Wojciech CHLEWICKI, Tomasz ROGALA
POLITECHNIKA SZCZECIŃSKA, WYDZIAŁ ELEKTRYCZNY, INSTYTUT ELEKTRONIKI, TELEKOMUNIKACJI I INFORMATYKI

On the application of content adaptive mesh image representation

Wojciech CHLEWICKI, Ph.D.

Graduated from the Faculty of Electrical Engineering, Szczecin University of Technology (1999, specialty: Electronics and Telecommunications). He also obtained a M.Sc. degree in Biomedical Engineering from National Technical University of Athens and University of Patras in Greece (2001). He received his Ph.D. degree from the faculty of Electrical Engineering Szczecin University of Technology in 2005 year. Since October 2005, he has been with Szczecin University of Technology, as an Assistant Professor. He does research in the field of medical and industrial imaging.

e-mail: Wojciech.Chlewicki@ps.pl

Tomasz ROGALA, Ph.D.

Graduated from the Faculty of Electrical Engineering, Szczecin University of Technology (2002, speciality: Electronics and Telecommunications). He received his Ph.D. degree from the Faculty of Electrical Engineering at Szczecin University of Technology in 2005. His scientific interests concern analysis and processing of bioelectrical signals and pattern recognition algorithms. He is currently engaged in a research project concerning imaging of low conductivity structures (including biological tissues) using magnetic inductance tomography.

e-mail: Tomasz.Rogala@ps.pl

Abstract

In this article we focus on image representation using Content Adaptive Mesh Models (CAMM). We discuss the idea of image representation using the triangular mesh and limitations of this method. The performance of the method is evaluated with two sample images representative for biomedical applications: brain reconstruction from Positron Emission Tomography (PET) scanner and Shepp-Logan head phantom. The conclusion is that the CAMM approach may be very effective representation for image reconstruction, but the current version of the algorithm is inappropriate for very low contrast data, such as the Shepp-Logan phantom. The main conclusion is that the node placement scheme should be corrected to prevent excess concentration of nodes in unimportant regions of high contrast and shortage of nodes in low-contrast parts of the image. It is postulated that contrast stretching could be a possible solution to that limitation.

Keywords: image representation, mesh generation, nonuniform sampling

O zastosowaniu adaptacyjnych modeli siatkowych do reprezentacji obrazu

Streszczenie

W artykule opisano zastosowanie reprezentacji obrazu w postaci adaptującej się do jego zawartości siatki elementów trójkątnych (ang. *Content Adaptive Mesh Models – CAMM*). Przeanalizowano skuteczność takiego podejścia i ograniczenia metody na przykładzie dwóch obrazów testowych, typowych dla zastosowań biomedycznych: rekonstrukcji skanu mózgu z wykorzystaniem tomografii emisyjnej (PET) oraz tzw. fantomu Sheppa-Logana. Na podstawie uzyskanych wyników wnioskuje się, że wykorzystanie siatki elementów trójkątnych może być bardzo efektywnym sposobem reprezentacji obrazu, jednak w swojej obecnej wersji algorytm nie sprawdza się w przypadkach w których najistotniejsza część obrazu cechuje się niskim kontrastem. Zasugerowano zastosowanie kompresji kontrastu w celu przezwyciężenia tego ograniczenia.

Słowa kluczowe: reprezentacja obrazu, generacja siatek, próbkowanie nierównomierne

1. Introduction

An effective image representation plays an important role in many applications which process graphical information. A way the image is represented in the computers memory greatly influences the performance of the image processing algorithm. It also determines the amount of memory used for storage of the image.

In image reconstruction from projections, which is the main field of our investigations, the image (or volume) can be simply represented using pixels (2D) or voxels (3D) [1]. This is the most intuitive approach for image representation. However, it does not fulfill the requirements of image reconstruction algorithms due to the following reasons. Firstly, the projection value of the rectangular (cubic) image element depends on the angular orientation. This makes the pixels not suitable for the considered task since the projections are oriented around the object at different angle. Secondly, the frequency response of the rectangular window has worse properties compared to the frequency responses of other interpolation kernels, such as bilinear or Gaussian [2]. This causes a high level of noise in the images reconstructed with pixel based algorithms.

More sophisticated way of representing the image is using basis functions [2]. The basis functions can be of two different kinds – radially symmetric or piecewise polynomial. The great advantage of the radially symmetric basis functions is that their projection value is independent of the angular orientation. The most popular radially symmetric basis function recently utilized in tomography is based on the Bessel-Kaiser function [3]. This basis function is often called blob. Utilizing blobs has shown to give more accurate reconstructions with lower level of noise. The advantage of blobs is that their bandwidth can be easily changed by determining one parameter. Appropriate setting of this parameter allows optimal reconstruction in terms of noise suppression - contrast recovery trade off [4].

Blobs, however, have one disadvantage. Image reconstruction from projections can be perceived as solving the system of linear equations. The unknown are the image values whereas the known are the measured projection values. The transient matrix of that system describes the contribution of the image element to each particular projection value. Using blobs increases the computational demands because the transient matrix is less sparse due to overlapping nature of blobs.

There are alternative ways of making the transient matrix sparser, which have been described in the literature. One of the solutions is using content adaptive mesh model [5] instead of uniform sampling. The local frequency content in an image varies spatially (image signals are spatially nonstationary). Taking this into account the employment of nonuniform sampling seems to be an efficient way of representing the image. Therefore, the second approach seems to have great potential for the development of efficient image reconstruction algorithms.

The mesh representation has been shown useful in other areas of image processing. It has been recently used for tracking of image sequences [6]. It has been applied to image compression [7] as well as image interpolation [8]. Even though, this work is dedicated to the analysis of content adaptive mesh model from the point of view of its usefulness in tomographic reconstruction the results presented here are (to some extent) valid in other fields.

2. Methods

In mesh-based model of the image, the data are represented as a set of intensity values of known points, called nodes. The nodes are connected together forming the mesh of image elements. The value of arbitrarily chosen image sample is interpolated from the nodal values of its corresponding element using a set of functions (called shape functions) associated with the element. This concept is taken directly from the numerical method of solving partial differential equations (PDE's) called finite elements method (FEM). Therefore, additional information concerning the mesh construction may be found in appropriate popular FEM literature, e.g. [9]. Another loose analogy between FEM and CAMM can be made: in both methods partitioning of the problem domain allows simplification of computations. For PDE's it transforms a differential equation into the limited set of algebraic equations. In tomographic reconstruction tasks, the use of CAMM simplifies and speeds up the reconstruction algorithm [5], since all the computations are performed only for nodal values.

In mesh model, the sought function $f(x)$ (where x denotes 2-D or 3-D vector) is interpolated over each element using the values of the relevant nodes [10]:

$$f(x) = \sum_{i=1}^{n} f(x_i)\varphi_i(x_i) \tag{1}$$

where: N – number of nodes in the element, φ_i – shape function associated with i-th node, x_i – value of f in i-th node.
The interpolating functions are called shape functions. For any point x (vector in two or three dimensional space):

$$\sum_{i=1}^{N} \varphi_i(x) = 1 \tag{2}$$

The shape functions values in the nodes fulfill the following property:

$$\varphi_i(x) = \begin{cases} 1 \ \ for\ i-th\ node \\ 0 \ \ elsewhere \end{cases} \tag{3}$$

For 2-D triangular elements with linear interpolation (see figure 1) the shape functions are defined as follows [9]:

$$\begin{aligned} \varphi_1(x,y) &= \frac{1}{2\Delta}\left[(x_2y_3 - x_2y_3) + (y_2 - y_3)x + (x_3 - x_2)y\right] \\ \varphi_2(x,y) &= \frac{1}{2\Delta}\left[(x_3y_1 - x_1y_3) + (y_3 - y_1)x + (x_1 - x_3)y\right] \\ \varphi_3(x,y) &= \frac{1}{2\Delta}\left[(x_1y_2 - x_2y_1) + (y_1 - y_2)x + (x_2 - x_1)y\right] \end{aligned} \tag{4}$$

where Δ denotes the area of the element. Figure 1 presents the schematic view of a single element and its mapping back into the pixel domain (used for displaying the interpolated solution). The quality of the approximation depends on a measure called element quality. In general, the closer to the equilateral is the triangle, the better is the approximation.

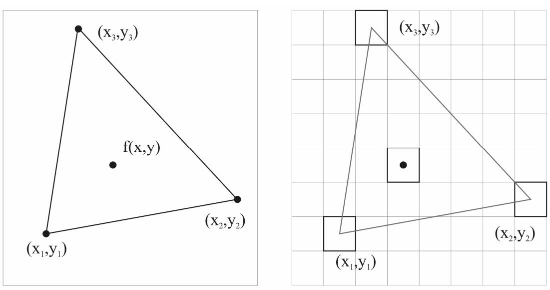

Fig. 1. The schematic view of a single element and its mapping back into the pixel domain

The first step conducted during mesh construction is the determination of the feature map. The feature map is an image obtained from the original image using a transformation which should detect significant features and surpass regions that contain less detail. As proposed by Yang et al [10] the feature map serves as a measure of the high frequency content in the particular area of the image. Such situation is further discussed using an example of biomedical imaging data, i.e. PET brain scan. The most significant information in that image - the presence of a tumor may have much lower contrast values than object boundaries and measurement noise. Another example considered here will be the Shepp-Logan phantom, in which most of the structures are of low contrast. In figure 3 the contrast has been increased for presentation purposes. In terms of grayscale values, the dots in the middle of the ellipse differ from the background by only 1%.

According to [10] the feature map $\sigma(x)$ is computed as follows:

$$\sigma(x) = \left[\max\left(\left|\frac{\partial^2}{\partial x^2} f(x)\right|, \left|\frac{\partial^2}{\partial x \partial y} f(x)\right|, \left|\frac{\partial^2}{\partial y^2} f(x)\right|\right)\right]^{\gamma} \tag{5}$$

where the parameter $\gamma > 0$ is used to adjust the sensitivity of the output image for edges in the input image. It is also claimed that the input image should be initially filtered using lowpass filter. Computation of directional derivatives involves using digital approximation of differentiation [10].

The feature map obtained in a way described above was used for determining the location of the nodes of triangular mesh. The number of nodes may be determined arbitrarily (considering the size of the image) or using statistical measure, as presented in [10]. In this paper it was assumed that the number of nodes should be equal to 5% of the number of pixels in the image.

The nodes were placed using Floyd-Steinberg (FS) dithering algorithm [11] with thresholding (to ensure given number of nodes). The algorithm originates from computer graphics. It is based on error dispersion. For any pixel in the image it finds the closest available color (e.g. black or white for 2-color dithering), and computes the difference between found color and original image. This difference is dispersed among the neighboring pixels according to given mask of weights. In cited works on CAMM it was used to determine the placement of mesh nodes. When the value of a pixel in the output image of FS algorithm exceeded assumed threshold, that pixel was chosen to be a mesh node.

The reconstruction accuracy was assessed using Peak Signal To Noise Ratio (PSNR) [10], defined as:

$$PSNR = 10\log\left(M \cdot N / \left\|\hat{f}(x) - f(x)\right\|^2\right) \tag{6}$$

where the denominator denotes vector norm of difference between original and reconstructed image.

3. Results

The CAMM models of two images typical for medical applications were created. The first one was a reconstruction from an 18F-fluoro deoxyglucose brain scan. The data were obtained using ECAT EXACT HR (Siemens/CTI, Knoxville, TN) Positron Emission Tomography (PET) scanner. The results are presented in figure 2. The plot a) is the original image in the pixel domain. Plot b) contains the location of the mesh nodes, obtained using the Floyd-Steinberg algorithm. Plot c) presents the triangular mesh, whereas the plot d) shows the image after conversion back into the pixel domain. It is clearly visible, that the mesh representation acts like a lossy compression – it suppresses small, low-contrast details. If these are only the effects of measurement noise (like in case of figure 2), all the important data are preserved.

For the image presented in figure 2, obtained PSNR equaled 55.54 dB. This value represents only the distortion introduced by the triangular representation. Any operations performed in triangular mesh domain would additionally decrease this value, introducing further errors.

However, there might be some situations when the small, low-contrast objects play crucial role during the diagnostic process. Popular artificial dataset representing such a case is called Shepp-Logan phantom. Figure 3 shows the mesh model and reconstructions for Shepp-Logan phantom, obtained analogously like in case of brain image.

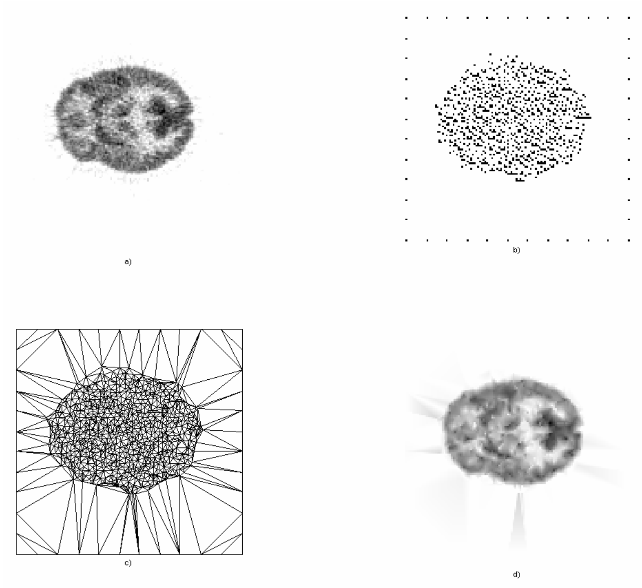

Fig. 2. Representation of PET brain reconstruction using triangular elements : a) tomographic reconstruction using Filtered Backprojection Algorithm; b) node placement; c) mesh structure; d) pixel reconstruction from the CAMM model

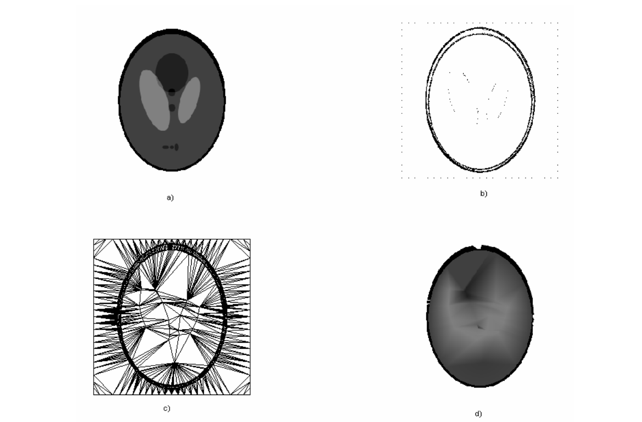

Fig. 3. Representation of Shepp-Logan phantom using triangular elements : a) original image; b) node placement; c) mesh structure; d) pixel reconstruction from the CAMM model

In both cases the number of nodes equaled to 5% of the total number of pixels in the input image. The ratio of PSNR measure for both images was summarized in table 1. The results suggest that the algorithm performed better for the Shepp-Logan phantom than for the brain image.

Tab. 1. Comparison of the PSNR ratio for both images

	whole image area	ROI (31x31 pixels)
PET brain	55 dB	39 dB
Shepp-Logan Phantom	67 dB	84 dB

However, the visual analysis of the image suggests contrary conclusion. Analysis of the plot 3c) explained the fact of the suppression of the details. The node-placement algorithm has put all the nodes on the boundary of the outer ellipse, since it produces the largest value of the feature map (due to high contrast). Relatively few nodes were placed in the interior area. This happened because of the low contrast values of the inclusions. The most significant regions of the image lie inside the triangles, and are therefore approximated by the linear functions. They loose their local character, being “smeared” all over the area of such a triangle. On the other hand, the PSNR measure is immune to the real significance of the image regions. Loosing the low contrast values (diagnostically important) does not lower the PSNR results as much as losing the high contrast parts (even if they are only the effect of noise). Therefore, another quality measure should be constructed for the images having similar character. Another problem is the triangle quality. The approximation of the inner values is the better; the more equilateral is the triangle [9]. The algorithm proposed by Yang et al [10] does not control this feature. Certain images may result in acceptable meshes, whereas the others may appear analogous to the Shepp-Logan case – contain many low quality elements. This error of approximation is added to the overall representation error. It would be advisable, if the algorithm could insert additional nodes when necessary and control the mesh quality.

4. Conclusions and future work

The CAMM model was analyzed in the paper. For both brain reconstruction and Shepp-Logan phantom their feature maps and Content Adaptive Mesh Models were created. Then the images were converted back into the pixel domain. The results were assessed using both numerical measure (PSNR) and visual inspection. The CAMM approach appeared successful for the brain image, due to the fact that it does not contain very low-contrast content of great significance. On the other hand, the mesh representation of the Shepp-Logan phantom looses crucial image elements. Additionally, it is not reflected in the numerical quality measure. The main conclusion is that the either CAMM approach should not be used for images with low-contrast regions of interest, or the node placement scheme should be corrected to prevent excess concentration of nodes in unimportant regions of high contrast and shortage of nodes in low-contrast parts of the image. It is postulated that contrast stretching could be a possible problem to that limitation. This is a matter of future studies.

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