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On possibility of applying the MFC idea to control the MIMO processes

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Abstract

The paper presents results of comparative analysis of the control system introducing dynamic decoupling and the robust two-degrees of freedom control system with the use of the decoupled model of the MIMO process. The model loop of the MFC is designed with the use of dynamic decoupling algorithm. The correcting controller in the secondary loop uses the control law based on the knowledge of difference between the model and the process outputs. It is a static diagonal matrix. Presented comparative results from the tracking processes of yaw angle and position of the drillship point the possibility to introduce the MFC scheme into the control system of such process. Model and the main control loop are derived for the nominal case. The results obtained for the other working conditions shows the advantages of proposed here solution.

Keywords: Dynamic decoupling, robust control, Model-Following Control, MIMO (multi-input multi-output), multivariable control systems.

O możliwości zastosowania układu MFC do sterowania procesami o wielu wejściach i wielu wyjściach

Streszczenie

Artykuł prezentuje wyniki analizy porównawczej układu regulacji dla obiektów o wielu wejściach i wielu wyjściach MIMO, zaprojektowanego z użyciem algorytmu dynamicznego odsprzęgania, oraz odpornego układu regulacji MFC; zakłada się przy tym, że w pętli głównej regulator wyznaczony jest w drodze syntezy z użyciem dynamicznego odsprzęgania dla nominalnego modelu liniowego, wyznaczonego dla wybranego punktu pracy obiektu, natomiast regulator korekcyjny, bazujący na informacji na temat różnicy pomiędzy wyjściami modelu i procesu ma postać diagonalnej macierzy wzmacnień. Przedstawione porównanie procesu regulacji pozycji i kąta kursowego statku wiertniczego dla omawianych rozwiązań wskazuje na możliwość wykorzystania układu MFC do sterowania tego typu procesem. Model oraz główna pętla sterowania obliczane są dla nominalnego kąta kursowego, natomiast wyniki uzyskane dla innej wartości tegoż kąta wykazują, że proponowane w pracy rozwiązanie zachowuje dobre właściwości dla znaczącej perturbacji kąta kursowego statku.

Słowa kluczowe: Dynamiczne odsprzęganie, regulacja odporna, regulacja typu model-following, układy MIMO, wielowymiarowe układy sterowania.

1. Introduction

For many years, and even before the term of robustness was used, control engineers were looking for control laws insensitive to the variations of the system to be controlled, later called perturbations. These control laws should improve the control quality in the case of wide class of processes, MIMO (multi-input multi-output) included.

Typical approach in the case of control of MIMO processes is dynamic decoupling.

As an idea of dynamic decoupling has been considered by many authors for many years there are many different decoupling methods available. In this paper however, an algorithm proposed in [1–4] is applied. It is designed and may be used for linear m -input l -output both invertible with $m = l$ and right invertible with $m > l$ plants described by proper rational full rank transfer matrix $T(s)$. Plants can be unstable, non-minimum phase or both.

The dynamic decoupling uses the nominal plant's model for the design of the controller. Obviously it has to control the real perturbed process. Those variations are typically bounded, but also – unknown. It can be seen that some perturbations led to loss of quality or/and even stability.

The controller designed with the use of dynamic decoupling is optimal only for the nominal case.

There is a lack of qualitative assessment of dynamic decoupling design procedure in the case of control of perturbed processes. Quantitative evaluation can be processed only for the particular cases.

Robust MFC system, proposed in [5], and then intensively developed [7, 8], is known of its outstanding robustness for plant parameter and/or structure perturbations [6], as well as the great disturbances' dumping at the input and at the output of the plant to be controlled. Also the influence of nonlinearity of the system is hardly reduced in the case of MFC usage.

The paper considers the employment of robust MFC for the perturbed MIMO system. As it is assumed, the perturbations are unknown but bounded. Another assumption is that the perturbations don't alter the process structure.

The paper is of debatable character and is the short summary of the first stage of research work on MIMO-MFC control system.

2. Dynamic decoupling

The goal of control is to maintain stability of the system and at the same time to satisfy many other requirements in order to achieve high performance of control processes. It may be very difficult to realize these requirements especially for complex multi-input multi-output plants, mainly due to coupling of the plant inputs with different outputs. This is why decoupling of the MIMO systems plays a very significant role in designing its control systems. It allows us to consider each decoupled loop independently of any other one. When the row-by-row (diagonal) decoupling is applied to the system a set of single-input single-output subsystems, which are easier to control, is obtained.

The goal we pursue is to obtain a decoupled control system in which each loop of a multipurpose system defined by pairs of reference and output signals $y_{oi}(t), y_i(t)$ for $i = 1, \dots, l$ could be controlled independently of other parts $j \neq i$. Moreover, each part of the system should be designed with individually supposed dynamic properties according to the given class of reference signals $y_{oi}(t) \in R^l$. All of the above mentioned goals can be achieved in

a control system structure presented in Fig. 1, which contains the dynamic feedforward compensator, the Luenberger observer with feedback matrix F and the decoupled diagonal controller.

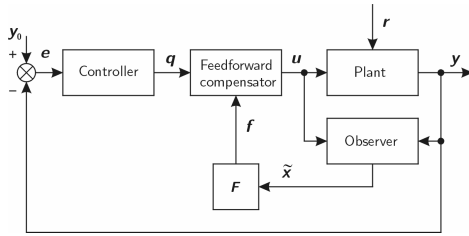


Fig. 1. Structure of the decoupled control system
Rys. 1. Struktura odprężonego systemu sterowania

We consider a controllable and observable linear LTI MIMO model of the plant defined by the state and output equations

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{E}\mathbf{r}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}\quad (1)$$

where $\mathbf{x}(t) \in R^n$, $\mathbf{u}(t) \in R^m$ and $\mathbf{y}(t) \in R^l$ ($m \geq l$) are the state, input and output vectors respectively. The vector $\mathbf{r}(t) \in R^r$ describes deterministic (non-diminishing) disturbances. In the polynomial matrix approach transfer matrices of all elements of the system are defined by pairs of polynomial matrices either relatively right prime (*r.r.p.*) for plants, or relatively left prime (*r.l.p.*) for other elements. Applying this approach, the plant model (1) can be transformed into the relatively prime matrix fraction description in the frequency s -domain as follows

$$\mathbf{y} = \mathbf{B}_1(s)\mathbf{A}_1^{-1}(s)\mathbf{u} + \mathbf{A}_3^{-1}(s)\mathbf{B}_3(s)\bar{\mathbf{r}} \quad (2)$$

where

$$\mathbf{B}_1(s)\mathbf{A}_1^{-1}(s) = \mathbf{C}(s\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (3)$$

and

$$\mathbf{A}_3^{-1}(s)\mathbf{B}_3(s) = \mathbf{C}(s\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{E}\mathbf{r}(s) \quad (4)$$

Since the transformed disturbance vector $\mathbf{r}(s)$ is included into the transfer matrix (4) the symbol $\bar{\mathbf{r}}$ in the eq. (2) denotes a "fictitious" impulsive input signal applied to the deterministic disturbance model.

Similarly, as in the disturbance vector $\mathbf{r}(s)$ case, the reference signal vector $\mathbf{y}_o(s)$ is generated from the reference model defined by (unstable) strictly proper transfer matrix functions (possible with different transfer functions for each reference signal)

$$\mathbf{y}_o(s) = \mathbf{A}_o^{-1}(s)\mathbf{B}_o(s)\bar{\mathbf{y}}_o \quad (5)$$

with the impulsive signal input $\bar{\mathbf{y}}_o$.

The feedback law, employed to decouple the system (the linear state variable feedback along with dynamic feedforward) is described by

$$\mathbf{u}(s) = \mathbf{G}^{-1}(s)\mathbf{L}_0(s)\mathbf{f}(s) + \mathbf{G}^{-1}(s)\mathbf{L}(s)\mathbf{q}(s) \quad (6)$$

where

$$\mathbf{f}(s) = \mathbf{F}(s)\mathbf{x}_p(s) \stackrel{\Delta}{=} \mathbf{F}\mathbf{x}(t) \quad (7)$$

$\mathbf{x}_p(s)$ is a Laplace transform of a partial state vector of the plant, $\mathbf{G}(s) \in R[s]^{m \times m}$, $\mathbf{L}(s) \in R[s]^{m \times l}$, $\mathbf{L}_0(s) \in R[s]^{m \times m}$, $\mathbf{F}(s) \in R[s]^{m \times m}$ – polynomial matrices such that $\mathbf{G}^{-1}(s)\mathbf{L}_0(s)$ and $\mathbf{G}^{-1}(s)\mathbf{L}(s)$ are proper and $\mathbf{F}(s)\mathbf{A}_1^{-1}(s)$ is strictly proper. Without any loss of generality the matrix $\mathbf{L}_0(s)$ may be taken as $\mathbf{L}_0(s) = \mathbf{I}_m$. Then the system has the structure presented in Fig. 2.

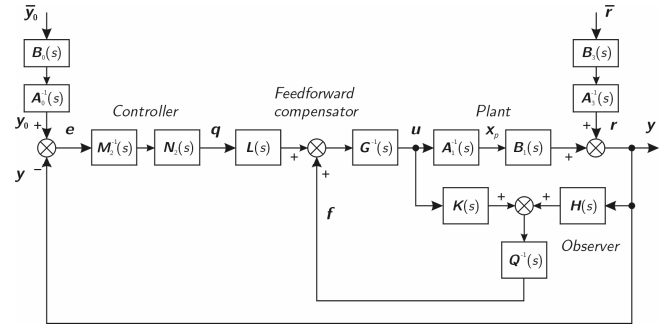


Fig. 2. Structure of the decoupled control system with inaccessible plant's state vector

Rys. 2. Struktura odprężonego systemu sterowania z niedostępnym wektorem stanu

According to this scheme the considered multipurpose control systems are suitably defined in s -domain by: proper and possible "low-order" transfer matrix $\mathbf{G}^{-1}(s)\mathbf{L}(s)$ for the dynamic feedforward compensator, strictly proper (or proper) transfer matrices $\mathbf{Q}^{-1}(s)\mathbf{H}(s)$ and $\mathbf{Q}^{-1}(s)\mathbf{K}(s)$ for the full (or reduced) order Luenberger observer along with a feedback matrix \mathbf{F} and a strictly proper transfer matrix $\mathbf{M}_2^{-1}(s)\mathbf{N}_2(s)$ for the decoupled controller. All of the above-mentioned polynomial matrix fractions should be relatively left prime (*r.l.p.*) with nonsingular, row-reduced, denominator matrices.

3. Model-Following Control for MIMO processes

The proposed here MIMO-MFC control system is shown in Figure 3. It can be regarded as a part of a more general class of systems called Model Based Control. The essential component of the plant input signal in the MFC structure is generated in the main control system containing a nominal model of the plant M and its controller in the feedback loop (designed here with the use of dynamic decoupling).

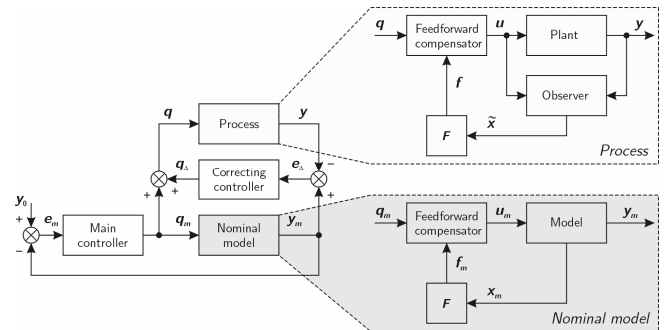


Fig. 3. MIMO-MFC system

Rys. 3. Struktura MIMO-MFC

It means that the output of the main controller acts on the input of the actual process plant. The second control loop of the MFC structure contains the correcting controller and the actual process P , where the difference e_Δ between the plant output y and the model output y_m is processed. Thus, the summed result of actions of both controllers, i.e. main and correcting controller, excites the input of the actual plant P . Thanks to the dynamic decoupling used for design of the main controller, the correcting controller in the MIMO-MFC system can take the diagonal form, i.e. static diagonal matrix.

Features exhibited by the MFC structure have been discussed in [6, 8] in detail.

Obviously in the model loop (Fig. 3) of the MFC the state variable \mathbf{x}_m is accessible. Then, there is not need to use an observer in the feedback. It may be necessary for the plant, however.

The applied algorithm, which full version may be found in [2, 3], guarantees free location of all poles of the system and guarantees that all designed elements (parts of the system) are proper (or strictly proper), so they are able to be physically realizable. So it suits perfectly all requirements which should be met by algorithm used in construction of the MFC control structure.

4. Numerical example

In order to illustrate the theoretical considerations an example of design of a MFC control system is presented. As a plant we choose a non-linear model ("non-dimensional") of a low-frequency motions of a drilling vessel „Wimpey Sealab” given in [5] which after linearization (for current velocity $v_c = 3$ knot with the course angle $x_{30} = 0^\circ$) is defined by the following matrices of the state and output equations (1)

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.0507 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0134 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.071 & -0.0275 \\ 0 & 0 & 0 & 0 & -0.2429 & 0 \end{bmatrix},$$

$$B = E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.958 & 0 & 0 \\ 0 & 0.544 & 0 \\ 0 & 0 & 9.4693 \end{bmatrix}, \quad (8)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

It has the poles $s_{1,2,3} = 0$, $s_4 = -0.1246$, $s_5 = 0.0537$, $s_6 = -0.0134$.

As a "real" plant in the simulations we used a model linearized for current velocity $v_c = 3$ knot and the course angle $x_{30} = 25^\circ$. The model has the same input, output and transition matrices (8) but the state matrix A of the form

$$A_{zm} = \begin{bmatrix} 0 & 0 & 0 & 0.9063 & -0.4226 & 0 \\ 0 & 0 & 0.0326 & 0.4226 & 0.9063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.0122 & -0.0012 & -0.0205 \\ 0 & 0 & 0 & 0.0238 & -0.1086 & -0.0249 \\ 0 & 0 & 0 & 0.1076 & -0.1864 & 0 \end{bmatrix}, \quad (9)$$

It has the poles $s_{1,2,3} = 0$, $s_4 = -0.1339$, $s_{5,6} = 0.0065 \pm 249i$. The "denominator" matrix $M_2(s)$ of the main controller was defined as

$$M_2(s) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix}.$$

Assuming the following values of poles:

– for the first loop: $s_1 = -0.1$, $s_2 = -0.25$, $s_3 = -0.8$,

– for the second loop: $s_4 = -0.08$, $s_5 = -0.2$, $s_6 = -0.5$,

– for the third loop: $s_7 = -0.1$, $s_8 = -0.5$, $s_9 = -1$,

we obtain:

– numerator matrix of the controller

$$N_2(s) = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.008 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}$$

– dynamic feedforward compensator $G^{-1}(s)L(s)$

$$L(s) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G(s) = \begin{bmatrix} 0.958 & 0 & 0 \\ 0 & 0.544 & 0 \\ 0 & 0 & 9.4697 \end{bmatrix}$$

and the feedback matrix

$$F = \begin{bmatrix} -0.305 & 0 & 0 & -1.1366 & 0 & 0 \\ 0 & -0.156 & -0.0395 & 0 & -0.709 & -0.0232 \\ 0 & 0 & -0.65 & 0 & 0.2429 & -1.6 \end{bmatrix}$$

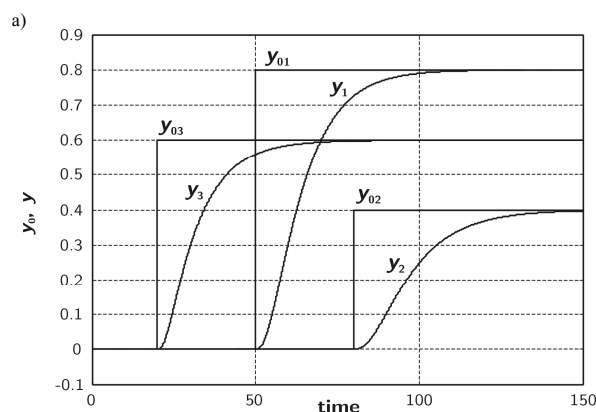
The "gain" matrix of the full order Luenberger observer (for plant) with the values of its poles assumed as $s_1 = -1.25$, $s_2 = -1.25$, $s_3 = -1$, $s_4 = -1$, $s_5 = -1.5$ is given as

$$L = \begin{bmatrix} 2.4866 & 0 & 0 \\ 0 & 1.929 & 0.0232 \\ 0 & -0.2429 & 3 \\ 1.5292 & 0 & 0 \\ 0 & 0.8698 & -0.0806 \\ 0 & -0.4686 & 2.2567 \end{bmatrix}$$

Experimentally derived gain matrix of the correcting controller has the simple form

$$C_\Delta = \begin{bmatrix} 3.0 & 0 & 0 \\ 0 & 3.0 & 0 \\ 0 & 0 & 3.0 \end{bmatrix}$$

Simulation of the control process of yaw angle and position of the drillship consists of three steps. Ship changes course angle (from 0 to 35 degrees). It began after 20 relative units of time (*r.u.t.*). When a new course angle was reached the ship began to be steering to the new drilling point: after 50 *r.u.t.* it starts to move about 76m forward, and about 38m right after 80 *r.u.t.* Results of simulation of the system are shown in Figs. 4 – 6.



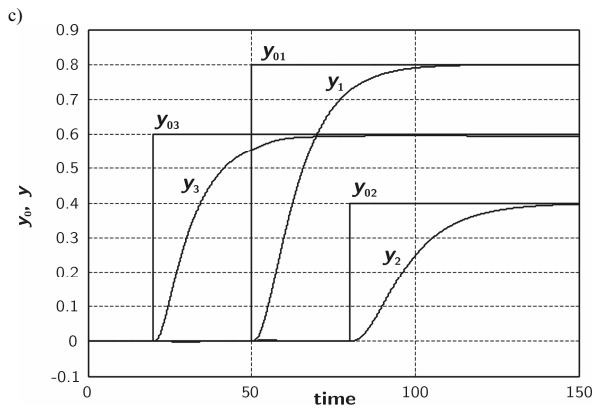
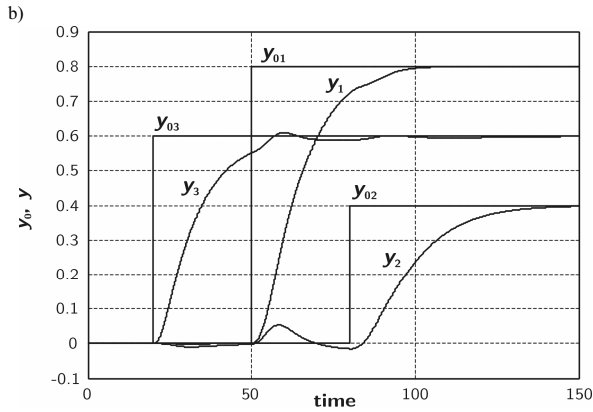


Fig. 4. Results of simulation; output signals in: nominal case (a), perturbed with DD controller (b), perturbed with MIMO-MFC (c)
 Rys. 4. Wyniki symulacji; sygnały wyjściowe – przypadek nominalny (a), obiekt perturbowany z regulatorem DD (b), obiekt perturbowany z regulatorem MIMO-MFC (c)

The following Figures show the control signals of compared control systems (Fig. 5) and the error e_A (Fig. 6).

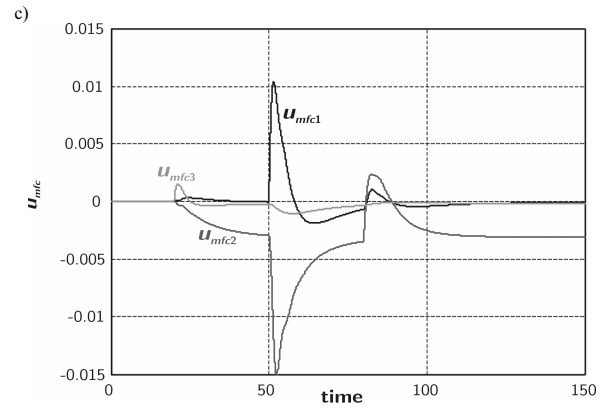
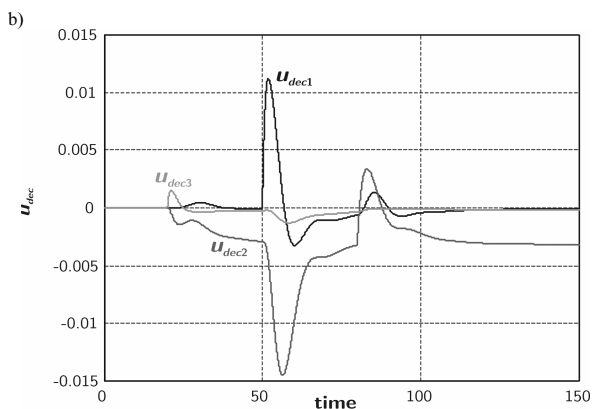
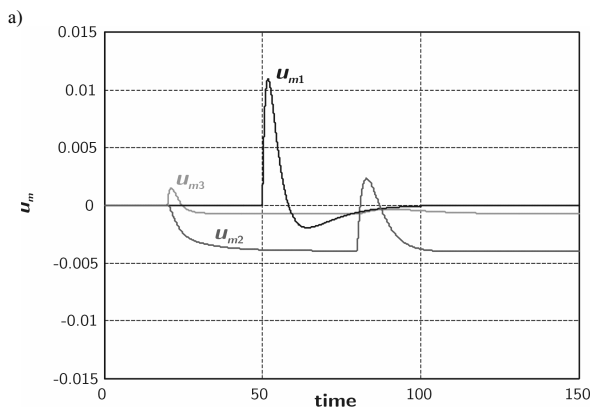


Fig. 5. Control signals (according to Fig. 4)
 Rys. 5. Sygnały sterujące (patrz rys. 4)

It can be seen, that in the case of dynamic decoupling (DD) controller cannot handle the perturbation of the controlled plant, while MIMO-MFC system works according to its main principle – plant outputs are following after the outputs of the nominal model.

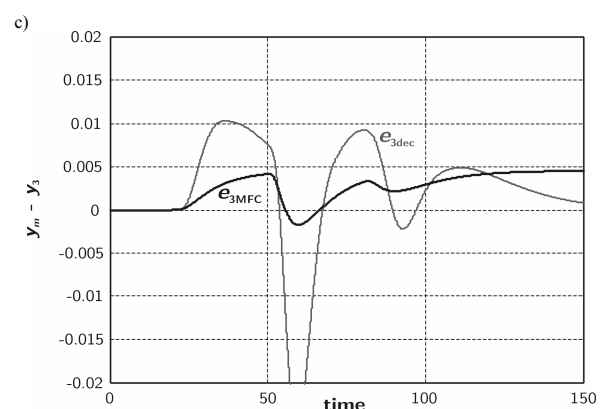
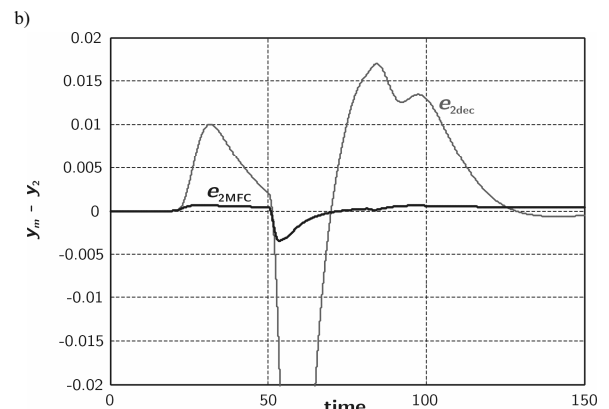
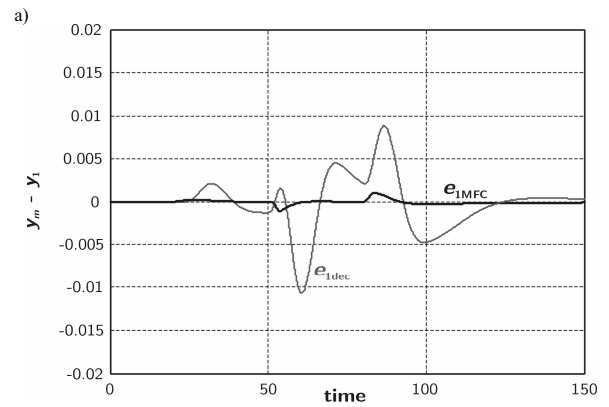


Fig. 6. Error signals (according to Fig. 4)
 Rys. 6. Sygnały błędów (patrz rys. 4)

5. Summary

As it was shown, MFC system can be applied for the control of MIMO processes. Proposed approach makes full use of the dynamic decoupling procedures for design of the main controller. Simulation results proved its usability for control of unstable system.

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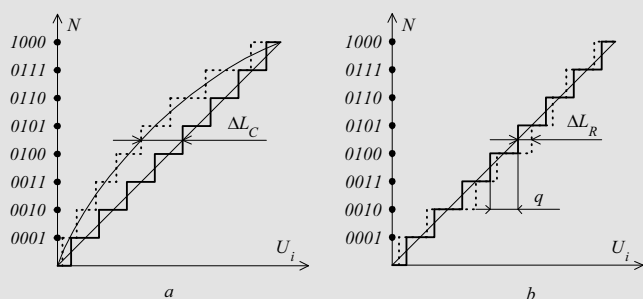
Artykuł recenzowany

INFORMACJE

O pewnym nieporozumieniu terminologicznym

W fachowej literaturze dotyczącej przetworników analogowo-cyfrowych i cyfrowo analogowych publikowanej w języku polskim powszechnie pojawiają się nazwy pewnych błędów takich przetworników, które moim zdaniem są „nomen omen”, również błędne.

Chodzi tu o błędy liniowości takich przetworników, powszechnie nazywane „błędami całkowymi” i „błędami różniczkowymi”. Definicje tych błędów ilustrują zamieszczone poniżej rysunki (rys. 1).



Rys. 1. Błędy liniowości przetwornika A/C

Zgodnie z definicją błąd ΔL_C , przedstawiony na rysunku *a*, określony jest jako: *największa różnica pomiędzy wartością napięcia dla której następuje zmiana słowa kodowego w przetworniku, a wartością napięcia jakiej należałoby się spodziewać w przypadku charakterystyki idealnej* [1]. Z kolei błąd ΔL_R , na rysunku *b*, jest określony jako: *największa różnica pomiędzy szerokością przedziału napięć, któremu przetwornik przypisuje słowo kodowe, a szerokością przedziału kwantowania q idealnego przetwornika* [1].

Definicje błędów są oczywiste i dobrze określają własności przetworników rzeczywistych, ale doprawdy nie mają nic wspólnego z operacjami matematycznymi całkowania i różniczkowania.

Ponieważ te nazwy tzn. „błąd całkowity” i „błąd różniczkowy” pojawiają się w polskim piśmiennictwie od lat, można jedynie domniemywać, że ktoś kiedyś niefrasobliwie przetłumaczył z języka angielskiego nazwy tych błędów, które w piśmiennictwie angielskojęzycznym nazywane są odpowiednio „*integral linearity error*” i „*differential linearity error*” [2], a następnii autorzy bezkrytycznie te nazwy przyjęli. Jedyłą znaną mi polskojęzyczną pozycją literaturową jest [1], gdzie autorzy nazwali te błędy odpowiednio „błędem liniowości całkowitej” i „błędem liniowości różnicowej”. Są to nazwy mające uzasadnienie merytoryczne i będące w zgodzie z pierwotnym znaczeniem słów „*integral*” i „*differential*”, które wg [3] określone są:

integral – 1. *necessary for completeness*, 2. *whole; having or containing all parts that are necessary for completeness.*

differential – *of, showing, depending on, a difference*

Propozycje nazw błędów przetworników A/C przedstawione w [1] są do przyjęcia, chociaż osobiście preferowałbym nazwy „całkowy błąd liniowości” i „różnicowy błąd liniowości”.

Mam nadzieję, że ten mój krótki tekst może pomóc chociaż w części wyeliminować to pleniące się nieporozumienie terminologiczne, które trwa już zaskakująco długo i ciągle jest powtarzane, czego doświadczyłem przeglądając ostatnio świeżą rozprawę habilitacyjną.

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