AERODYNAMIC AND DYNAMIC MODEL OF COAXIAL ROTOR BLADES

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For the purpose of determination of aerodynamic and dynamic loads of the bottom rotor blade of a coaxial rotor system a model of calculation would be based on the blade-element-momentum theory with taking into consideration the effect of top blade tip vortex, the unsteady effects due to the unsteady flow around the blade foils, the rigid motion as flapping and feathering, the bending deformation and the effect of control system too. The blade will have a generalized equilibrium state, which can be calculated as an asymptotic solution of the set of non-linear, ordinary differential equations as the non-linear differential equation of the flapping, and feathering motion, the modal equations – with generalized masses and forces - of the elastic deformation, and the equations of aerodynamic forces are calculated by combination of the impulse, blade element and vortex method.

1. INTRODUCTION

My aim was to establish a technical-mathematical model for write the aerodynamic-dynamic-aero elastic behaviour of coaxial rotor system in a steady linear flight. For reaching the goal I had to analyse the rigid and elastic blade motions, the flow area above the rotors and the aerodynamic forces acting on blades with taking into consideration the effect of top rotor and unsteady effects of variable flow too.

The base of calculation is the combined blade-element momentum theory with the ONERA model [1] for unsteady flow effects, with the effect of top blade tip vortexes and the effect of control system too. By this way the induced velocity field and the unsteady effects can be calculated. In case of success, these results can be used by the calculation of helicopter performance, equilibrium states, and finally but not at least for the investigation of rotor blade's life time with the calculation of the loads. The combined blade element momentum theory in English abbreviation BEMT or CBEMT, is a well known theoretical method [2, 3]. The zero resulted effects of blade tip vortices can be taken into consideration with using of the vortex theory complement added by me, which missed from the original theory. The present calculation method could be checked by the application of the results of KA-26 helicopter investigations have been implemented at 1990 in a co-operation of RWTH Aachen and TUB [4].

2. THE MODEL AND RESULTS

For reaching the aim I have had to consider my strongly limited computational capacity and measuring possibility as well as the relatively simple programming ability and satisfactory accuracy, I have chosen the base of model the combined blade element momentum theory supplemented with effect of trailing vortices. Increasing the accuracy I have considered thee movements of rotor blade, the flow above the rotor, the aerodynamic forces on the blade, the effects of top rotor, the unsteady flow around the profiles, the elastic deformations and the effect of tip vortices. For the solution of problem I have developed the model of Tamás Gausz Ph.D. written in version 3.5 of Power-Basic [5] and based on the combined blade-element momentum theory for single rotor case. I have used the MATLAB too mainly for filtering of the results of measurements and Microsoft Excel for completions of those processes were not programmed, and for the graphics. In case of using the momentum theory the cross section of stream tube of the coaxial rotor system, what I have given as a function of the place along the rotor disc. By the way on a coordinated place knowing the distant flow velocity, the pressure and the density, the induced velocity could be simply determined by using of momentum theory. Using the blade-element theory in case of bottom rotor we have to consider even the classical components of the flow or the normal and tangential induced velocities of top rotor, the velocities are induced by the tip vortex of top rotor on the bottom rotor disc, the effect of tip vortex on the lift near the tips (the lift coefficient was decreased to zero with a polynomial by the tips) and the effect of unsteady flow on the profile characteristic with the ONERA model. By considering of the effects of top rotor those area is on the bottom rotor disturbed by the flow of top rotor had to be determined in the function of advance ratio [6]. Here the setting back of the stream tube was considered. The flapping and feathering motions were considered with their simplified classical differential equation by this way consider with the control law and the effect of flapping compensation. In the computation the bending deformation was considered with the linear combination of the first four free vibration with their azimuthally coefficients, those azimuthally coefficients were very necessary for the calibration of the model and for the strength analysis too.

The computation process have two parts: In the first part the program calculate the induced velocity-distribution, thrust, horizontal and side forces of the top rotor. The steps:

1. The program reads the geometrical, structural and aerodynamic dates.
2. Computation the preliminary induced velocity distribution on the base of Glauert's approximation and calculation the thrust force of the whole helicopter with classical method.
3. Numerically integration of differential equations of flapping and bending motions, during one revolution. The calculation of the flapping motion and bending deflections has included the unsteady-compressible lift coefficient, and the equation of the connection between the flapping and feathering motion. This calculation uses polar coordinate system.
4. The force distribution over the rotor surface is known the corresponding (new) induced velocity distribution can be calculated in a Descartes coordinate system. On the base of these calculations – in order to investigate the equilibrium state of the helicopter – can be determined the horizontal, side and trust force of the rotor.
5. After these steps the program goes back to flapping calculation – while the rotor blade turns to the generalised equilibrium state. This can be realised practically after 10 revolutions. If the equilibrium state is not reached, then the P_0; P_1; P_2 parameters can be changed.
6. We have to store the values of the equilibrium state.

The second part is same as the first one, only by the 2nd step we have take into consideration the foregoing computed and conformal positioned induced velocities of the top rotor and expand with the velocities induced by the top rotor tip vortices.

For the calibration of model I have used the results of a measurement already was published in the [7] and was analysed in my previously papers [8] [9] [10]. Without detailing the base of the measurement was the signs of tensiometric stamps calibrated for unit-moment values on a bottom rotor blade of a Ka-26, flapping angle transmitter and rotation per minute transmitter were transmitted to the earth with a telemetric system during steady level flight with different advance ratio. The differential equation of flexible chord can be easily solved numerically with the linear combination of the above mentioned azimuthally coefficients and free vibrations. So giving same operation parameters near the measured moment values could be calculated with the model, in a given azimuth and place along the rotor blade. For determination of the deviation between the measured and with a dipole Chebisev filtered and model computed moment values I have used a deviation function with the space of quadratic integral able functions, computed by scalar product and well usable in case of any constant approximation [11]. The Table 1 shows the above mentioned deviations in case of $\mu = 0,15$ advance ratio. On the base of results above I have looked at the model as a valid one. The whole both the normal and tangential induced velocity field of the helicopter was calculated by different advance ratios as an aerodynamic application of model. One of these results (the induced velocity field of bottom rotor) is shown by the

Tab. 1. Relative deviations of the two results on the measuring places

Meas. Places	No1	No2	No3	No4	No6	No7	No8
Deviation [%]	14,4	15,01	24,38	22,04	18,59	21,77	23,22

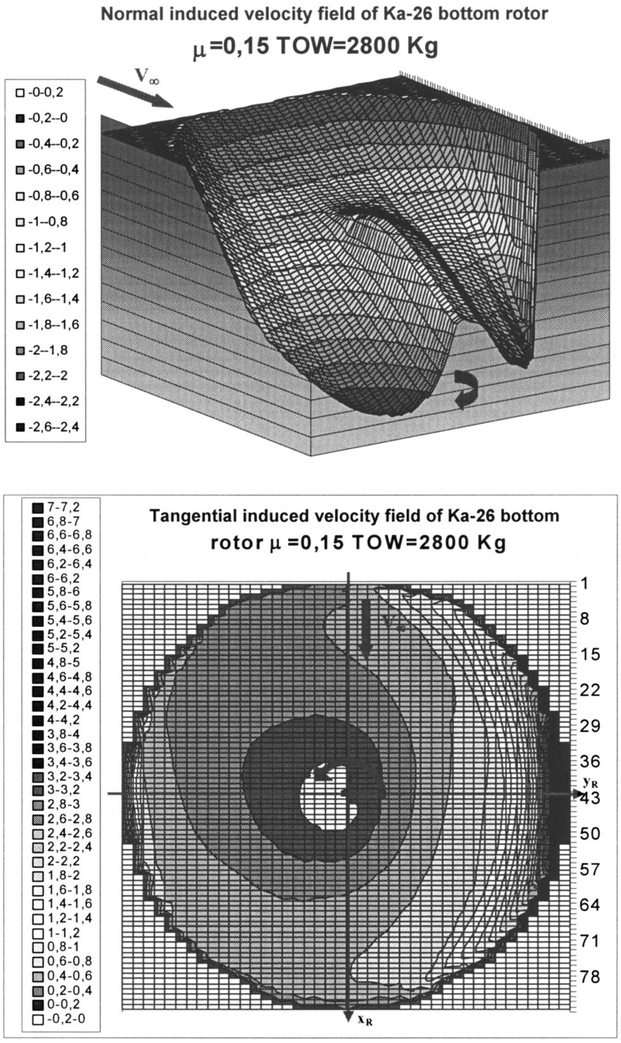


Fig. 1. The normal and tangential induced velocity field of bottom rotor [m/s]

The whole calculation of rotor blade's loads would be calculated with those movements were determined by the model so by this reason it would be possible the calculation of load on the base of static – with superposition of the external loads - and on the base of dynamic – calculation of the internal loads as to be in balance with external ones.

By the calculation of static bending load the following effects had been considered the moments and forces from the lift, the moments and forces from the lift centrifugal force, the moments and forces from the mass forces.

The internal stresses from the elastic and rigid bending motion were considered as the base of dynamic bending load. The following equations coming – from differential equation of flexible chord – were used to calculate the dynamic bending load as a bending moment:

$$M_{DIN}(x_l, \psi) = IE(x_l)Y''(x_l, \psi) \qquad (1)$$

and from this moment the stress in the outermost cord of the bended structure – in this case the bottom outermost fibre of the spar of rotor blade – can be calculated with the following equation:

$$\sigma_{DEF} = \frac{\partial^2}{\partial x^2}\sum_{i=2}^{4}\Phi_i(x_l)H_i(\Psi)Ee(x_l) \qquad (2)$$

The reduced stress values in the table 2 by both -dynamic and static too – cases were calculated with the following equation [12]:

$$\sigma_{red} = \sqrt{\sigma_r^2 + 4\tau_Y^2} \qquad (3)$$

where in static case σ_r is a summary of follows: the bending moment from lift force, the bending moment from centrifugal force, the bending moment from mass force, tensile stress from centrifugal force. In case if shear only the lift was considered and related to the shear strength.

Tab. 2. Relative stresses and elongations

Adv. Ratio	$\sigma_{din}/\sigma_{stat}$	$\sigma_{redStat}/\sigma_B$	σ_{redDin}/σ_B	ε_{Stat} (m/m)	ε_{Din} (m/m)	τ_Y/τ_B
μ=0,025	12/85 (14,1%)	135/420 (32,14%)	56/420 (13,33%)	0,0045	0,0019	2,8/40 (7%)
μ=0,15	15/180 (8,3%)	180/420 (42,85%)	60/420 (14,28%)	0,006	0,002	2,6/40 (6,5%)
μ=0,25	19/325 (5,8%)	330/420 (78,57%)	66/420 (15,71%)	0,011	0,0022	3,2/40 (8%)

In dynamic case σ_r is a summary of follows: internal stresses from elastic deformations due to dynamic forces, tensile stress from centrifugal force as above. The table 2 shows the absolute and relative values of the stresses and elongations. The strength properties of rotor blade material were sourced from the literature [13].

The figure 2 and 3 show the distributions of the static and dynamic reduced stresses along the rotor disc.

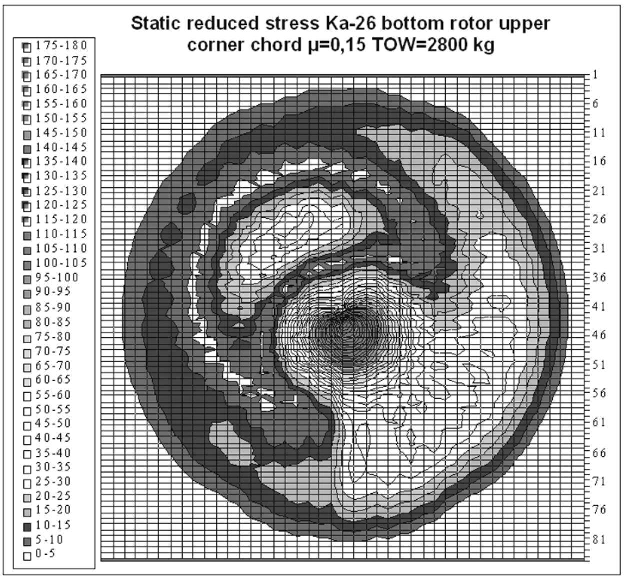


Fig. 2. Distribution of static reduced stress [MPa]

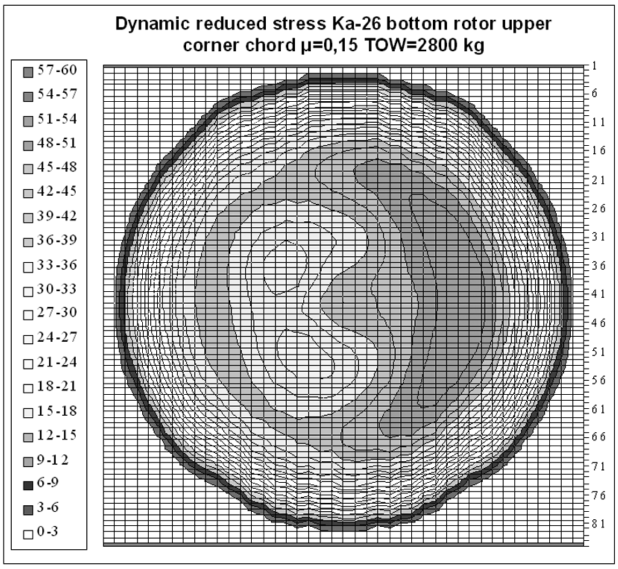


Fig. 3. Distribution of the dynamic reduced stress [MPa]

3. CONCLUSIONS

Using this model the following could be state able:

1. The flow around the bottom rotor blades are not changed significantly due to the effect of induced velocity field of top rotor.
2. The diameter and place of the stream tube of top rotor is changed in the function of advance ratio and there is always an area to be not disturbed by the flow of top rotor.
3. The top blade's tip vortices act on the load of bottom blades only in case of medium advanced operation and the effect of the induced velocities is the higher in case of small advance or hanging. On the other hand in case of high speed operation the effects of top rotor on the loads of bottom one is insignificant. This is showed by the 4÷6 figures what shows the effects of top rotor in the relative radius of 0,75 (is place is the place of No. 3 of the measure rotor blade) The symbols in the figures are the follows:
   - No. 3 AERO: Results are calculated with the model considered with the most effects,
   - ÖN: Results are calculated with the model without the effect of top blade tip vortices,
   - FN: Results are calculated with the model without any effects of the top rotor.

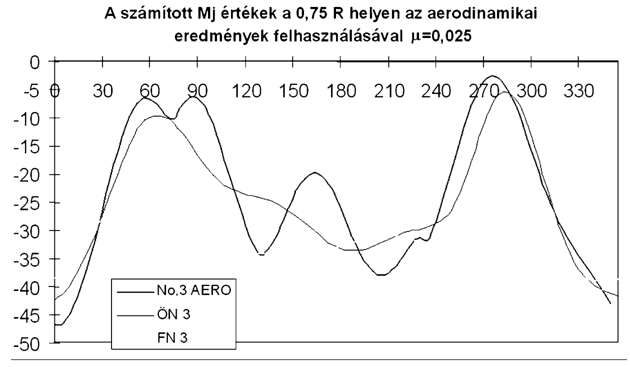


Fig. 4. The effects of top rotor on the bending moment of bottom blades by small advance

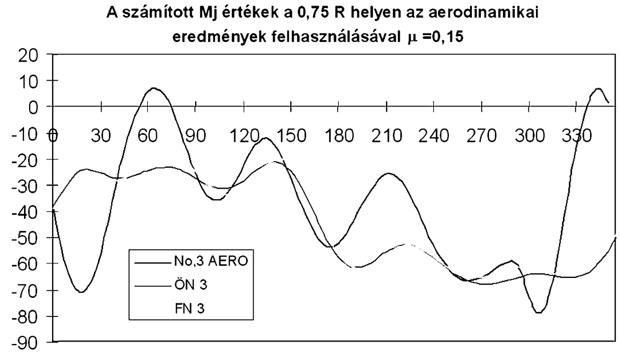


Fig. 5. The effects of top rotor on the bending moment of bottom blades by medium advance

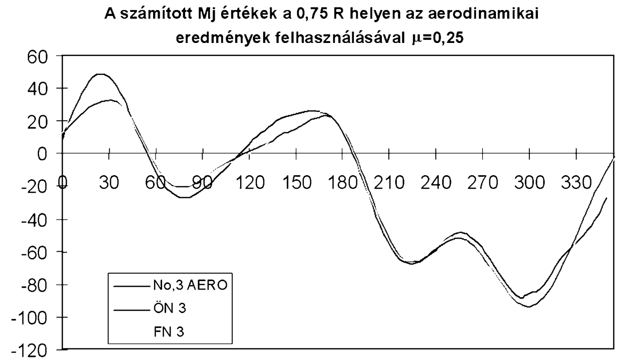
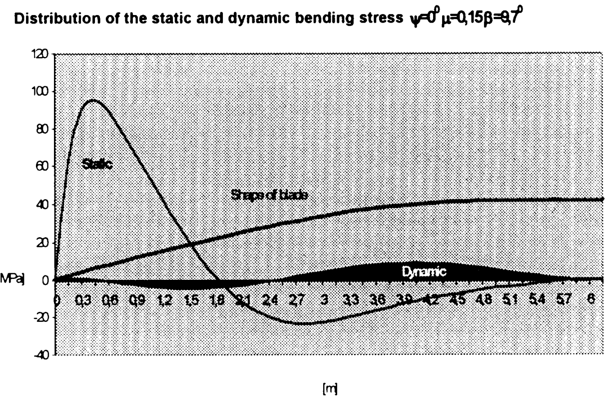


Fig. 6. The effects of top rotor on the bending moment of bottom blades by high advance

4. It can be well seen on the base of values of the table 2 that values calculated on the base of external loads and called for static are much more higher than those values are calculated on the base of deformations as internal stress (called dynamic) and really existing. This goes to show in real operation situation due to the fast change of loads the structure has no enough time to carry those loads were calculated on base of static point of view. The figure 7 as azimuthally intercepts well show the rate of static and dynamic stresses.



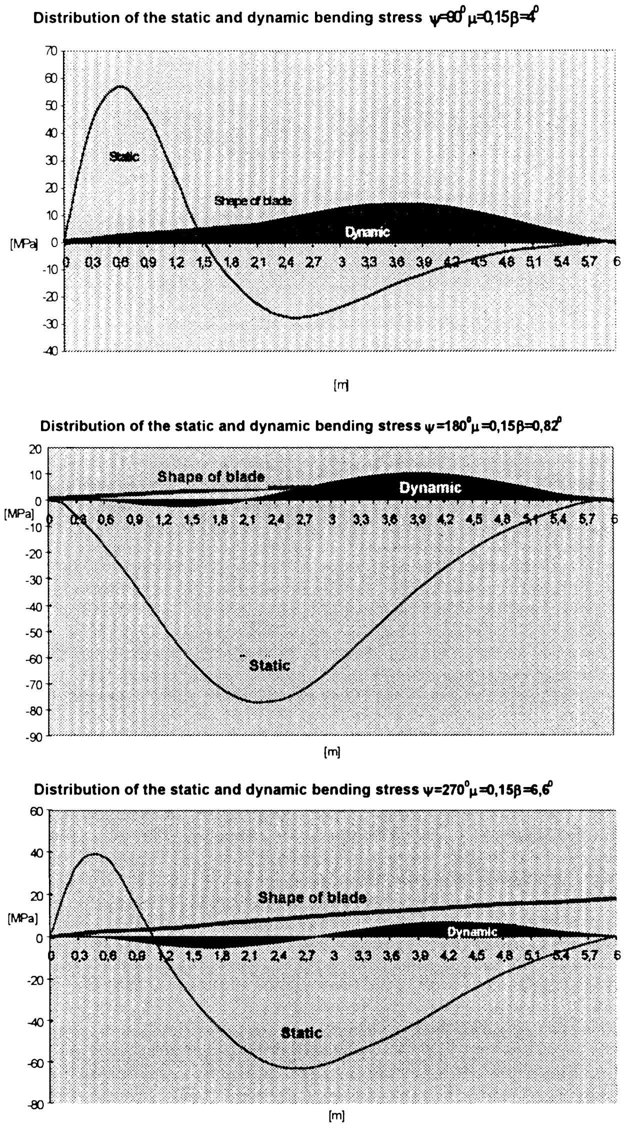


Fig. 7. Azimuthally intercept of shape of blade and strength distributions

5. By this reason the rotor blades are constructed on the base of static aspect (forasmuch as the rotor blade of Ka-26 rotorcraft was constructed at 1959 and then were not so computational background as were able to determine the dynamic loads) are exaggerated structures.
6. Using dynamic loads new construction limits could be determined as a more economic solution or the life time of these structures could be lengthen on this base.
7. It is ascertainable that the relative elongation of rotor blades of Ka-26 helicopter exceeds nowhere the 0,004 value [14] [15] [16] [17] so by this way the life time of blade goes to the infinite when the mechanic loads are considered only. On this base (considered with practical experiences in the subject [18]) the life time of the blades would be not infinite due to the environmental effects, but it is expectable this life time will be very high.

4. SUMMARY

The paper describes a technical-mathematical model for write the aerodynamic-dynamic-aero elastic behaviour of coaxial rotor system in a steady linear flight. This model considers with a limited process capacity, the accuracy and fastness, and almost total effects (rotor blade movements, flow above the rotor, aerodynamic and dynamic forces, and unsteady flow around the profiles, elastic deformations and effect of tip vortices too). The base of this model is the combined blade element momentum theory supplemented with effect of trailing vortices. This model could be used to determine inducted velocity field, dynamic and aerodynamic forces along the area of both rotor of the coaxial rotor system, to determine blade's forces, stress and dynamical characteristics of coaxial rotor system in equilibrium state, to investigate the equilibrium state of the whole helicopter.

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D. Szilágyi

AERODYNAMICZNY I DYNAMICZNY MODEL ŁOPAT WIRNIKÓW

Streszczenie

W celu wyznaczenia aerodynamicznych i dynamicznych obciążeń łopaty dolnego wirnika układu współosiowego na bazie teorii pędu elementu łopaty został zbudowany model obliczeniowy z uwzględnieniem: wiru u wierzchołka łopaty, zjawisk przejściowych wskutek niestabilnego opływu profili łopat, ruchów składowych jak wahanie pionowe i przekręcenie wokół osi łopaty, odkształcenia giętnego oraz wpływu układu sterującego. Łopata będzie się znajdować w ogólnym stanie równowagi, który można policzyć jako rozwiązanie asymptotyczne układu równań różniczkowych, równań różniczkowych nieliniowych ruchu w przypadku wahań pionowych i przekręcenia łopaty wokół osi, równań modalnych odkształceń elastycznych z uogólnionymi masami i siłami. Równania sił aerodynamicznych rozwiązano przez kombinację impulsów, elementu łopaty oraz metody wirowej.

Д. Шилаги

АЭРОДИНАМИЧЕСКАЯ И ДИНАМИЧЕСКАЯ МОДЕЛЬ ЛОПАСТЕЙ СООСНЫХ РОТОРОВ

Резюме

С целью определить аэродинамические и динамические нагрузки лопасти нижнего ротора соосной системы на базе теории количества движения элемента лопасти была построена рассчётная модель с учётом: вихря на конце лопасти, переходных процессов связанных с нестабильным обтеканием профилей лопастей, добавочных движений как вертикальное колебание и скручивание, деформации изгиба и влияния системы управления. Лопасть будет находиться в состоянии общего равновесия, которое можно рассчитать как асимптотическое решение системы нелинейных, обычных, диференциальных уравнений как нелинейное, диференциальное уравнение движения для вертикальных колебаний и скручивания, модальных уравнений эластических деформаций с обобщёнными массами и силами. Уравнения аэродинамических сил были решены с использованием комбинации методов: импульса, элемента лопасти и вихревого.