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TELECOMMUNICATIONS AND COMPUTER SCIENCE

# Multidimensional training of iterative image reconstruction algorithm

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### Abstract

Iterative image reconstruction methods are currently a matter of extensive research. These methods have usually parameters which have to be optimized. For instance, choice of appropriate values of such parameters in each iteration has a great effect on the performance of many iterative image reconstruction algorithms. The search for the optimal coefficients is usually done in "step-by-step" manner. Alternatively, multidimensional optimization of such coefficients was proposed in this paper. The trained algorithm was Row Action Maximum Likelihood Expectation Maximization (RAMLA). Controlled Random Search was employed during the training procedure. The experiments were carried out using mathematically defined phantoms and their projections. Simulation study using two "numerical observers" (training measure and evaluation measure) showed that higher reconstruction accuracy can be obtained when multidimensional optimization is applied. The initial results indicate the potential of using multidimensional optimization in training of the iterative image reconstruction algorithms.

### Streszczenie

Iteracyjne metody rekonstrukcji obrazu są obecnie przedmiotem intensywnych badań. Metody te mają przeważnie parametry, które należy zoptymalizować. Dla przykładu, dobór odpowiednich wartości współczynnika relaksacji w każdej iteracji ma znaczący wpływ na właściwości wielu algorytmów rekonstrukcji obrazów. Poszukiwanie optymalnych współczynników jest przeważnie wykonywane „krok po kroku”. Alternatywnie, w artykule przedstawiona została propozycja użycia wielowymiarowej optymalizacji współczynników. Trenowanym algorytmem był tzw. Row Action Maximum Likelihood Expectation Maximization (RAMLA). Podczas procedury treningu użyte zostały kontrolowane przeszukiwanie losowe. Eksperymenty zostały przeprowadzone z użyciem matematycznie zdefiniowanych fantomów oraz ich projekcji. Badania symulacyjne z użyciem dwóch „numerycznych obserwatorów” (miara treningowa oraz miara oceniająca) pokazały, że wyższa dokładność rekonstrukcji może być uzyskana przy stosowaniu optymalizacji wielowymiarowej. Wstępne wyniki wskazują potencjał, który daje optymalizacja wielowymiarowa przy treningu iteracyjnych metod rekonstrukcji obrazów.

## 1. Introduction

Inversion of the Radon transform, known as image reconstruction from projections is a problem arising in many fields of science, medicine and technology. The image reconstruction algorithms were initially designed for medical imaging purposes. However, these methods are becoming more and more useful in industrial imaging. In particular, the reconstruction methods have found their applications in industrial process control and non-destructive industrial testing.

Reconstruction of an image or volume from projections is in fact the problem of retrieving a planar or spatial distribution of certain

physical entity. Depending on the imaging medium it could be e.g. linear attenuation coefficient of X-rays. Recovery of its distribution requires measuring multiple line or area integrals, called projections.

There are two basic approaches to the reconstruction problem: analytical and algebraic approach. Analytical methods are based on determining a continuous formula for inversion of the Radon transform. Such formula is discretized during the process of implementation. In iterative methods, on the other hand, the detector system as well as the reconstructed volume is considered discrete from the beginning.

Iterative image reconstruction methods are currently a matter of extensive research. Such algorithms are capable of producing higher quality images than analytical methods. The iterative methods, however, have usually parameters which have to be optimized. Problem of task specific optimization of the free parameters in iterative image reconstruction algorithms is one of the drawbacks which causes problems in implementation of this kind of algorithms in commercial scanners.

One of such parameters is relaxation coefficient which has great influence on the speed of convergence of the algorithm as well as image quality. Seeking the optimal values of such parameter is called training. Because setting the same value of the relaxation coefficient in each iteration does not ensure satisfactory results the optimal coefficient has to be found for each iteration. This can be done in "step-by step" manner as reported by Herman and Meyer [1]. The authors also pointed out that the second important aspect determining the performance of iterative image reconstruction algorithm is an order of projections applied during the process of iteration.

In this article employing multidimensional global optimization during the training process was proposed. The usefulness of the proposed methodology was assessed through numerical evaluation based on simulated projection data.

## 2. Row Action Maximum Likelihood

The algorithm of which relaxation coefficient was optimized in this study was Row Action Maximum Likelihood Algorithm (RAMLA) of Browne and De Pierro [2]. It is defined by the following correction formula

$$c_j^{(k+1)} = c_j^{(k)} + \lambda^{(k)} c_j^{(k)} \left( \frac{P_i}{\sum_{n=1}^J a_{i,n} c_n^{(k)}} - 1 \right) a_{i,j}$$

where  $c_j$  represents image coefficient and  $p_i$  denotes the projection element. The coefficients  $a_{ij}$  in the system matrix represent the contribution of  $j$ -th image coefficient to  $i$ -th ray-integral (projection).

In principle, iterative image reconstruction algorithm consists of series of subsequent forward/backward projections until convergence. Several projection/backprojection operators have been proposed in the literature. Splatting [3] with radially symmetric basis functions was employed here to perform projection and backprojection operations. This operator was proved to be fast and to provide high accuracy. The projections were ordered according to Weighted Distance Scheme (WDS) [4]. This algorithm groups the projections in a manner which maximizes the angular distance of a newly selected projection with respect to an extended sequence of previously applied projections. This results in more accurate images with less noise like artifacts. WDS was originally designed for Algebraic Reconstruction Techniques (ART) [1]. It should be mentioned that RAMLA is similar to ART, i.e. both are row action algorithms. Therefore, using WDS with RAMLA seems to be appropriate.

### 3. Optimization and evaluation procedure

Optimization was carried out using two randomly generated phantoms. Each phantom contained five elliptical objects (hot and cold spots) placed inside the uniform background of the circular shape. The images of the phantoms are presented in Figure 1. The projections of such phantoms were simulated using dedicated software [5].

A set of 45 projections uniformly distributed around 180 degree was generated. The projection for each angular orientation consisted of 65 elements. Accordingly, the size of the reconstructed image was 65x65.

Normalized root mean square error between the original and the reconstructed image coefficients was chosen as the training measure. The measure is expressed in the following way

$$NRMS = \sqrt{\frac{\sum (\bar{c}_j - c_j)^2}{\sum (c_j - \bar{c}_j)^2}}$$

where  $\bar{c}_j$  and  $c_j$  are the image coefficients of the reconstructed and the digitized phantom respectively.  $\bar{c}_j$  is average value of the image coefficients of the original phantom. It was computed inside randomly chosen regions of interest.

More detailed information about using Controlled Random Search [6] procedure in image reconstruction from projections can be found in [7] as well as in [8]. The studies with Controlled Random Search was previously described in [9].

The quality of the obtained reconstructions was assessed by structural inaccuracy. It was computed using the formula below

$$STRINACC = \frac{1}{B} \sum_{b=1}^B |\bar{c}_b - c_b|$$

where B denotes the number of image coefficients inside region of interest. On the other hand this measure was computed for all the image coefficients within the original and the reconstructed phantom.

### 4. Results

Figures 2 and 3 present optimal relaxation coefficients for phantom 1 and 2 respectively. In "step-by-step" optimization the coefficients are selected as to minimize the training measure for each iteration. However this does not mean that the optimal set for the first or the second iteration will be the most appropriate when the algorithm stops after five iterations. This can be seen when observing the curves for the multidimensional training. They are of a different shape. In this case all the coefficients are set as to minimize the training measure after all five iterations.

The superiority of the second approach can be seen in Table 1. Structural inaccuracy for multidimensional training is lower in both cases.

Tab. 1. Structural inaccuracy of the reconstructed phantoms.

Phantom	Structural inaccuracy	
	Step-by-step training	Multidimensional training
1	0.0487	0.0420
2	0.0334	0.0308

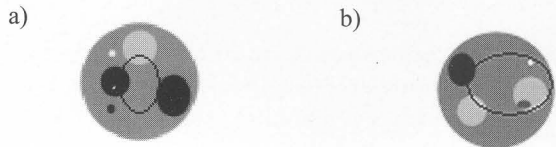


Fig. 1. Images of two randomly generated phantoms. Size and position of each region of interest was randomly generated as well.

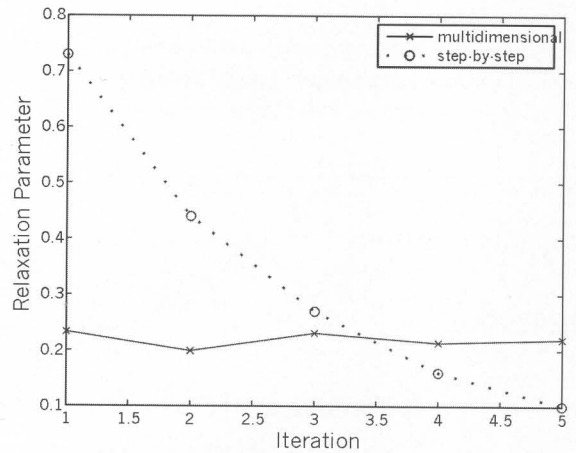


Fig. 2. Optimal relaxation parameters for RAMLA based on the first phantom.

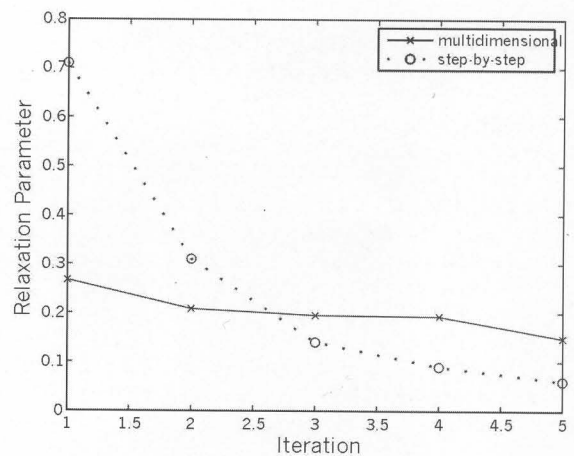


Fig. 3. Optimal relaxation parameters for RAMLA based on the second phantom.

### 5. Conclusions and future work

Multidimensional global optimization during the training stage of the iterative image reconstruction algorithm was proposed in this study. Initial comparative evaluation shows that optimizing the coefficients for each iteration provides better performance of the reconstruction algorithm than the "step-by-step" optimization.

More sophisticated statistical analysis of the usefulness of the proposed training is required. Further study will include



assessment of statistical significance of the presented initial results. This will be done based on large number of randomly generated phantoms as presented in [10].

## 6. References

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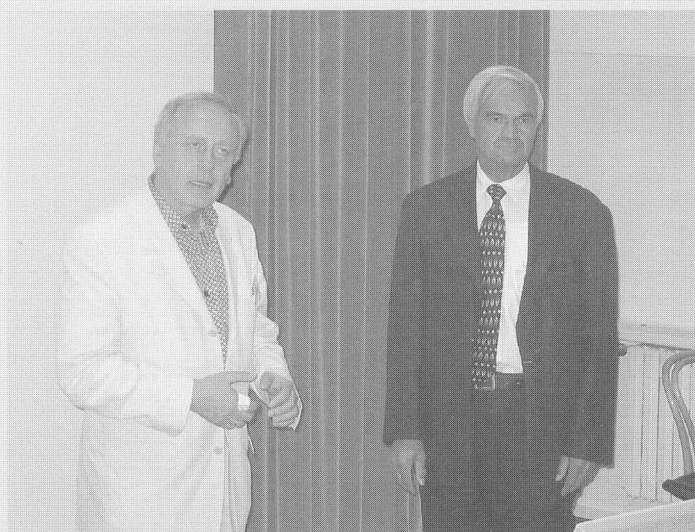
**Tytuł:** Wielowymiarowy trening iteracyjnego algorytmu rekonstrukcji obrazu.

*Artykuł recenzowany*

## INFORMACJE

20-24.06.2005 Politechnika Warszawska

### Szkoła Letnia "Podstawy Konstrukcji, Technologia i Aplikacje Mikroczujników i Mikrosystemów"



Przewodniczący prof. R. Jachowicz anonsuje wykład prof. H. Meixner'a z Siemens AG, Niemcy

W dniach 20-24 czerwca 2005 odbyła się międzynarodowa tygodniowa Szkoła Letnia pod nazwą „Podstawy Konstrukcji, Technologia i Aplikacje Mikroczujników i Mikrosystemów“. Szkoła była organizowana przez Centrum Doskonałości „Centre of Microsystems, Design and Technology - COMBAT“ w Politechnice Warszawskiej.

W Szkole Letniej, wzięło udział 56 uczestników z całej Polski i z Finlandii. Zajęcia były prowadzone w języku angielskim.

Wykłady były prowadzone przez wielu niezwykle doświadczonych i znakomych wykładowców z zagranicy i z kraju a w tym 2 profesorów Niemiec, 3 profesorów z Węgier, 1 z Holandii i 1 z Hiszpanii oraz z 14 wykładowców krajowych (w tym 1 z Politechniki Wrocławskiej, 1 ze Śląskiej, 2 wykładowców z Instytutu Technologii Elektronowej i 10-ciu profesorów z Politechniki Warszawskiej).

W programie Szkoły należy wyróżnić dwie zasadnicze części. W pierwszej części (2 dniowej) zapoznano słuchaczy z podstawowymi konstrukcjami mikroczujników i mikrosystemów, najbardziej rozpowszechnionymi procesami technologicznymi niezbędnymi do ich wytwarzania oraz z zasadami i metodami modelowania mikrostruktur zarówno w zakresie elektrycznym, mechanicznym, chemicznym jak i cieplnym. Druga część Szkoły (3 dniowa) pozwoliła słuchaczom poznać najnowsze na świecie rozwiązania konstrukcyjne i systemowe mikrosensorów, mikrośiowników i mikrosystemów stosowanych w medycynie, w samochodach, w przemyśle i w badaniach naukowych.

Wszyscy uczestnicy otrzymali Certyfikat dokumentujący ukończenie Szkoły. Szkoła Letnia uzyskała bardzo wysoką ocenę jej uczestników.