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Enhancement of ordered subsets iterative image reconstruction methods

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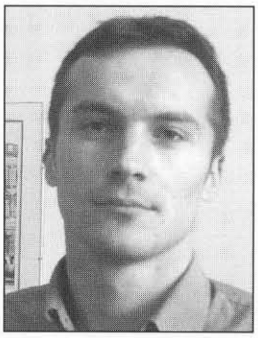

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Abstract

Using ordered subsets is an efficient way of accelerating iterative image reconstruction techniques. Ordered Subsets – Expectation Maximization (OS-EM) is recently a very popular reconstruction method and is incorporated into many imaging systems particularly in emission tomography. Compared to the most popular reconstruction algorithm – Filtered Backprojection (FBP), it is more computationally demanding but in many cases it outperforms FBP in terms of image quality. This work aimed at increasing a rate of convergence and accuracy of the OS-EM as well as reducing its overall computational cost. This was achieved by utilizing the weighted distance projection access scheme originally proposed for ART and using spherically symmetric image elements (blobs) for image representation. Evaluation based on synthetic projection data showed the improvement of the image quality of the obtained reconstructions and increase in the convergence speed.

Streszczenie

Użycie uporządkowanych podzbiorów rzutów jest wydajnym sposobem przyspieszania iteracyjnych technik rekonstrukcji obrazów. Metoda maksymalizacji wartości oczekiwanej z uporządkowanymi podzbiorami projekcji (OS-EM) jest obecnie bardzo popularnym algorytmem rekonstrukcji i jest stosowana w wielu systemach obrazowania, zwłaszcza w tomografii emisyjnej. W porównaniu do najbardziej popularnego algorytmu – wstecznej projekcji filtrowanych rzutów (FBP), OS-EM jest bardziej wymagający obliczeniowo, ale w wielu przypadkach przewyższa FBP w kategoriach jakości obrazu. Celem pracy było zwiększenie zbieżności oraz dokładności metody OS-EM, a także zredukowanie jej złożoności obliczeniowej. Zostało to osiągnięte poprzez użycie schematu dostępu do rzutów z ważonym dystansem (WDS) oryginalnie zaproponowanym dla metody ART oraz użycie sferycznie symetrycznych elementów obrazu (blobów) do reprezentacji obrazu. Ocena bazująca na symulowanych rzutach wykazała poprawę jakości uzyskanych rekonstrukcji oraz przyspieszenie zbieżności algorytmu.

I Introduction

Image reconstruction from projections (IRP) is an interdisciplinary field of research combining variety of subjects. The design and implementation of image reconstruction algorithms involve techniques from electrical engineering (particularly, signal processing), computer science (data structures, software engineering), physics (modeling of radiation transport and detection processes), mathematics (functional analysis, optimization, numerical analysis), and statistics (random processes, statistical estimation theory) [1]. It is predominantly utilized in medical imaging applications such as computed tomography (CT), positron emission tomography (PET), single photon emission tomography (SPECT), 3D digital angiography (DA), and microscopy, IRP was found useful also in nondestructive evaluation, electrical capacitance tomography, electron paramagnetic resonance imaging (EPRI), and few other industrial imaging techniques. Surprisingly, tomographic algorithms have been even applied to study geophysical phenomena, i.e. in ocean acoustic tomography and whole-earth imaging.

In general, IRP problem is to recover the spatial distribution of a certain entity based on its projections measured at different angular orientations. Strictly speaking, a projection at a given angle is the line integral of the image in the direction specified be that angle. In other words by the projection it is meant the information derived from the transmitted (emitted) energies when an object is illuminated (or transmits energy) from a particular angle. The first mathematical formulation concerning a function reconstruction from its projections has been introduced by Radon as early as in 1917. Presently, the field of IRP is rapidly emerging.

Filtered Backprojection (FBP) is currently the most common IIR algorithm. Inasmuch as it is linear, fast and robust the method is generally accepted despite producing images with streaky artifacts. FBP is noise sensitive; therefore image quality of the images obtained using that algorithm may be limited. This particularly concerns nuclear medicine where the projections are very noisy and 3D digital angiography due to extremely limited number of available projections.

An alternative to methods based on backprojection of filtered data is the Iterative Image Reconstruction (IIR) approach. IIR have been proven to produce relatively higher quality reconstructions than obtained with FBP, particularly in terms of noise reduction and artifacts. The iterative methods are generally more computationally demanding then FBP. However, constant technological development makes IIR promising solution even for computationally intensive tasks such as volumetric imaging [2].

Several iterative reconstruction methods have been proposed. Among them, two main groups can be distinguished: algebraic methods – Algebraic Reconstruction Techniques (ART), Simultaneous ART (SART), Multiplicative ART (MART), Simultaneous Iterative Reconstruction Technique (SIRT) [3] and statistical methods: Maximum Likelihood – Expectation Maximization (ML-EM), Ordered Subsets EM (OS-EM), Weighted Least Squares (WLS), Image Space Reconstruction Algorithm (ISRA), Space Alternating Generalized Expectation Maximization (SAGE) [1], and many others. The methods within the former group are usually used in CT whereas the algorithms from the latter group are utilized in nuclear medicine.

There are common aspects for all the IIR algorithms. One of them is an image representation that is related to forward and backprojection operators. These on the other hand are two basic constituents of any IIR algorithm having a great effect on speed and accuracy. Enhancements introduced for one algorithm may be incorporated into another one even when the methods are not members of the same group. Such situation happened here where the Weighted Distance Projection Access Scheme (WDS) [4] introduced for ART was incorporated into OS-EM.

Thus, the purpose of this work was to realize and numerically evaluate the application of WDS into OS-EM. Additionally using more sophisticated than traditional pixels image elements was assessed with emphasis on efficient implementation of forward and backward projection.

II Principles of the iterative reconstruction methods

Usually in IIR (opposed to analytical approach) the detector system so as the projection data are considered discrete. Taking into account that the resulting image function is represented by a set of numbers IIR problem can be formulated as solving a system of linear equations

$$P = A \cdot F \tag{1}$$

where P is the projection data matrix, F is the image matrix and A is the transfer (system) matrix describing the relation between P and F. The A matrix depends on factors such as geometry of the imaging system, the detector efficiency etc. Specifically, in emission tomography the coefficients of the transfer matrix represent the probability of the emission from a particular place in the image and its acquisition in a chosen detector (detector pair in PET). In transmission tomography these coefficients represent the contribution of a particular image element to the total attenuation measured in the detector element.

There is a plethora of inversion formulas derived for solving such a system. They are based on different assumptions about the imaging modality properties. In emission tomography, for example, the Poisson likelihoods or data-weighted least-squares criterions are utilized. ML-EM takes into account the Poisson nature of a radioactive decay. OS-EM is an accelerated version of ML-EM, which is commonly applied to the imaging system software in PET and SPECT. This method provides a reconstruction imposing a natural positivity condition, which is not always the case for additive schemes such as ART. The OS-EM formula is as follows [5]

$$\lambda_j^{(n+1)} = \frac{\lambda_j^{(n)}}{\sum_{d \in D_k} a_{dj}} \sum_{d \in D_k} \frac{p_d a_{dj}}{\sum_{j'=1}^{J} \lambda_{j'}^{(n)} a_{dj'}} \tag{2}$$

where λ_j represents mean counts at image element j, p denotes the acquired number of counts at detector element d, D_k represents a partition of the projection space into m subsets, $n = 0,1,2,3...$ is the subiteration number and $k=n \bmod m$. A cycle of m-subiterations constitutes a complete iteration. The coefficients a_{dj} in the system matrix represent the probability of emission in the image element j being detected in detector unit d.

III Image representation

An intuitive way of representing digital images is through dividing the image into set of rectangular elements. Thus the object function has a constant value within each element (pixel). The discontinuous nature of that representation and the fact that the pixel projection depends on the angular orientation of the ray-line made that approach not suitable for IIR.

The development of algebraic methods resulted in other approaches for image representation. One of them is using so called basis functions, i.e. spatially limited elements that may overlap. Mathematically this type of representation is expressed by the formula below [1], [6]

$$\hat{f}(x,y) = \sum_{j=1}^{J} c_j b_j(x - x_j, y - y_j) \tag{3}$$

where $\{(c_j)\}$ is a set of coefficients of the image representation and $\{(x_j,y_j)\}$ is a set of J points in 2D space that are the nodes of a uniform grid over a region of the space. In this work a Cartesian grid was utilized. However, other grids were proven to give satisfactory results and could be used as well.

Using the image representation presented above the line integral p_d of f along the line d can be computed in the following way

$$\hat{p}_d = \sum_{j=1}^{J} a_{dj} c_j \tag{4}$$

where a_{dj} is the line integral, along the line i, of the shifted basis function with the center in (x_j, y_j).

The core of every projection (backprojection) process can be realized in several ways. It can be computed in spatial domain (ray casting, splatting) or in Fourier domain. In the work of Mueller et al. [7] the already described continuous representation of discrete images has been incorporated into projection/backprojection operators called splatting. Apart from high accuracy those operators can be sufficiently fast so that they are practical even in volumetric tomography. The implementations of splatting are footprint based what means that the blob projection values are preintegrated and stored in the computer memory. Storing preintegrated projection values has several advantages [7]: i) the ray integrals are calculated very accurately, since each footprint table entry can be integrated analytically or with good quadrature, ii) the complexity for interpolation is reduced, i. e. fast incremental algorithms can then be used to index the footprint tables in image space (in ray-driven splatting) or projection space (in voxel-driven splatting), iii) kernels with superior frequency characteristics such as Bessel-Kaiser function can be used despite their computational complexity. The computational speed up is additionally achieved through various kinds of caching depending on the correction scheme and the available amount of memory.

Among the basis functions proposed, the family of Bessel-Kaiser functions (blobs) introduced to the field of image reconstruction by Lewitt [6] has been proven to be a favorable choice. The blob is defined in the following way

$$b(m,\alpha,a;r)=\frac{I_m(\alpha\sqrt{1-(r/a)^2})}{I_m(\alpha)}\left(\sqrt{1-(r/a)^2}\right)^m \qquad (5)$$

where r is the radial distance from the blob center ($0\leq r\leq a$), I_m denotes the modified Bessel function of order m, a determines the extent (radius) of the blob, and α is a parameter controlling the blob shape. The most useful properties of blobs are: (i) they are spatially and band limited with possibility of easy tuning, (ii) there is an analytical formula for computing their projection, (iii) their frequency characteristics, which can be also expressed analytically, have more suitable properties for IIR compared to other basis functions.

Two blobs were considered in this study. The first one had radius 2 (pixel size unit) whereas the extent of the second one was 3. For both blobs the? parameter was set in such a way as to place the first minimum of the frequency characteristic close to the grid sampling frequency (see Fig. 2). That ensured the highest possible accuracy of the constant function representation. Spatial plots of the blobs already described are presented in Fig. 1.

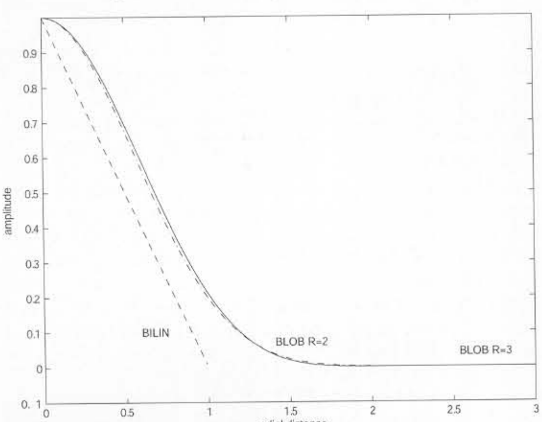

Figure 1: Spatial plots of the basis functions considered in this study: BILIN – bilinear element, BLOB R=2 – blob of radius r=2 and α=10.80, BOLB R=3 – blob of radius r=3 and α=27.58.

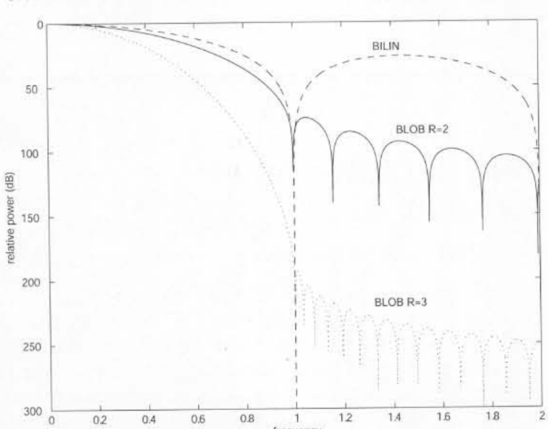

Figure 2: Frequency characteristics of the basis functions considered in this study: BILIN – bilinear element, BLOB R=2 – blob of radius r=2 and α=10.80, BOLB R=3 – blob of radius r=3 and α=27.58.

IV The projection ordering scheme

Hudson and Larkin [5] in their original work describing OS-EM mentioned that the order in which projections are processed is arbitrary, though it may be advantageous to the quality of the reconstruction provided to choose a special order. In the simulation study presented by the authors the projections were ordered in opposing pairs. For the number of 64 projections they utilized the following order: 0^0, 90^0, 45^0, 135^0, 22.5^0, 112.5^0, 67.5^0, etc. Yet, a general projection selection algorithm was not specified.

The WDS projection-ordering method is based on two postulates which ensure minimizing correlation in projection access. The postulates are [7]: (i) a series of subsequently applied projections is evenly distributed across a wide angular range, (ii) at no time is there an angular range that is covered more densely than others. While all of the existing projection ordering algorithms tend to be strong in one of the two aspects WDS compromises both postulates in a weighted manner. It is also important that the projection selection scheme provides a smooth transition between iterations. This is achieved by including projections applied in previous iteration, which are stored in the buffer. In conclusion, the WDS maintains a large angular distance among the whole set of used projections and prevents clustering of projections around a set of main view orientations.

V Numerical simulation results

The simulation study was performed using 90 parallel projections of the Shepp-Logan head phantom [3] uniformly distributed within 180-degree arc. The detector size as well as the image size was equal to 129. The set of projections ordered using WDS was grouped into 18 subsets. The discrete footprint table had 300 subdivisions. Apart from high accuracy and smaller computation cost of ray driven splatting over pixel driven splatting the former seemed to be more suitable for OS-EM that is projection oriented. The caching scheme on a ray level (see Mueller et al. [7]) was used in the ray driven splatting for the purpose of additional gain in speed.

The accuracy versus number of iteration characteristics is presented in Fig. 3. It is expressed in term of NRMS error between the pixels inside the discretized and the reconstructed Shepp-Logan phantom [3] excluding the exterior high contrast shell, which represents a skull. Considering the rate of convergence much improvement can be observed (Fig. 3) for OS-EM with WDS. That happened for all the image elements used in this study. Visual inspection of the images (Fig. 4) reveals similar positive influence of WDS on the OS-EM reconstructions compared to the sequential projection access scheme (SAS). The presented results are in line with those obtained by Mueller et al [3] for ART. This happens when the number of subsets is large but it is supposed (and was actually experimentally confirmed) that with decreasing number of subsets that influence would diminish.

It is clearly seen in Fig. 4 that the low contrast structures of the phantom reconstructed using bilinear interpolation kernel is not visually appealing independently of the projection ordering method. This is not surprising when one analyses the frequency plots (Fig. 2). The high frequency content is not sufficiently suppressed by the bilinear interpolation, resulting in noise occurrence. Another way of decreasing the noise in subsequent iterations is using various kinds of regularization. That issue has been recently studied by Chlewicki et al. in [8].

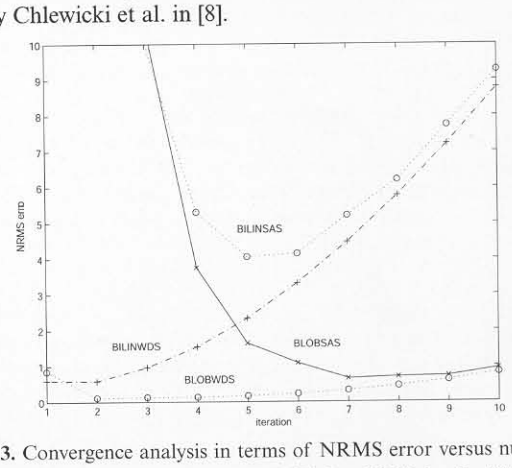

Figure 3. Convergence analysis in terms of NRMS error versus number of iterations. The symbols denote: BILIN-SAS – OS-EM with bilinear elements using SAS, BILIN-WDS – OS-EM with bilinear elements using WDS, BLOB-SAS – OS-EM with blobs (r=2, α=10.8) using SAS, BLOB-WDS – OS-EM with blobs (r=2, α=10.8) using WDS.

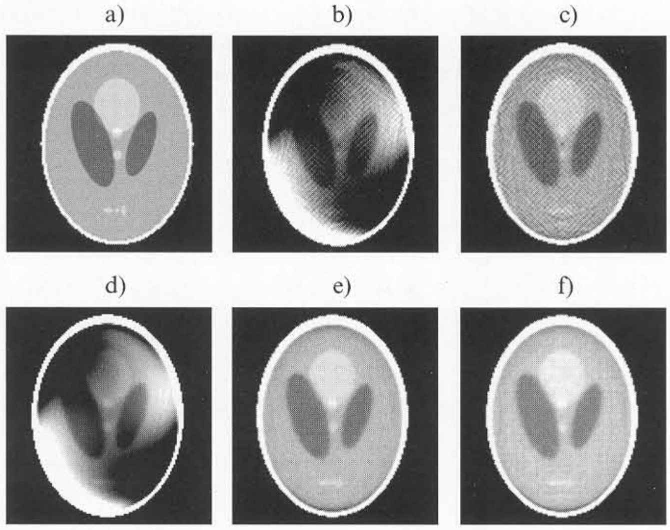

Figure 4. The images of the original Shepp-Logan head phantom and its reconstructions using the various options of OS-EM (2 iterations): a) original phantom, b) OS-EM with bilinear elements using SAS, c) OS-EM with bilinear elements using WDS, d) OS-EM with blobs (r=2, α=10.8) using SAS, e) OS-EM with blobs (r=2, α=10.8) using WDS, f) OS-EM with blobs (r=3, α=25.78) using WDS.

VI Conclusions and future work

The performance of OS-EM with blobs in which the projections were ordered using WDS was evaluated in this work. WDS exhibits more uniform projection access space sampling and applied to OS-EM improves its convergence rate. Using splatting with blobs was proven to be an efficient way of implementing accurate forward and backward projection operators. This was confirmed numerically as well as through visual inspection. Ordered subsets iterative methods with blobs, which make use of WDS projection-ordering algorithm, seem to be efficacious approach in various image reconstruction tasks.

While the OS-EM produces the reconstructions which quantitatively feet the acquired data it is not always possible to establish relevant diagnosis based on the visual inspection of the images. For instance, restrictions on the allowed dose of radioisotope used or the acquisition time constraints may cause the low signal-to-noise ratio (SNR) in the projection data due to insufficient number of counts. It is postulated here that a very useful solution to tackle this problem would be using an artificial intelligence algorithm [9] together with the task specific optimization of the reconstruction method. This is, however, a matter of future investigations.

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Tytuł: Rozszerzenie uporządkowanych podzbiorów iteracyjnych technik rekonstrukcji obrazów.

Artykuł recenzowany

RECENZJE

MEDICAL IMAGE UNDERSTANDING TECHNOLOGY

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W książce Autorzy przedstawili nowe, unikalne podejście do analizy obrazów, ze szczególnym uwzględnieniem zastosowań w medycynie. Jako pierwsi przedstawili bardzo dojrzałą koncepcję automatycznego rozumienia obrazów. Rozumienie obrazów jest według Autorów kolejnym po przetwarzaniu, analizie i rozpoznawaniu, etapem w procesie komputerowej obróbki obrazu. Autorzy podjęli w książce niezmiernie trudny i ważny problem automatyzacji interpretacji obrazów medycznych. To nowe podejście przedstawione jest w książce w sposób przejrzysty i bardzo dydaktyczny. Na bazie znanych pojęć z zakresu przetwarzania, analizy, segmentacji i klasyfikacji obrazów Autorzy wprowadzili pojęcia związane z technologia zrozumienia obrazów, ilustrując je przykładami z zakresu zastosowań w medycynie. Technologia przedstawiona w książce umożliwia rozwiązywania problemów analizy obrazów dla przypadków do tej pory bardzo trudnych lub nawet niemożliwych do automatyzacji. Taki przypadkami były specjalnie kontrastowane obrazy rentgenowskie szczególnie trudnego diagnostycznie narządu, jakim jest trzustka (tak zwane obrazy ERCP), a także inne obrazy medyczne z tomografii komputerowej, koronografii serca czy urografii układu moczowego człowieka. Obrazy te cechuje duża zmienność osobnicza, oraz warunkowana zmiennością form i zakresu patologii niepowtarzalność diagnostycznie ważnych cech obrazu. Autorzy przytoczyli przykłady obrazów, które dla danej (identycznej) jednostki chorobowej różniły się znacznie w zależności od cech morfologicznych badanego pacjenta. Problem rozpoznawania obrazów w tych przypadkach jest bardzo trudny, a w wielu przypadkach niemożliwy do rozwiązania metodami klasycznymi.

Autorzy zaproponowali nową technologie rozwiązania tego typu problemów nazwana przez nich technologię rozumienia obrazów. Technika ta nawiązuje do metod lingwistyki matematycznej i metod opisywania struktury obrazu z wykorzystaniem specjalnych gramatyk oraz dostosowanych do nich parserów. Według mnie autorzy dokonali znacznego postępu w dziedzinie rozpoznawania obrazów, stworzyli nową jakość w tych technikach, która może być wykorzystana do rozwiązywania wielu problemów w innych dziedzinach, wszędzie tam gdzie zachodzi konieczność podejmowania decyzji na podstawie analizy obrazu. Stosując zaproponowana technologie badacz zwolniony jest od żmudnych operacji związanych z interpretacją wyników przetwarzania obrazów, a zaimplementowana technologia rozumienia obrazów podaje mu w wyniku ostrzeżenia i alarmy. Treść publikacji podzielona została na sześć rozdziałów oraz spis bibliografii z zakresu rozpoznawania i rozumienia obrazów. Bardzo liczne przykłady zastosowań uwiarygodniają przedstawione rozważania teoretyczne oraz opisane procedury.

Książkę tą polecam uwadze wszystkim, którzy w swojej działalności badawczej lub inżynierskiej zajmują się rozpoznawaniem obrazów w zakresie rozwoju techniki, jak również zastosowań. Może ona być też przydatna tym, którzy interesują się możliwościami automatyzacji rozumienia w innych jak rozpoznawanie obrazów dziedzinach nauki i techniki.

Tadeusz Uhl