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Estimation of equivalent power system impedance in normal conditions

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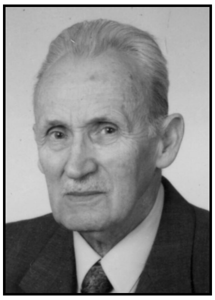

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Abstract

The paper presents method of estimation of equivalent power system impedance. The method based on the correlation method which uses natural variability of the voltage level in the network nodes. To find the value of equivalent power system resistance and reactance the Lyapunov exponents were used. Equivalent impedance of power system is the base to determination of short-circuit currents and short-circuit power. On-line control over the short-circuit power in key nodes of power system might provide system dispatchers with an important piece of information.

Keywords: Power system, short-circuit power, equivalent impedance, on-line control.

Wyznaczanie impedancji zastępczej systemu elektroenergetycznego w warunkach normalnych

Streszczenie

Ważnym parametrem systemu elektroenergetycznego (SEE) jest poziom mocy zwarciowej w poszczególnych węzłach. Znajomość wartości mocy zwarciowej pozwala na określenie wartości spodziewanych prądów zwarciowych. Jest to niezbędne do prawidłowego doboru urządzeń w fazie projektowania systemu oraz do doboru nastaw automatyki zabezpieczeniowej. Moc zwarciowa zależy od impedancji zastępczej SEE. Wartość tej impedancji zależy od liczby generatorów, transformatorów, linii elektroenergetycznych oraz ich mocy, impedancji i układu połączeń między nimi. Wartość tej impedancji nie jest wielkością stałą w czasie. Zmiany mogą wynikać zarówno ze zmiany układu połączeń w systemie (zmiany krótkookresowe) jak też z rozwoju SEE (zmiany długookresowe). W artykule przedstawiono koncepcję wyznaczania impedancji zastępczej SEE w warunkach ruchowych. Przedstawiona metoda bazuje na zmienności poziomu napięcia w węzłach pomiarowych SEE spowodowanej przez losowe zmiany prądu obciążenia wynikające z losowej zmiany obciążenia. Zgodnie z proponowaną metodą do wyznaczenia impedancji systemu konieczny jest pomiar wartości skutecznej prądu, napięcia, mocy czynnej i dodatkowo wyznaczenie sumy wykładników Lapunowa.

Słowa kluczowe: System elektroenergetyczny, moc zwarciowa, impedancja zastępcza, monitoring.

1. Introduction

To estimate properly the power system performance it is necessary to recognize the equivalent system impedance in operating conditions. The impedance may vary from a value being close to short-circuit impedance, considered for dynamic changes condition up to the fixed value characteristic for normal operation state of power system. Short-circuit impedance value is the base to determine the short-circuit currents. The short-circuit currents in networks depend on many factors, i.e.: the number of nodes feeding the networks, the power and the short-circuit reactance of autotransformers, transformers and the generators feeding particular nodes, the number of lines converging in particular nodes, the lengths of those lines and double-track lines contribution. Some performance experiments have proved that short-circuit currents, and simultaneously the short-circuit power are time varying values. Considering those changes for few years time period the increasing nature of them can be observed, since they are connected with the increase of total power installed of all the devices mentioned. The common worldwide trend is to create some standard levels of short-circuit currents. The standards result from technical and economical analysis. Different media are used (mainly network sectioning) to avoid exceeding the limits. Except for current increase tendency for few-years time period the changes of short-circuit current values following system changes may be observed.

The design phase considers most often the maximum short-circuit current values that may occur in given system point. It frequently causes redimensioning the devices resulting in additional expenses. The tests carried out in USA [1] have proved that an average short-circuit current value was 36% of the maximum value whereas there were no values higher than 85% of maximum obtained. Among the others it arises from the fact that most short-circuits occur along the lines and mostly they are the ground and two-phase short-circuits. On the other hand the question that follows is how the maximum permissible short-circuit current value changes in particular system node in real operating conditions?

The paper presents a method of on-line estimation of equivalent power system impedance. The method is based on the natural variability of the voltage level in the network nodes. To find the value of equivalent power system resistance and reactance the Lyapunov exponents are used. The statistic analysis is led using the correlation.

2. Method of determining the equivalent power system impedance

Most methods providing determination of equivalent power system impedance using intermediate measurements are based on forced voltage change and enable determining the transient primary power values [2]. Establishing system impedance may also be based on measuring little voltage and current changes, enabling obtaining the value referred to normal operating conditions of the system shown at Fig. 1.

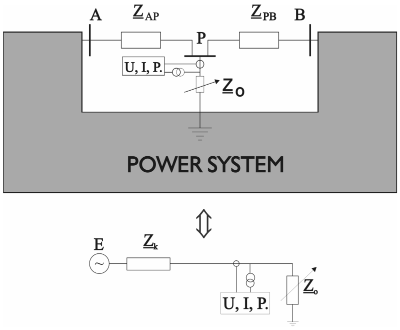

Fig. 1. Power system with measuring point P
Rys. 1. System elektroenergetyczny z punktem pomiarowym P

The record of stochastic voltage changes caused by stochastic current changes is made during normal system performance. The main effort with this method is transferred from the measuring operations to elaboration works over the voltage and current recordings. It may be assumed that processes of power system in short time periods (20s) are of stationary nature. Moreover, when system operational stability is concerned then the linearity of the system may be also assumed.

In the tests suggested in this paper the dynamic characteristics of impedance is not considered as important as its mean value Z_k, for particular period of time equal to time of measurements of n series. Basing on impedance value Z_k the approximate short-circuit power can be evaluated. It is assumed that during that period the system impedance Z_k is constant and the voltage changes result from load current changes. Then the analysis of static current and voltage distributions can be brought to calculate mean values, variances, covariances and correlation coefficient.

The voltmeter and ammeter readings for network node P shown at Fig. 1 are the following:

$$\underline{U} = E\frac{\underline{Z}_o}{\underline{Z}_k + \underline{Z}_o}, \quad \underline{I} = \frac{E}{\underline{Z}_k + \underline{Z}_o}. \tag{1}$$

After the change of load impedance $\Delta\underline{Z}_O$ the readings are the following:

$$\underline{U}_1 = E\frac{\underline{Z}_o + \Delta\underline{Z}_o}{\underline{Z}_k + \underline{Z}_o + \Delta\underline{Z}_o}, \quad \underline{I}_1 = \frac{E}{\underline{Z}_k + \underline{Z}_o + \Delta\underline{Z}_o}. \tag{2}$$

Thus:

$$\underline{Z}_k = \frac{\underline{U} - \underline{U}_1}{\underline{I}_1 - \underline{I}}. \tag{3}$$

Voltage and current measurements provided inside the transmission system enable calculating the impedance towards the source.

The created software enables to record the effective current and voltage in selected network node measurements in a disk file. For an arbitrary series of n measurements the n values of effective current and voltage are recorded. To avoid the requirement of determining the power coefficient value the low voltage system has been loaded with an active power. The calculation of average values for the relevant series followed as:

$$U_a = \frac{1}{n}\sum_{i=1}^{n} U_i, \quad I_a = \frac{1}{n}\sum_{i=1}^{n} I_i. \tag{4}$$

Next the variance values of voltage $\sigma^2(U)$ and current $\sigma^2(I)$ have been calculated:

$$\sigma^2(U) = \frac{1}{n}\sum_{i=1}^{n}(U_i - U_a)^2, \quad \sigma^2(I) = \frac{1}{n}\sum_{i=1}^{n}(I_i - I_a)^2 \tag{5}$$

and covariance cov(U,I):

$$\mathrm{cov}(U,I) = \frac{1}{n}\sum_{i=1}^{n}(U_i - U_a)(I_i - I_a). \tag{6}$$

To establish the interrelation between the current and voltage values the correlation coefficient R(U,I) has been calculated, defined as:

$$\mathrm{R}(U,I) = \frac{\mathrm{cov}(U,I)}{\sigma(U)\sigma(I)} \tag{7}$$

The values of current and voltage should show negative correlation: R(U,I)∈(-1, -0.5). Using the least squares method it can be expressed as follows:

$$\sum_{i=1}^{n}(\Delta U_i - Z_I \Delta I_i)^2 = \min \text{ or } \sum_{i=1}^{n}(\Delta I_i - Z_U^{-1}\Delta U_i)^2 = \min. \tag{8}$$

The expressions (8) allows to obtain:

$$Z_I = \frac{\mathrm{cov}(U,I)}{\sigma^2(I)} \text{ or } Z_U = \frac{\sigma^2(U)}{\mathrm{cov}(U,I)}. \tag{9}$$

Multiplying these two equations (9) the third relation is obtained as:

$$Z_k = \sqrt{Z_I Z_U} \tag{10}$$

When R(U,I) ≈ -1, then $Z_k \approx Z_I \approx Z_U$ (the subscripts "U" and "I" let to recognize the way of calculation). The measurement series have been carried out periodically to determine the time variability of system impedance value. During testing some parameters for single series were being established, i.e. number of measurements n, single measurement duration τ and series duration T, to obtain the most reliable results. The algorithm controlling computer work allowed an arbitrary suitable selection of the above parameters. This method is useful for low voltage network, where impedance of system fulfill the relation $Z_k \approx R_k$ [3]. This method applied in medium voltage and high voltage network would introduce too high error. In order to determine R_k and X_k this method should be modernized.

3. Method of determining the equivalent power system impedance using Lyapunov exponents

Lyapunov exponents are a generalization of the eigenvalues at an equilibrium point and of characteristic multipliers. They are used to determine the stability of any type of steady-state behavior including quasi-periodic and chaotic solutions [4, 5].

Two nearby points (X_0 and $X_0 + \Delta x_0$) in a space (Fig. 2) will generate orbits in that space using some equation or system of equations. The Lyapunov exponent can be defined as follows:

$$\lambda = \lim_{\substack{t\to\infty \\ |\Delta x_0|\to 0}} \frac{1}{t}\ln\frac{|\Delta x(X_0,t)|}{\Delta x_0}. \tag{11}$$

It works for both discrete and continuous systems. Lyapunov exponent is useful for distinguishing among the various types of orbits:

- $\lambda < 0$ - the orbit attracts to a stable fixed point or stable periodic orbit,
- $\lambda = 0$ - the orbit is a neutral fixed point or an eventually fixed point,
- $\lambda > 0$ - the orbit is unstable and chaotic.

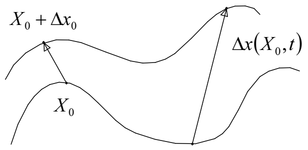

Fig. 2. Graphic interpretation of the Lyapunov exponent
Rys. 2. Graficzna interpretacja wykładników Lyapunowa

There are as many Lyapunov exponents as dimensions of the system. If the system has more than one Lyapunov exponents, for an attractor, contraction must outweigh expansion so

$$\sum_{i=1}^{n} \lambda_i < 0 . \tag{12}$$

The system can be described by the following formula

$$L_\Sigma \frac{di}{dt} + R_\Sigma i = E \sin(\omega t) . \tag{13}$$

It can be shown [4] that the sum of Lyapunov exponents of system (13) is as follows

$$\sum_{i=1}^{n} \lambda_i = -\frac{R_\Sigma}{L_\Sigma} . \tag{14}$$

The methods of calculating the Lyapunov exponents from a time series and systems of equations are well known [6, 7].

For load impedance $\underline{Z}_1 = R_1 + \mathrm{j}X_1$ the voltage U_1, current I_1, active power P_1 and sum of Lyapunov exponents λ_1 are measured. Likewise for another load impedance $\underline{Z}_2 = R_2 + \mathrm{j}X_2$ the voltage U_2, current I_2, active power P_2 and sum of Lyapunov exponents λ_2 are measured. Now the following formulas can be written:

$$\begin{aligned} R_1 &= \frac{P_1}{I_1^2}, \quad X_1 = \frac{\sqrt{U_1^2 I_1^2 - P_1^2}}{I_1^2} \\ R_2 &= \frac{P_2}{I_2^2}, \quad X_2 = \frac{\sqrt{U_2^2 I_2^2 - P_2^2}}{I_2^2} \end{aligned} . \tag{15}$$

The model of a power system in Fig. 1 is described by the formula (13). The sums of Lyapunov exponents are as follows:

$$\begin{cases} \lambda_1 = -\dfrac{R_k + R_1}{L_k + L_1} \\ \lambda_2 = -\dfrac{R_k + R_2}{L_k + L_2} \end{cases} . \tag{16}$$

Using (15) and (16) the reactance and resistance of the power system can be obtained:

$$\begin{cases} R_k = -\dfrac{\left(\dfrac{R_1}{\lambda_1} - \dfrac{R_2}{\lambda_2}\right) + (L_1 - L_2)}{\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right)} \\ L_k = -\dfrac{(\lambda_1 L_1 - \lambda_2 L_2) + (R_1 - R_2)}{(\lambda_1 - \lambda_2)} \end{cases} . \tag{17}$$

To lead the statistic analysis a large number of measurements must be made. Afterwards, using the least squares method, it can be shown, that the resistance and reactance of the power system are as follows:

$$\begin{cases} R_k = \dfrac{\operatorname{cov}\left(\dfrac{R}{\lambda}, \dfrac{1}{\lambda}\right) + \operatorname{cov}\left(L, \dfrac{1}{\lambda}\right)}{\sigma^2\left(\dfrac{1}{\lambda}\right)} \\ L_k = \dfrac{\operatorname{cov}(\lambda L, \lambda) + \operatorname{cov}(R, \lambda)}{\sigma^2(\lambda)} \end{cases} \tag{18}$$

where: $\operatorname{cov}(x, y)$ - covariance values of x and $y, \sigma^2(x)$ - variance values of x.

System of equations (18) enables to determine equivalent impedance of power system $\underline{Z}_k = R_k + \mathrm{j}X_k$. This impedance can be used for calculation of short-circuit current I_k“ and short-circuit power S_k”:

$$I_k'' = \frac{cU_n}{\sqrt{3} Z_k}, \quad S_k'' = \frac{cU_n^2}{Z_k} . \tag{19}$$

The further modernization of this methods can be application of artificial neural network (ANN) to quick identification of equivalent impedance [8]. As input of ANN modules and arguments of bus voltage and current from two sampling cycles are given. Bus equivalent impedance can be obtained from the output.

4. Conclusions

Presented methods make possible to determine the equivalent impedance of power system. The first method, which uses only modules of bus voltage and current is useful only for typical low voltage network.

The method, which uses Lyapunov exponents is more universal. It can be used in low voltage, medium voltage and high voltage installations.

Equivalent impedance of the power system is the base to determination the short-circuit currents and short-circuit power. On-line control over the short-circuit power in key nodes of the power system might provide system dispatchers with an important piece of information.

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