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Positive bilinear discrete-time systems with delays

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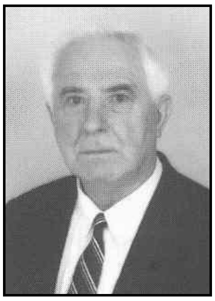

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Abstract

Necessary and sufficient conditions are established for the positivity of bilinear discrete-time systems with delays. Solutions to positive bilinear discrete-time systems are derived. The controllability to zero of the bilinear positive systems with delays is addressed. Necessary and sufficient conditions for the controllability to zero of positive bilinear systems with delays are formulated and proved. The considerations are illustrated by examples.

Keywords: bilinear system, controllability, delay, discrete-time, positive system.

Dodatnie biliniowe układy dyskretne z opóźnieniami

Streszczenie

Podano warunki konieczne i wystarczające dodatności biliniowych układów dyskretnych z opóźnieniami w wektorze stanu i w wymuszeniu. Podano również metodę wyznaczania rozwiązania tych biliniowych układów dyskretnych z opóźnieniami. Wyprowadzono warunki konieczne i wystarczające sterowalności do zera tej klasy układów dodatnich. Rozwiązanie ogólne zostało zilustrowane przykładem liczbowym.

Słowa kluczowe: biliniowość, sterowalność, opóźnienie, dodatni układ dyskretny.

1. Introduction

In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear systems behaviour can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of state of the art in positive systems theory is given in the monographs [3, 4]. Recent developments in positive systems theory and some new results are given in [5].

The controllability of bilinear discrete-time standard (nonpositive) systems has been considered in the monograph [7].

The controllability of positive continuous-time bilinear systems has been investigated in [1, 2, 8, 9]. Necessary and sufficient conditions for the positivity of bilinear discrete-time systems and for the controllability to zero and the reachability of the systems have been established in [6].

In this paper the posivity, solutions and controllability to zero of positive bilinear systems with delays will be addressed.

To the best knowledge of the author the positive bilinear discrete-time systems with delays has not been considered yet.

2. Positive bilinear systems

Let $R_+^{n\times m}$ be the set of $n\times m$ matrices with real nonnegative entries and $R_+^n = R_+^{n\times 1}$. The set of nonnegative integers will be denoted by Z_+ and the $n\times n$ identity matrix by I_n.

Consider the discrete-time bilinear system described by the equations

$$x(i+1) = A\left(I_n + \sum_{j=1}^{m} B_j u_j(i)\right)x(i) + B_0 u(i) \tag{1a}$$

$$y(i) = Cx(i) + Du(i) \tag{1b}$$

where $x(i)\in R^n$, $u(i)\in R^m$, $y(i)\in R^p$ are the state, input and output vectors, $u_j(i)$ is the jth component of $u(i)$ and $A, B_j \in R^{n\times n}, j=1,...,m, B_0\in R^{n\times m}, C\in R^{p\times n}, D\in R^{p\times m}$.

For $\overline{B}_j = AB_j$, $j=1,...,m$ the equation (1a) takes the form

$$x(i+1) = \left(A + \sum_{j=1}^{m} \overline{B}_j u_j(i)\right)x(i) + B_0 u(i) \tag{2}$$

Definition 1. The bilinear system (1) is called (internally) positive if for every $x(0)\in R_+^n$ and all inputs $u(i)\in R_+^m, i\in Z_+$, $x(i)\in R_+^n$ and $y(i)\in R_+^p$ for $i\in Z_+$.

Theorem 1. The bilinear system (1) is positive if and only if

i) $A\in R_+^{n\times n}, B_0\in R_+^{n\times m}, C\in R_+^{p\times n}, D\in R_+^{p\times m}$

ii) $AB_j \in R_+^{n\times n}$ for $j=1,...,m$

The proof is given in [6].

Corollary 1. If

$$\begin{aligned} &A\in R_+^{n\times n}, B_j\in R_+^{n\times n}, j=1,..., m, B_0\in R_+^{n\times m},\\ &C\in R_+^{p\times n}, D\in R_+^{p\times m}\end{aligned} \tag{3}$$

then the system (1) is positive.

Corollary 2. For the system (2) the condition ii) of Theorem 1 takes the from

$$\overline{B}_j \in R_+^{n\times n} \text{ for } j=1,...,m \tag{4}$$

Definition 2. The bilinear positive system (1) is called controllable to zero in n steps if and only if there exists a sequence of inputs $u(i) \in R_+^m, i = 0,1,...,n-1$ such that $x(n) = 0$ for any $x(0) \in R_+^n$.

Theorem 2. The bilinear positive system (1) is controllable to zero in n steps if and only if A is a nilpotent matrix, i.e.

$$A^n = 0 \tag{5}$$

Moreover the sequence $u(i) \in R_+^m, i = 0,1,...,n-1$ that steers the state of the system to zero has the form

$$u(i) = 0 \text{ for } i = 0,1,...,n-1 \tag{6}$$

The proof is given in [6]

3. Positive bilinear systems with delays

Consider the discrete-time bilinear system with delays

$$\begin{aligned} x(i+1) &= \left(A_0 + \sum_{j=1}^m B_{0j} u_j(i) \right) x(i) + \\ &+ \left(A_1 + \sum_{j=1}^m B_{1j} u_j(i) \right) x(i-1) + \\ &+ B_0 u(i) + B_1 u(i-1) \end{aligned} \tag{7a}$$

$$y(i) = Cx(i) + Du(i) \tag{7b}$$

where $x(i) \in R^n$, $u(i) \in R^m$, $y(i) \in R^p$ are the state, input and output vectors, $u_j(i)$ is the jth component of $u(i)$ and $A_0, A_1, B_{0j}, B_{1j} \in R^{n\times n}, j = 1,...,m, B_0, B_1 \in R^{n\times m}, C \in R^{p\times n}, D \in R^{p\times m}$.

Definition 3. The bilinear system with delays (7) is called (internally) positive if for every initial condition $x(k) \in R_+^n, u(-1) \in R_+^m$ and all inputs $u(i) \in R_+^m, i \in Z_+$, $x(i) \in R_+^n$ and $y(i) \in R_+^p$ for $i \in Z_+$.

Theorem 3. The bilinear system with delays (7) is positive if and only if

$$\begin{aligned} A &= \begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix} \in R_+^{2n\times 2n}, \\ B_j &= \begin{bmatrix} B_{0j} & B_{1j} \\ 0 & 0 \end{bmatrix} \in R_+^{2n\times 2n}, j = 1,...,m, \\ B_k &\in R_+^{n\times m}, k = 0,1, C \in R_+^{p\times n}, D \in R_+^{p\times n} \end{aligned} \tag{8}$$

Proof. Defining the new vectors $\bar{x}(i) = \begin{bmatrix} x(i) \\ x(i-1) \end{bmatrix}, \bar{u}(i) = \begin{bmatrix} u(i) \\ u(i-1) \end{bmatrix}$ and the matrix $B = \begin{bmatrix} B_0 & B_1 \\ 0 & 0 \end{bmatrix}$ and using (8) we may write the equations (7) in the form

$$\bar{x}(i+1) = \left(A + \sum_{j=1}^m B_j u_j(i) \right) \bar{x}(i) + B\bar{u}(i) \tag{9a}$$

$$y(i) = [C \ \ 0]\bar{x}(i) + [D \ \ 0]\bar{u}(i) \tag{9b}$$

Applying the Theorem 1 to the system (9) we obtain the conditions (8).

4. Solutions of positive bilinear systems with delays

The solution to the equation (7a) for the given initial conditions

$$x(-k) \in R^n \text{ for } k = 0,1 \text{ and } u(-1) \in R^m \tag{10}$$

can be computed in the following way.

Let as defined $T_k(i) \in R^{n\times n}$ for $k = 0,1,2$ and $i = 0,1,...$ as follows

$$\begin{aligned} T_0(i+1) &= \left(A_0 + \sum_{j=1}^m B_{0j} u_j(i) \right) T_0(i) + \\ &+ \left(A_1 + \sum_{j=1}^m B_{1j} u_j(i) \right) T_0(i-1) \\ T_0(0) &= I_n \text{ and } T_0(1) = A_0 + \sum_{j=1}^m B_{0j} u_j(0) \end{aligned} \tag{11}$$

$$\begin{aligned} T_1(i+1) &= \left(A_0 + \sum_{j=1}^m B_{0j} u_j(i) \right) T_1(i) + \\ &+ \left(A_1 + \sum_{j=1}^m B_{1j} u_j(i) \right) T_1(i-1) \\ T_1(0) &= 0, T_1(1) = A_1 + \sum_{j=1}^m B_{1j} u_j(0) \end{aligned} \tag{12}$$

and

$$\begin{aligned} T_2(i+1) &= \left(A_0 + \sum_{j=1}^m B_{0j} u_j(i) \right) T_2(i) + \\ &+ \left(A_1 + \sum_{j=1}^m B_{1j} u_j(i) \right) T_2(i-1) + B_0 u(i) + B_1 u(i-1) \\ T_2(0) &= 0, T_2(1) = B_0 u(0) + B_1 u(-1) \end{aligned} \tag{13}$$

Theorem 4. The solution to (7a) with (10) is given by

$$x(i) = T_0(i)x(0) + T_1(i)x(-1) + T_2(i) \tag{14}$$

for $i = 0,1,...$

Proof. The proof will be accomplished by induction with respect to i.

The hypothesis is true for $i = 0,1$ since from (14) for $i = 1$ we have

$$\begin{aligned} x(1) &= T_0(1)x(0) + T_1(1)x(-1) + T_2(1) = \\ &= \left(A_0 + \sum_{j=1}^m B_{0j} u_j(0) \right) x(0) + \\ &+ \left(A_1 + \sum_{j=1}^m B_{1j} u_j(0) \right) x(-1) + B_0 u(0) + B_1 u(i-1) \end{aligned}$$

The same result we obtain from (7a) for $i = 0$.

Assuming that the hypothesis is true for $i = k$ we shall show that it is also valid for $i = k+1$.

From (14) and (11)-(13) for $i = k+1$ we have

$$x(k+1) = T_0(k+1)x(0) + T_1(k+1)x(-1) + T_2(k+1) =$$
$$\left[\left(A_0 + \sum_{j=1}^{m} B_{0j}u_j(k)\right)T_0(k) + \left(A_1 + \sum_{j=1}^{m} B_{1j}u_j(k)\right)T_0(k-1)\right]x(0) +$$
$$\left[\left(A_0 + \sum_{j=1}^{m} B_{0j}u_j(k)\right)T_1(k) + \left(A_1 + \sum_{j=1}^{m} B_{1j}u_j(k)\right)T_1(k-1)\right]x(-1) +$$
$$\left(A_0 + \sum_{j=1}^{m} B_{0j}u_j(k)\right)T_2(k) + \left(A_1 + \sum_{j=1}^{m} B_{1j}u_j(k)\right)T_2(k-1) +$$
$$+ B_0u(k) + B_1u(k-1) = \left(A_0 + \sum_{j=1}^{m} B_{0j}u_j(k)\right)$$
$$\left[T_0(k)x(0) + +T_1(k)x(-1) + T_2(k)\right] +$$
$$\left(A_1 + \sum_{j=1}^{m} B_{1j}u_j(k)\right)\left[T_0(k-1)x(0) + T_1(k-1)x(-1) + T_2(k-1)\right] +$$
$$+ B_0u(k) + B_1u(k-1) = \left(A_0 + \sum_{j=1}^{m} B_{0j}u_j(k)\right)x(k) +$$
$$+\left(A_1 + \sum_{j=1}^{m} B_{1j}u_j(k)\right)x(k-1) + B_0u(k) + B_1u(k-1)$$

The proof has been completed.

5. Controllability to zero of positive bilinear systems with delays

Definition 4. The positive bilinear positive system with delays (7) is called controllable to zero in q steps if and only if there exists a sequence of inputs $u(i) \in R_+^m$ for $0,1,...,q-1$ such that $x(q) = 0$ for any $x(k) \in R_+^n, k = 1,0, u(-1) = 0$.

In farther considerations the following two lemmas will be used.

Lemma 1. If A has the form (8) then

$$A^k = \begin{bmatrix} A_{11}^k & A_{12}^k \\ A_{21}^k & A_{22}^k \end{bmatrix} = \tag{15a}$$
$$= \begin{bmatrix} A_0A_{11}^{k-1} + A_1A_{11}^{k-2} & A_0A_{12}^{k-1} + A_1A_{12}^{k-2} \\ A_{11}^{k-1} & A_{12}^{k-1} \end{bmatrix}$$

for $k = 2,3,..$

where by definition

$$A_{11}^0 = I_n, A_{11}^1 = A_0 \text{ and } A_{12}^0 = 0, A_{12}^1 = A_1 \tag{15b}$$

Proof. The proof will be accomplished by induction with respect to k.

From (15a) for $k = 2$ we have

$$A^2 = \begin{bmatrix} A_0^2 + A_1 & A_0A_1 \\ A_0 & A_1 \end{bmatrix}$$

The same result we obtain multiply A by A. Thus, the hypothesis is true for $k = 2$. Assuming that the hypothesis is true for $i \geq 2$ we shall show that it is also true for $i+1$. Using (15a) we obtain

$$A^{i+1} = \begin{bmatrix} A_0 & A_1 \\ I & 0 \end{bmatrix}\begin{bmatrix} A_{11}^i & A_{12}^i \\ A_{11}^{i-1} & A_{12}^{i-1} \end{bmatrix} = \begin{bmatrix} A_0A_{11}^i + A_1A_{11}^{i-1} & A_0A_{12}^i + A_1A_{12}^{i-1} \\ A_{11}^i & A_{12}^i \end{bmatrix} = \begin{bmatrix} A_{11}^{i+1} & A_{12}^{i+1} \\ A_{21}^{i+1} & A_{22}^{i+1} \end{bmatrix}$$

Therefore, the hypothesis is true for $k = 2,3,..$

It is assumed that the nilpotent matrices $A_k, k = 0,1$ can be transformed by the similarity to the following forms

$$PA_kP^{-1} = \begin{cases} \begin{bmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & \cdots & a_{2n} \\ \hdashline 0 & 0 & \cdots & 0 & a_{n-1,n} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ a_{21} & 0 & \cdots & 0 & 0 \\ \hdashline a_{n-1,1} & a_{n-1,2} & \cdots & 0 & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn-1} & 0 \end{bmatrix} \end{cases} k = 0,1 \tag{16}$$

where $P \in R_+^{n \times n}$ is the permutation matrix obtained from I_n by any number of interchanges of its rows and columns.

Remark 1. Note that the nilpotency index is independent of the particular value of nonzero entries of the matrices. Thus it is enough to consider matrices with entries equal to 0 and 1.

Definition 5. Let

$$A_1 = \left[a_{ij}^1\right] \in R_+^{n \times n}, A_2 = \left[a_{ij}^2\right] \in R_+^{n \times n}, i, j = 1,2,...,n$$

It is said that $A_1 \leq A_2$ if $a_{ij}^1 \leq a_{ij}^2$ for $i, j = 1,2,...,n$

Lemma 2. Let $A_k = \left[a_{ij}^k\right] \in R_+^{n \times n}$ for $k = 0,1$.

Then the matrix

$$A = \begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix} \tag{17}$$

is nilpotent if and only if the matrix $A_s = (A_0 + A_1)$ is nilpotent.

Proof. ($\Leftarrow$) If the matrix $A_s = (A_0 + A_1)$ is nilpotent then the matrix A (defined by (17)) is nilpotent.

We have

$$A = \begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix} \leq \begin{bmatrix} A_s & A_s \\ I_n & 0 \end{bmatrix} = \overline{A} \tag{18}$$

and

$$\overline{A}^2 = \begin{bmatrix} (A_s^2 + A_s) & A_s^2 \\ A_s & A_s \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ 0 & A_s \end{bmatrix}\begin{bmatrix} A_s + I_n & A_s \\ I_n & I_n \end{bmatrix}$$

$$\overline{A}^{2\upsilon_{A_s}} = \begin{bmatrix} A_s & 0 \\ 0 & A_s \end{bmatrix}^{\upsilon_{A_s}} \begin{bmatrix} A_s + I_n & A_s \\ I_n & I_n \end{bmatrix}^{\upsilon_{A_s}} = 0$$

Hence the matrix $\overline{A}$ is nilpotent and from (18) the matrix A is nilpotent too.

($\Rightarrow$) If the matrix A is nilpotent then the matrix $A_s = (A_0 + A_1)$ is nilpotent.

The nilpotency of matrix A implies that for all

$$k \in R_+, k \geq \upsilon_A : A^k = \begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix}^k = 0 \tag{19}$$

Note that all the expands of the power of the sum $A_s^{\upsilon_A} = (A_0 + A_1)^{\upsilon_A}$ occur as components in the elements of the matrices:

$$\begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix}^{\upsilon_A}, \begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix}^{\upsilon_A+1}, \begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix}^{\upsilon_A+2} \cdots \tag{20}$$

For $A_0, A_1 \in R_+^{n\times n}$, all the products of A_0, A_1, A_0A_1 and A_1A_0 are nonnegative matrices. From (19) and (20) we have $A_s^{\upsilon_A} = (A_0 + A_1)^{\upsilon_A} = 0$.

Hence the matrix A_s is nilpotent.

Lemma 3. The nilpotency indices of the matrices $A_0, A_1, A_s = (A_0 + A_1)$ and A are related by

$$\upsilon_A \begin{cases} = \upsilon_{A_0} + 1 & for \quad A_1 = 0 \\ = 2\upsilon_{A_1} & for \quad A_0 = 0 \\ \leq 2\upsilon_{A_s} & otherwise \end{cases} \tag{21}$$

Proof. Using (8) for $A_1 = 0$ it is easy to show that

$$A^k = \begin{bmatrix} A_0^k & 0 \\ A_0^{k-1} & 0 \end{bmatrix} \text{ for } k = 1,2,...$$

Hence $\upsilon_A = \upsilon_{A_0} + 1$ for $A_1 = 0$.

If $A_0 = 0$ then by induction it can be easily show that

$$A^{2k} = \begin{bmatrix} A_1^k & 0 \\ 0 & A_1^k \end{bmatrix} \text{ for } k = 1,2,...$$

and

$$A^{2k+1} = \begin{bmatrix} 0 & A_1^{k+1} \\ A_1^k & 0 \end{bmatrix} \text{ for } k = 0,1,...$$

We have

$$A^{2\upsilon_{A_1}-1} = \begin{bmatrix} 0 & A_1^{\upsilon_{A_1}} \\ A_1^{\upsilon_{A_1}-1} & 0 \end{bmatrix} \neq 0 \text{ cause } A_1^{\upsilon_{A_1}-1} \neq 0$$

$$A^{2\upsilon_{A_1}} = \begin{bmatrix} 0 & A_1^{\upsilon_{A_1}} \\ A_1^{\upsilon_{A_1}} & 0 \end{bmatrix} = 0 \text{ cause } A_1^{\upsilon_{A_1}} = 0$$

Hence $\upsilon_A = 2\upsilon_{A_1}$ for $A_0 = 0$.

The upper bound $\upsilon_A \leq 2\upsilon_{A_s}$ follows from the previous equalities and the proof of Lemma 2.

Theorem 5. The bilinear positive system with delays (7) is controllable to zero in υ_A steps if and only if one of the following equivalent condition is satisfied

i) the matrix A (defined by (17)) is nilpotent.

ii) the matrix $A_s = (A_0 + A_1)$ is nilpotent.

Moreover the sequence of inputs $u(i) \in R_+^m$ $i = 0,1,...,\upsilon_A - 1$ that steers the state of system to zero has the form $u(i) = 0$ for $i = 0,1,...,\upsilon_A - 1$.

Proof. The condition i) follows from Theorem 2 applied to the equivalent system without delays (9). The equivalence of the conditions i) and ii) follows from Lemma 2.

Remark 3. Note that the controllability to zero of (7) does not depend on the matrices B_{kj} and B_k for $k = 0,1; j = 1,...,m$.

Example 1. Consider the bilinear positive system with delays (7) with the matrices:

$$A_0 = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \tag{22}$$

The nilpotency indices of the matrices are $\upsilon_{A_0} = 3$ and $\upsilon_{A_1} = 2$ since

$$A_0^3 = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$A_1^2 = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The nilpotency index of the matrix

$$A = \begin{bmatrix} A_0 & A_1 \\ I_n & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 3 \\ 1 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

is equal $\upsilon_A = 5$ since $A^5 = 0$.

For $A_0 = 0$ we have $\upsilon_A = 2\upsilon_{A_1} = 4$ and for $A_1 = 0$ we obtain $\upsilon_A = \upsilon_{A_0} + 1 = 4$. The sane results we obtain from (21).

6. Concluding remarks

The necessary and sufficient conditions for the internal positivity of the bilinear discrete-time systems with delays have been established (Theorem 3). Solutions to positive bilinear discrete-time systems are derived (Theorem 4). The necessary and sufficient conditions for the controllability to zero of the positive bilinear systems with delays have been formulated and proved (Theorem 5). The reachability of the positive bilinear systems with delays can be investigated by the method proposed for positive bilinear systems without delays in [6]. Extension of these considerations for bilinear continuous-time systems is possible but is not trivial.

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Artykuł recenzowany