struct timezone {
int tz_minuteswest; /* mins W of Greenwich */
int tz_dsttime;     /* dst correction */
};
```

b) using *usleep()* function that suspends execution for microsecond intervals:

```
#include <unistd.h>
void usleep(unsigned long usec);
```

c) using *nanosleep()* function that pauses execution for a specified time:

```
#include <time.h>
int nanosleep(const struct timespec *req, struct
timespec *rem);
```

where the structure *timespec* is defined as:

```
struct timespec {
time_t tv_sec;  /* seconds */
long   tv_nsec; /* nanoseconds */
};
```

Another function is *get_time()* that returns microseconds using *timeval* structure (t.tv.sec * 1000000.0 + t.tv_usec) of the *gettimeofday()* function. Since the times of reading counters and saving data to a file varies and last approximately 68 μs (mean value obtained for 10000 samples), it introduces a sampling time error. In spite of the fact, that the a) method loads the CPU up to 80-90%, the best results (smallest dispersion) was obtained just for this method – Tab. 1. The function *usleep()* (method b) for intervals about 10 ms requires using value 0 in place of its argument. Similar situation is for *nanosleep()* (method c) – here, one needs to put ts.tv_sec=0 and ts.tv_nsec=0 instead of desired nanoseconds. If the parameters are greater than 0, interval times “jump” to about 20 ms level. The lack of smoothness of adjusting parameters used in these functions involves zero-valued variables that gives smaller values of delays in the main loop.

Tab. 1. Mean and standard deviation values for 3 delaying methods
Tab. 1. Wartości średnie oraz odchylenia standardowe dla 3 metod opóźniania

| | method a) | method b) | method c) |
|---|---|---|---|
| mean [μs] | 10000.78 | 9984.88 | 9980.42 |
| std dev. [μs] | 56.32 | 324.58 | 306.37 |

A direct compensation of the reading time (approx. 68 μs) by subtraction of this value in a consecutive steps of the delay loop leads to an unstable behavior. Assuming that the delay interval in the first step is 200 μs and the delay time introduced by reading counters and writing data to a file is exactly 60 μs, the first steps of this procedure are:

```
Step 0: diff[0]=INT       = 10000,  v[0]=2*INT-diff[0] = 10000
Step 1: diff[1]=v[0]+200 = 10200,  v[1]=2*INT-diff[1] =  9800
Step 2: diff[2]=v[1]+60  =  9860,  v[2]=2*INT-diff[2] = 11140
Step 3: diff[3]=v[2]+60  = 11200,  v[3]=2*INT-diff[3] =  9800
Step 4: diff[4]=v[3]+60  =  9860,  v[4]=2*INT-diff[4] = 11140
Step 5: diff[5]=v[4]+60  = 11200,  v[5]=2*INT-diff[5] =  9800
...
```

Thus we have two relations: $diff_{n+1} = v_n + 60$ and $v_{n+1} = 2 \cdot INT - diff_{n+1}$. We can rewrite them as:

$$v_{n+1} + v_n = 2 \cdot INT - 60 = a \cdot \tag{1}$$

*Z* transform for this difference equation have the following form:

$$z \cdot [V(z) - v_0] + V(z) = \frac{a \cdot z}{z-1} . \tag{2}$$

The solution using inverse Z transform gives:

$$v_n = (-1)^n \cdot (v_0 - a/2) + a/2 . \tag{3}$$

One can see, that the solution has two alternating values: $v_0$ for even *n* and $a - v_0$ for odd *n*. If randomness is added, behavior of the process can be very unstable.

An improvement of the stability and the accuracy of measures was obtained by comparing current time to the reference time. If a delay time for next step exceeds some threshold, a measurement takes place. The mean value for this method is 10000.01 μs and a standard deviation equals only 5.35 μs (very good result comparing to the previous results from Tab. 1). This technique have been used for all measurements at Technical University of Szczecin.

In the case of University of Szczecin (UoS) all headers from the packets that pass the main gateway have been saved using *tcpdump* 3.6.2-9 program working on Linux with kernel 2.4.20 (January 2003). Based on saved data (first snaplen=96 bytes of each packet), interarrival times have been analyzed. The percentage of captured packets was about 99.5%. 0.5% of packets dropped by kernel was caused by temporary buffer overflows corresponding to file saving process. An accuracy of timestamps in *tcpdump* depends on current load of multitasking system (Linux). The simplest method of checking the accuracy of saved timestamps in multitasking system was used. It was based on reading the arrival times of packets sent at equally spaced moments. The accuracy varied with the system load. The results of sample tests was similar for intervals 10 ms and 0.5 ms, performed on P2 800 MHz that has worse performance than the firewall-PC (P3 800 MHz). Mean standard deviation for 10 ms and 0.5 ms were 3.01 μs and 4.69 μs as respectively. When we compare the results to the analyzed time scales, one can notice that the timestamp errors do not significantly impact the results of the estimation of Hurst exponent.

To improve an effectiveness of data analysis based on a text output (using -r option) of *tcpdump*, when data have to be processed twice, another method based on binary representation of captured data was developed. Since all packets were the IP packets – a firewall do not forward any other packets, thus all data needed for the analysis are just in IP header, after which headers for higher network layers can be put: UDP, TCP, ICMP, IGMP, etc. In order to determine a self-similarity level, randomly chosen fragment (1048576 samples with number of packets in unit time saved every 10 ms) has been analyzed. It corresponds to the following measure time in a rush hours (11-15): 2:54:45.76. Date and time of the chosen fragment of SUoT traffic that will be analyzed in detail is: 2002-03-04 11:42:29, and for UoS: 2003-01-28 11:35:04. In order to compare the results for different methods of estimation of Hurst exponent, the same traffic length/duration will be analyzed. Additionally, for comparison purposes, the results for traditional Poisson model will be also presented.

## 2. Meaning of self-similarity in network traffic

Experiments in local area networks revealed, that the traffic is better represented by self-similar than traditional, Markovian models. Some of the reasons that are responsible for this phenomenon are:

- a mechanism of retransmission in TCP protocol [4]
- distribution of length of files stored at WWW servers [5]
- response times („user think time") [5]

Research on revealing other reasons are still in progress.

Self-similarity in network traffic can be characterized as the preservation of some properties at different time scales. Let *Y*(*t*) be a stationary stochastic process with continuous time parameter *t*. *Y*(*t*) is called self-similar with self-similarity parameter *H* (Hurst exponent), if for any positive stretching factor *c*, the rescaled process with time scale *ct*, $c^{-H}Y(ct)$, is equal in distribution to the original process *Y*(t) [6]:

$$c^{-H}Y(ct) \to_d Y(t), \quad H > 0 , \tag{4}$$

where $\to_d$ denotes convergence in the sense of distribution. This means that, for any sequence of time points $t_1, \ldots, t_k$ and any positive constant factor *c*, the rescaled sequence $\{c^{-H}Y(ct_1), c^{-H}Y(ct_2), \ldots, c^{-H}Y(ct_k)\}$ has the same distribution as $\{Y(t_1), Y(t_2), \ldots, Y(t_k)\}$. Since the analysis of Hurst exponent in network traffic is based on packet count measurements, the increment process of $Y(t)$ should be considered, i.e. $X_n = Y(n+1) - Y(n)$, $n \in N$. The main difference between traditional and modern models of network traffic is that the autocorrelation function of $X_n$ that decays very slowly and is not summable.:

$$r(k,H) \approx H(2H-1)\sigma^2 k^{2H-2} \quad \text{and} \quad \sum_{k=0}^{\infty} r(k,H) \to \infty , \tag{5}$$

where *H* is the Hurst exponent – measure of self-similarity degree. The larger *H* value the slower decay of the autocorrelation function.

The value of Hurst exponent for network traffic is very important from the point of view of network (more precisely – queueing systems, such as routers or switches) performance, i.e. delays and packet losses. In Fig. 1 there are delays for the single server queueing system fed by measured traffic with general packet (GP) distribution and general input (G/GP/1), two classical systems with Poisson arrivals and deterministic (M/D/1), exponential (M/M/1) and real packet length distribution (M/GP/1) service times. Performance of the measured traffic queueing systems is significantly worse than for these with Poisson arrivals.

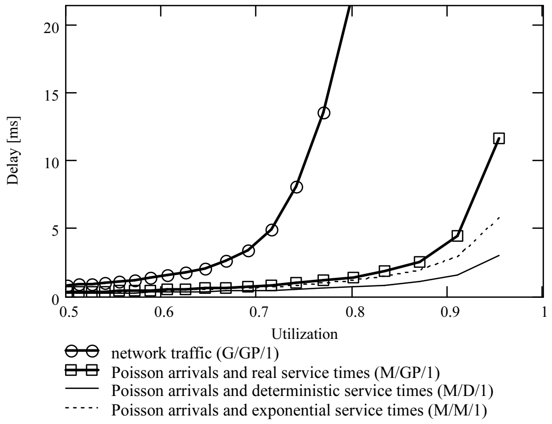

Fig. 1. Mean delay for analyzed data and different traffic models
Rys. 1. Średnie opóźnienie dla analizowanych danych i różnych modeli ruchu w sieci

## 3. Estimation of the degree of self-similarity

### 3.1. R/S analysis

One of the historically first method of determining Hurst exponent is the method of rescaled ranges called R/S analysis, proposed in 1951 by H. E. Hurst. A set of observations consisted of $N$ elements is divided on $n$-elements subsequences so that the number of subsequences $L = N/n$ would be an integer. As a result of averaging over L subsequences one obtains $E(R/S)_n$. In the case when a time series is a realization of independent and normally distributed random variables, the following formula derived by Anis and Lloyd in [7] holds:

$$E\left(\frac{R}{S}\right)_n = \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\cdot\sqrt{\pi}}\cdot\sum_{i=1}^{n}\sqrt{\frac{n-i}{i}}\,, \tag{6}$$

where $\Gamma(\cdot)$ is the gamma function. For $n \to \infty$ we obtain the following asymptotic:

$$E\left(\frac{R}{S}\right)_n = c \cdot n^H\,, \tag{7}$$

where $c$ is some constant, one can conclude that for normally distributed random variables the Hurst exponent is 0.5. The conclusion works only for $n \to \infty$, however for $n$ values that are finite the Hurst exponent is significantly different from 0.5. Peters in [8] introduced his 'empirical correction', cited later in [9] and [10], that caused some kind of confusion. In [11] it was proven that the correction has weak basis and is useless.

Taking the logarithm of (7), we have:

$$r_n = \log c + H_n \log n\,. \tag{8}$$

Assuming that the value of Hurst exponent is constant for subsequences of length: $n$-1, $n$ and $n$+1, from (8) we get:

$$H_n = \frac{r_{n+1} - r_{n-1}}{\log(n+1) - \log(n-1)}\,. \tag{9}$$

Basing on (9) relation, sample values of Hurst exponent for different lengths of subsequences $n$ were obtained (Tab. 2). All values are greater than 0.5, thus concluding only from the result values of R/S analysis, ignoring lengths ($n$) of subsequences, is unfounded.

Tab. 2. H values for different lengths of subsequences
Tab. 2. Wartości H dla różnych długości podciągów

|  | n=10<br>L=100000 | n=20<br>L=50000 | n=100<br>L=10000 | n=1000<br>L=1000 |
|---|---|---|---|---|
| H | **0.6275** | **0.5927** | **0.5438** | **0.5144** |

The linear regression can be applied to (8) in order to limit errors that arise from randomness. Increased number of observations for linear regression affects stability of $H$ results. It means that we should estimate $\widehat{H}$ rather for some range $\langle n-n_0, n+n_0\rangle$ than for particular $n$. Analyzing bias and variance of $\widehat{H}$ for different $n$, we can deduce that increasing the number of points in linear regression, the value of variance decreases, but on the other hand the bias becomes bigger (overestimated empirical value of $H_n$ comparing to theoretical value). As a result we can observe the minimum of mean-square error, that is the sum squared bias and the variance, for particular value of $n_0$.

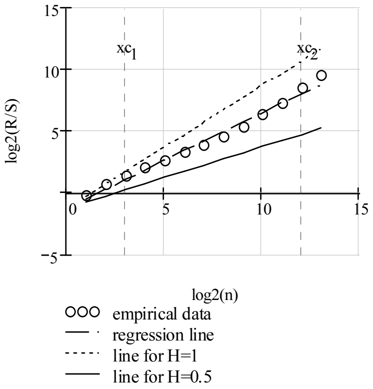

Fig. 2. R/S analysis for SuoT traffic, H=0.773
Rys. 2. Analiza R/S dla ruchu PS

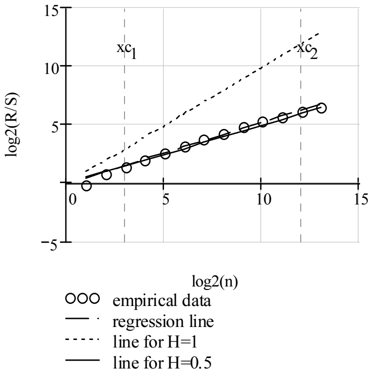

Fig. 3. R/S analysis for Poisson model, H=0.523
Rys. 3. Analiza R/S dla modelu Poissona

### 3.2. Variance-time (VT) analysis

Aggregating the increment process of $Y(t)$: $X(i) = Y(i) - Y(i-1)$ by using the following formula [3]:

$$X^{(m)}(i)=\frac{1}{m}\sum_{t=m(i-1)+1}^{mi}X(t), \qquad (10)$$

we get $X(t)$ that is partitioned into nonoverlapping blocks of size $m$ (aggregation level). Their values are averaged, and $i$ is used to index these blocks. To simplify notation, let $X=X(1)$ and $X^{(m)}=X^{(m)}(1)$, then $X^{(m)}$ can be viewed as computing a sample mean [6]:

$$X^{(m)}=\frac{1}{m}\sum_{t=1}^{m}X(t)=m^{H-1}X. \qquad (11)$$

When viewed as a sample mean where the samples are drawn independently, $\mathrm{var}\left(X^{(m)}\right)$ reduces to $\sigma^2 m^{-1}$ if $H$=0.5, but if $H\neq 0.5$, in particular, $0.5<H<1$, then:

$$\mathrm{var}\left(X^{(m)}\right)=\sigma^2 m^{-\beta}, \qquad (12)$$

where $0<\beta<1$ and $H=1-\beta/2$. Variance of aggregated random variable decays more slowly than the rate $m^{-1}$. It is caused by a dependency structure in the samples.

Averaging according to (10) causes big dispersion of observations around a regression line (Fig. 4). It is proposed to use slightly different method of averaging which leads to a more stable results (Fig. 5):

$$X'^{(m)}(i)=\frac{1}{m}\sum_{t=i\cdot M}^{i\cdot M+m}X(t). \qquad (13)$$

The difference between literature and proposed method lies in the fact that the latter averages samples globally until the block of size $M$ is not full. This method causes, that the variance of observations is smaller.

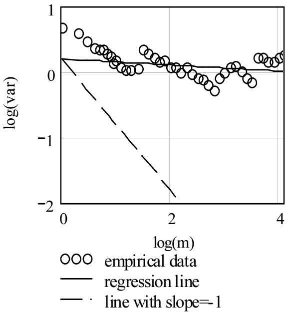

Fig. 4. VT analysis for SUoT traffic – literature method, H=0.977
Rys. 4. Analiza VT dla ruchu PS – metoda literaturowa

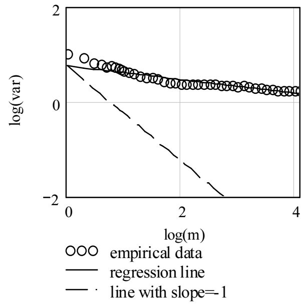

Fig. 5. VT analysis for SUoT traffic – proposed method, H=0.929
Rys. 5. Analiza VT dla ruchu PS – proponowana metoda

### 3.3. Index of dispersion for counts (IDC) analysis

In this method, for a given time period $L$, index of dispersion for counts is defined as relation of variance of aggregated random variable to its expected value [6]:

$$IDC(L)=\mathrm{var}\left(\sum_{j=1}^{L}X_j\right)\Big/E\left(\sum_{j=1}^{L}X_j\right)\approx cL^{2H-1}, \qquad (14)$$

where $c$ is a positive value independent from $L$. Drawing $\log(IDC(L))$ against $\log(L)$ we can fit a regression line, for which the slope equals $2H-1$. IDC method as well as the variance-time analysis is very fast in comparison with the R/S method. Furthermore, it gives more stable results using proposed ‘global’ aggregation of random variable.

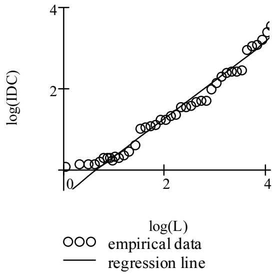

Fig. 6. IDC analysis for SUoT traffic – literature method, H=0.973
Rys. 6. Analiza IDC dla ruchu PS – metoda literaturowa

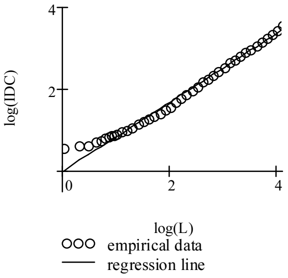

Fig. 7. IDC analysis for SUoT traffic – proposed method
Rys. 7. Analiza IDC dla ruchu PS – proponowana metoda

## 4. Conclusions

Measurement part of our experiments introduces problems with correct timing while capturing traffic data. It stems from the fact that 10 ms period used in measurements corresponds to the frequency of timer interrupt, that is 100 Hz for kernel 2.4.x. We are not able to use functions like *usleep()* or *nanosleep()* below this value, but it is possible to replace them with delay loop, that leads to decreasing the time interval. However, if the value of measured time interval is too small, a relative measurement error may be too big. On the other hand we are limited by the computation time that cannot be too long especially for real-time working systems. Furthermore, for the very small time scales there are significant irregularities in the results of Hurst exponent estimations, that can be reduced by using cut-off points for all estimation methods. A method that has the worst performance is R/S analysis: high computational complexity, strong dependency of Hurst exponent on the length of subsequence $n$, lowered values of $H_n$ compared to the other methods of estimation (Fig. 8).

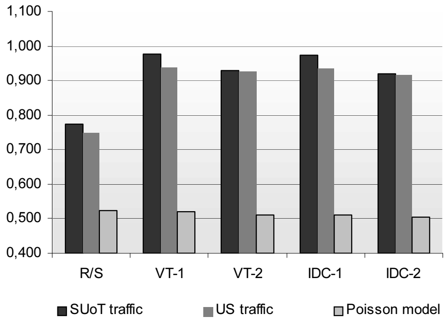

Fig. 8. Results of estimation of Hurst exponent
Rys. 8. Wyniki estymacji wykładnika Hursta

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*Artykuł recenzowany*

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## INFORMACJE

# Studia Podyplomowe

Wydział Elektryczny Politechniki Śląskiej w Gliwicach, Instytut Metrologii, Elektroniki i Automatyki
ogłasza nabór na Dwusemestralne Zaoczne Studia Podyplomowe

## Organizacja i Akredytacja Laboratoriów

### Cel studiów

Celem studiów jest pogłębienie wiedzy w zakresie systemu jakości laboratoriów wzorcujących, problematyki zapewnienia jakości wyposażenia pomiarowego, walidacji metod pomiarowych, metodyki tworzenia budżetów niepewności i opracowania wyników badań zgodnie z obowiązującymi przepisami oraz przygotowanie słuchaczy do samodzielnej pracy w zakresie organizowania i prowadzenia laboratorium akredytowanego. Przedstawione zostaną podstawy automatyzacji pomiarów i organizacji systemów pomiarowych. Problemy analizowane będą na przykładach, z uwzględnieniem niezbędnych podstaw teoretycznych oraz aktualnych przepisów.

### Profil uczestnika studiów

Studia przeznaczone są dla pracowników o różnych specjalnościach, zajmujących się organizacją laboratoriów oraz wykonywaniem badań i kalibracji w zakładach, firmach lub jednostkach naukowo-badawczych. Studia adresowane są do osób z wyższym wykształceniem zajmujących się realizacją pomiarów i opracowywaniem wyników badań w różnych dziedzinach. Ich ukończenie pozwoli uczestnikom na podwyższenie kwalifikacji niezbędnych do efektywnego opracowywania i dokumentowania procesów pomiarowych. Absolwent studiów otrzymuje Świadectwo Ukończenia Studiów Podyplomowych w zakresie objętym nazwą studiów.

Studia prowadzone są na Wydziale Elektrycznym Politechniki Śląskiej w Gliwicach, w systemie zaocznym w każdą sobotę lub w co drugi weekend (do wyboru) przez dwa semestry. Zajęcia prowadzone są przez nauczycieli akademickich ze stopniem co najmniej doktora oraz przez zaproszonych Gości o uznanym dorobku i autorytecie. Studia obejmują 200 godzin dydaktycznych. Rozpoczęcie Studiów nastąpi po skompletowaniu odpowiedniej liczby kandydatów na dany rodzaj studiów.

**Warunki przyjęcia na studia:**

1. Na studia mogą być przyjęte osoby posiadające dyplom magistra lub inżyniera, posiadające podstawową wiedzę z zakresu wybranych studiów.
2. Warunkiem uruchomienia studiów jest przyjęcie odpowiedniej liczby Kandydatów na podstawie złożonych dokumentów.
3. Dokumenty składane przez Kandydatów:
   - Kwestionariusz Osobowy – Karta Zgłoszenia (do pobrania ze strony internetowej). Przyjmowane na bieżąco: e-mailem, pocztą lub osobiście.
   - Kopia/odpis dyplomu ukończenia studiów wyższych.
4. Kandydaci odbywają rozmowę kwalifikacyjną. Termin ustalony i podany zostanie po skompletowaniu odpowiedniej liczby Kandydatów.
5. Po spełnieniu warunków Kandydaci wnoszą opłatę zgodnie z zawartą umową w wysokości 3 800 złotych za cały okres studiów.

**Organizator studiów:**

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