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# The project and construction of group of wheeled mobile robots

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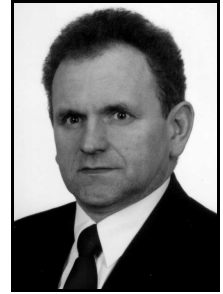
Jest pracownikiem w Katedrze Robotyki i Dynamiki Maszyn AGH. Jego zainteresowania skupiają się na mechatronice oraz wykorzystaniu sztucznej inteligencji. Jest autorem prac nad zastosowaniem powyższych technik do zadań związanych z modelowaniem, identyfikacją i sterowaniem mobilnymi robotami.



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Jest kierownikiem Katedry Robotyki i Dynamiki Maszyn, Akademii Górniczo-Hutniczej w Krakowie. W swoich pracach zajmuje się zagadnieniami dynamiki konstrukcji, a zwłaszcza ich analizy modalnej. Jego zainteresowania obejmują także układy aktywnej redukcji drgań, układy sterowania i szeroko pojętą mechatronikę. Jest autorem 15 książek i kilkuset artykułów dotyczących wspomnianych zagadnień.



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### Abstract

This work discusses the possibility of wheeled mobile robots design for cooperation in group of robots. The paper presents problems connected with the modelling of kinematics and dynamic motion equations derived on the basis of Maggi equations for a 2-wheeled mobile robot.

## Projekt i konstrukcja grupy mobilnych robotów kołowych

### Streszczenie

W artykule problemy związane z projektowaniem mobilnych robotów kołowych zdolnych do współpracy w grupie. Przedstawiono zagadnienia związane z modelowaniem kinematyki i dynamiki tego typu układów oraz przedstawiono rozwiązanie konstrukcyjne grupy mobilnych robotów kołowych.

## 1. Introduction

Now the main problem in robotics is to control more than 1 robot in the same time. We can distinguish swarm of robots and a group of robots. If we talk about a swarm it means that we have more than 100 robots to control. The group of robots consists of maximum 100 robots. In this paper we are trying to present wheeled mobile robot project capable of cooperating in a group. We decided to build 3 robots capable to communicate. Those robots can be used in indoor localization (fig. 1).

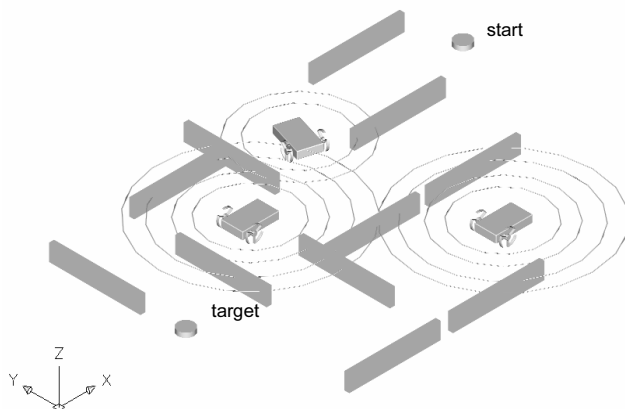


Fig. 1. The group of robots used in indoor localization

In those mechatronic systems we have several problems to solve. First problem is connected with communication between robots, also to find robots' position by themselves is an essential problem but it's not easy to solve. If we take into consideration a possible scenario for a group of robots we have (fig. 2):

### Scenario I

The Robots starting point is known  
The obstacle position is known  
The target position is known

### Scenario II

The Robots starting point is known  
The obstacle position is unknown  
The target position is known

### Scenario III

The Robots starting point is unknown  
The obstacle position is unknown  
The target position is known

### Scenario IV

The Robots starting point is unknown  
The obstacle position is unknown  
The target position is unknown

Fig. 2. The possible scenarios

In our project we assumed that the group of robots must be able to operate in all those scenarios from fig. 2.

## 2. The modelling of the kinematics of the 2-wheeled mobile robot

The analysis of the kinematics has been carried out for the model of a robot which has been shown in a schematic mode in fig. 3.

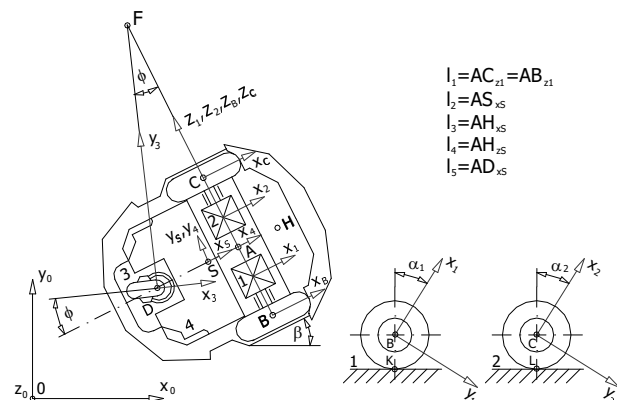


Fig. 3. Computational model of a mobile robot

The basic elements of the model are: wheel unit drive of wheels 1 and 2, self-adjusting supporting wheel 3 and the frame of unit 4.

When describing motion phenomena for a complex system such as mobile 2-wheeled robot, it is beneficial to attach the coordinates system to the particular robot's elements. System coordinates  $x_4 y_4 z_4$  have been attached to the element 4. The system coordinates  $x_0 y_0 z_0$  is motionless and based.

The systems of coordinates  $x_1 y_1 z_1, x_2 y_2 z_2$  have been attached to the unit drives 1 and 2. The next system of coordinates can be transformed to the previous coordinates system by means of three elementary translations and three elementary rotations, which is connected with the transformation matrix description. In order to describe a mobile robot, the kinematics equations for characteristic points have been applied with the use of Denavit-Hartenberg notation. After appropriate computations the kinematics equations for circular motion have been received in the following form (Giergiel, *et al.*, 2002):

$$\begin{aligned}
 \dot{x}_A - l_3 \dot{\beta} \sin(\beta) - R \dot{\phi} \cos(\phi) &= 0 \\
 \dot{y}_A + l_3 \dot{\beta} \cos(\beta) - R \dot{\phi} \sin(\phi) &= 0 \\
 \dot{x}_A - v_A \cos(\beta) &= 0 \\
 \dot{y}_A - v_A \sin(\beta) &= 0 \\
 \dot{x}_A - r_1 \dot{\alpha}_1 \cos(\beta) + l_1 \dot{\beta} \cos(\beta) &= 0 \\
 \dot{x}_A - r_2 \dot{\alpha}_2 \cos(\beta) - l_1 \dot{\beta} \cos(\beta) &= 0 \\
 \sqrt{l_5^2 \dot{\beta}^2 + v_A^2} - r_3 \dot{\alpha}_3 &= 0
 \end{aligned} \tag{1}$$

The above system of seven equations give possibility to calculate basic kinematics parameters under the condition that we know the velocity of the characteristic point A and the radius R of circular motion. Equations (1) are obligatory for circular motion, if we take into consideration an assumption that  $\beta = 0, \phi = 0, R = 0$ , then we receive equations for straight motion (Giergiel, *et al.*, 2002).

$$\begin{aligned}
 \dot{x}_A - V_A &= 0 \\
 \dot{x}_A - r_1 \dot{\alpha}_1 &= 0 \\
 \dot{x}_A - r_2 \dot{\alpha}_2 &= 0
 \end{aligned} \tag{2}$$

When designing a complex trajectory, it is necessary to analyze motion phenomena in separate intervals, practically there are straight and circular section.

### 2.1. The test rig of kinematics equations

Due to the nonlinearity of kinematics equations describing wheeled mobile robot it is necessary to make computer simulations of kinematics for example in Maple and Matlab environment. For our simulation we assumed the following parameters:

Tab. 1. The basic parameters for kinematics computations

$v_A$ [m/s]	$m_1$ [kg]	$m_2$ [kg]	$l_1$ [m]	$l_2$ [m]	$l_3$ [m]	$l_4$ [m]
0.150	0.800	0.800	0.163	0.140	0.060	0.700
$l_5$ [m]	$m_4$ [kg]	$r_1$ [m]	$r_2$ [m]	$r_3$ [m]	$R$ [m]	$\beta$
0.270	5.670	0.042	0.042	0.023	1.5	0-17°

Following the description above we assumed a trajectory for our robot and prepared time courses of velocity  $v_a$  and trajectory signal presented in fig. 4.

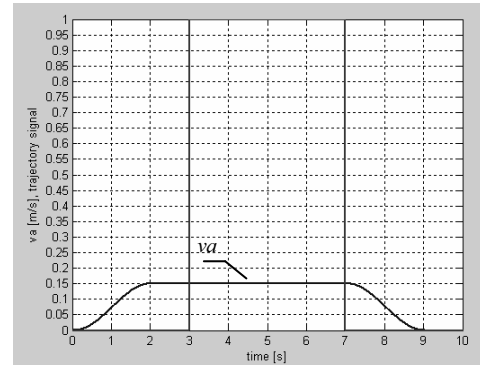


Fig. 4. The time courses of velocity  $v_a$  and trajectory signal

In Matlab-Simulink environment kinematics simulation system has been prepared and presented in fig. 5.

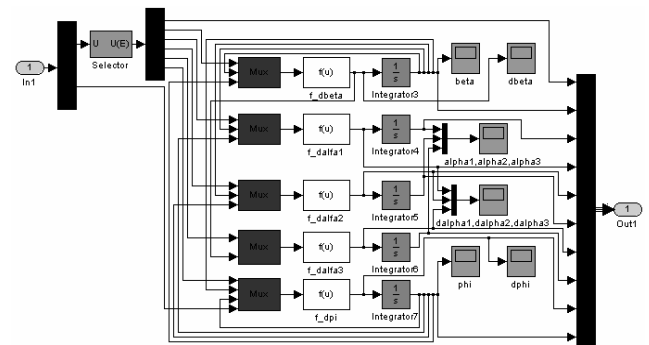


Fig. 5. The kinematic simulation system made in Matlab-Simulink environment

After test rig with the use of kinematics simulator (fig. 5) we received time courses of kinematics parameters. In the fig. 6 have been presented time courses of radius way for particular wheels.

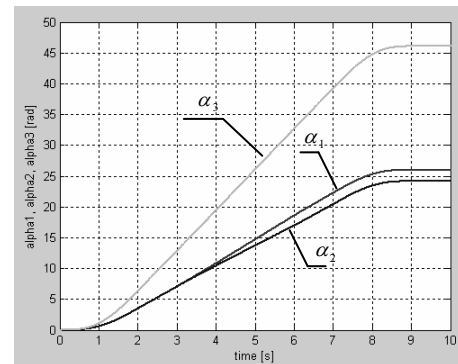


Fig. 6. The time courses of radius way for particular wheels

If we make operation of radius way derivative we receive time courses of angular velocity for particular wheels, presented in fig. 7.

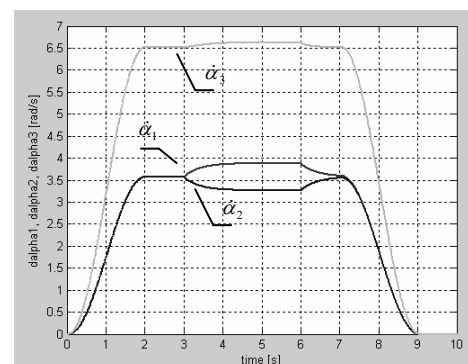


Fig. 7. The time courses of angular velocity for particular wheels

Another important kinematics parameter is connected with frame 4 and describing rotation  $\beta$  of this frame. The time courses of angular  $\beta$  and its derivative have been presented in fig. 8.

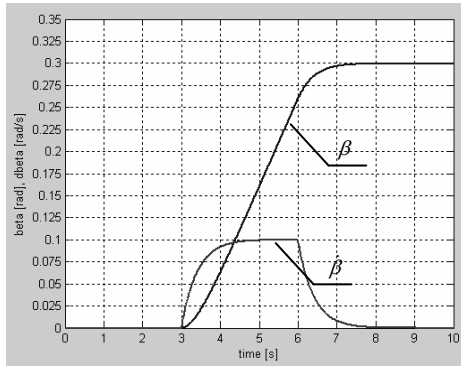


Fig. 8. The time courses of angle  $\beta$  and its derivative for particular wheels

Those kinematics parameters received after simulation of inverse kinematics will be used in dynamic simulations.

### 3. Modelling of the dynamics of the 2-wheeled mobile robot

As a result of symbolic computations of dynamic motion equations based on Maggi equations for the model taken have been received. In these equations the influence of the mass of the self-adjusting supporting wheel has not been taken into consideration (Żylski, *et al.*, 2002).

$$\begin{aligned} a_2 \ddot{\alpha}_2 + a_1 \ddot{\alpha}_1 + a_3 \dot{\alpha}_2^2 - a_3 \dot{\alpha}_1 \dot{\alpha}_2 &= M_1 - a_4 \operatorname{sgn}(\dot{\alpha}_1) \\ a_2 \ddot{\alpha}_1 + a_1 \ddot{\alpha}_2 + a_3 \dot{\alpha}_1^2 - a_3 \dot{\alpha}_1 \dot{\alpha}_2 &= M_2 - a_5 \operatorname{sgn}(\dot{\alpha}_2) \end{aligned} \quad (3)$$

where:

$$\begin{aligned} a_1 &= \frac{1}{4} \frac{r^2 m_4 l_1^2 + r^2 m_4 l_2^2 + r^2 I_{z_4} + 2 r^2 I_{x_1} + 4 r^2 m_1 l_1^2 + 4 I_{z_1} l_1^2}{l_1^2} \\ a_2 &= \frac{1}{4} \frac{-r^2 m_4 l_2^2 + r^2 m_4 l_1^2 - r^2 I_{z_4} - 2 r^2 I_{x_1}}{l_1^2} \\ a_3 &= \frac{1}{4} \frac{r^3 m_4 l_2}{l_1^2} \quad a_4 = N_1 f_1 \quad a_5 = N_2 f_2 \end{aligned} \quad (4)$$

however:

- $m_2 = m_1, m_4$  – mass of particular unit element.
  - $I_{x_2} = I_{x_1}, I_{z_2} = I_{z_1}, I_{z_4}$  – moments of inertia in relation to particular axis.
  - $M_1, M_2$  – drive moments.
  - $N_1, N_2$  – the pressure forces of particular wheel.
  - $f_1, f_2$  – rolling friction factors of wheels 1 and 2.
  - $\alpha_1, \alpha_2$  – angles of rotation wheels 1 and 2.
- Taking into consideration state variables as follows:

$$\alpha_1 = x_1, \dot{\alpha}_1 = \dot{x}_1 = x_2, \alpha_2 = x_3, \dot{\alpha}_2 = \dot{x}_3 = x_4 \quad (5)$$

the dynamic motion equations (1) have been written as (Żylski, *et al.*, 2002):

$$\dot{x} = Ax + B[f(x, a) + G(x, a)u] \quad (6)$$

where:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$G(x, a) = \begin{bmatrix} 0 & 0 \\ \frac{a_1}{a_1^2 - a_2^2} & -\frac{a_2}{a_1^2 - a_2^2} \\ \frac{a_2}{a_1^2 - a_2^2} & \frac{a_1}{a_1^2 - a_2^2} \end{bmatrix}$$

$$f(x, a) = \begin{bmatrix} 0 \\ 0 \\ \frac{a_2 a_5 \operatorname{sgn}(x_4) + a_2 a_3 x_2^2 - a_2 a_3 x_2 x_4 + a_1 a_3 x_2 x_4 - a_1 a_3 x_4^2 - a_1 a_4 \operatorname{sgn}(x_2)}{a_1^2 - a_2^2} \\ \frac{-a_1 a_5 \operatorname{sgn}(x_4) - a_1 a_3 x_2^2 + a_1 a_3 x_2 x_4 - a_2 a_3 x_2 x_4 + a_2 a_3 x_4^2 - a_2 a_4 \operatorname{sgn}(x_2)}{a_1^2 - a_2^2} \end{bmatrix}$$

Vector  $u$  stands for the driving moments  $M_1$  and  $M_2$  which can be available by measurement or which can be generated on the basis of the dynamic motion equations. This paper contains computations based on motor torques which have been received by means of computer simulations.

The form of the dynamic motion equations for 2-wheeled mobile robot as (6) allows for identification of the mathematical model of the robot. Driving moments of the wheels have been calculated solving the inverse dynamic problem for the assumed model of robot which moves with the velocity  $v_a$  (velocity of point A in fig. 4). In Matlab-Simulink environment dynamics simulation system has been prepared and presented in fig. 9 (Buratowski, *et al.*, 2002).

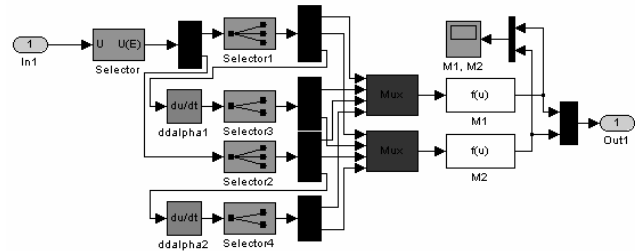


Fig. 9. The dynamics simulation system made in Matlab-Simulink environment

Exemplary time courses of driving moments for the movement model: starting, driving straight, driving on circular curve with turning axis of frame  $\beta$  and radius of wheels  $r$ , braking and construction data included in tables 1, 2 have been presented in fig. 10.

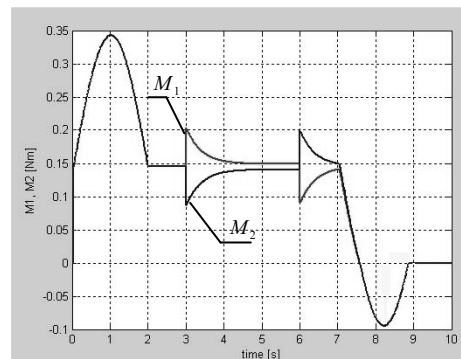


Fig. 10. The time courses of driving moments for particular wheels from simulation

Tab. 2. The basic parameters for dynamic computations

$I_{z1}[\text{kgm}^2]$	$I_{x1}[\text{kgm}^2]$	$N_1 [\text{N}]$	$f_1 [\text{m}]$	$I_{z2}[\text{kgm}^2]$	$I_{x2}[\text{kgm}^2]$	$N_2 [\text{N}]$
0.0005	0.0058	7.850	0.005	0.0005	0.0058	7.850
$f_2 [\text{m}]$	$I_{z4}[\text{kgm}^2]$	--	--	--	--	--
0.005	0.040	--	--	--	--	--

The kinematics parameters generated on the basis of a simulation and the driving moments were used to choose appropriate drives.

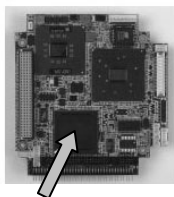
#### 4. The construction of mobile robot

On the basis of kinematics and dynamic simulations we decided to build a group of identical 3 2-wheeled mobile robots, presented in fig. 11.

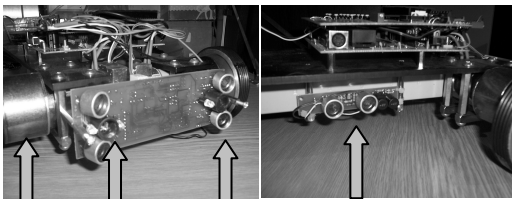


Fig. 11. The construction of mobile robot

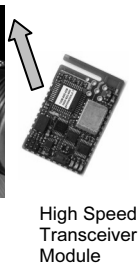
Our robot except for the physical layer is equipped with the sensor layer. This layer consists of infrared and ultrasonic sensors capable of collecting data about the surrounding environment including obstacles (fig. 12). Each robot will be equipped with a PC computer and High Speed Transceiver Module based on Bluetooth technology and it is a part of transmission layer in our project.



PC computer  
Modular PC104



DC Motor GM9434H127  
Infrared sensor  
The front ultrasonic sensor  
The side ultrasonic sensor



High Speed Transceiver Module

Fig. 12. The robots basic equipment

The group of robots' movement presentation and obstacle avoidance have been presented in fig. 13.



Fig. 13. The robots in indoor exploration

#### 5. Summary and conclusions

Each robot successfully passes the test connected with the movement on trajectory applied for simulation.

Another important problem for group of robots is cooperation. In this project we have been trying to find appropriate algorithms related with localization, communication and data exchange between each robot.

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#### 6. References

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