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Bessel filters with varying parameters

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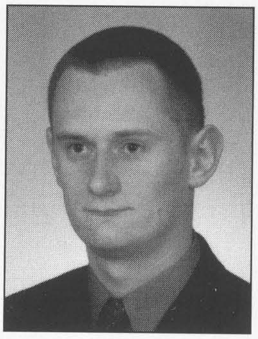

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Abstract

The paper presents the methodology how to vary in time values $\omega_{0i} = \omega_{0i}(t)$ in selected links of a structure of low-pass Bessel filters. In examinations of the filters a significant shortening of the transient state is achieved at the cost of slight increase in the oscillations. It allows to shorten time intervals of filtered signals. Typical filtration results are presented which are obtained by simulations of selected structures of filters with varying parameters.

Streszczenie

Wprowadzenie zmiennych w czasie parametrów ma na celu skrócenie czasu ustalania sygnału na wyjściu filtru. Ma to szczególne znaczenie w przypadku, gdy filtrację należy przeprowadzić na dużej liczbie sygnałów lub wielokrotnie ją powtarzać oraz w sytuacjach, gdy ważna jest prędkość wypracowywania sygnału wyjściowego. Szczególnie interesującym przypadkiem są dolnoprzepustowe filtry Bessela, które najlepiej nadają się do przenoszenia sygnałów prostokątnych. W artykule przedstawiona jest koncepcja wprowadzania zmiennych w czasie współczynników dla filtrów Bessela oraz badania symulacyjne w programie MATLAB.

Keywords: Bessel filters, time-varying systems.

1. Introduction

The design methods of filters are described in detail in the rich literature [1, 3, 8, 9, 10, 12] and concern mainly filters with constant parameter. These filters are well-known and their applications in particular measurement, diagnostic or control systems lead to satisfactory results. However they may also have some drawbacks. In every analog system with constant parameters when one designs the filter structure under prescribed assumption on the frequency specifications there is no control on the transient state time. This follows from the system structure and values of its parameters which in turn are determined by the frequency requirements. The indeterminacy principle holds here and makes it impossible to shorten the transient state of constant parameter filters with the frequency characteristics given apriori.

The Bessel filters enjoy the best properties among all filters when the passing through of rectangular impulses is considered. The condition for that is the constant group delay for possibly large frequency band which means that in this frequency band the phase is proportional to the frequency.

The filter structure is described by the transfer function (operator transmittance) which is the product of 2-nd order systems for even filter orders or the product of 2-nd order systems and one 1-st order system for odd filter orders. This can be written as follows:

- for even orders $n = 2i$

$$K_u(s)=\frac{k_u}{\prod_i\left(1+a_is+b_is^2\right)}=\frac{k_u}{\prod_i\left(\frac{1}{\omega_{0i}^2}s^2+\frac{2\beta_i}{\omega_{0i}}s+1\right)} \tag{1}$$

- for odd orders $n = 2i+1$

$$K_u(s)=\frac{k_u}{\prod_i\left(1+a_is+b_is^2\right)}=\frac{k_u}{(1+sT)\prod_i\left(\frac{1}{\omega_{0i}^2}s^2+\frac{2\beta_i}{\omega_{0i}}s+1\right)} \tag{2}$$

The computed values of the parameter ω_{0i}, β_i, T for filters from the 2-nd up to 6-th order of the transfer functions (1) and (2) are presented in Table 1.

Table 1. Values of the parameters T, β_i, ω_{0i} for individual elements occurring in the structure of Bessel filters from 2-nd up to 6-th order.

n	T	β_1	ω_{01}	β_2	ω_{02}	β_3	ω_{03}
2	–	0,8660	1,2720				
3	0,7560	0,7235	1,4476				
4	–	0,9580	1,4302	0,6207	1,6033		
5	0,6656	0,8875	1,5564	0,5456	1,7555		
6	–	0,9798	1,6040	0,8180	1,6891	0,4982	1,9048

It is easy to notice that the parameter values of individual elements differ from each other only slightly. As it is known the step response oscillations determine the degree of the damping coefficient β_i. The smaller values the bigger oscillations which means that overshot of the step response is higher. The rising time of the step response is determined by characteristic frequency ω_{0i}. The smaller values the longer rising time.

2. The varying parameters

For constant parameter filters there are only small possibilities of shortening the transient state and this is because the filter parameters are calculated on the base of the assumed approximation method of the frequency characteristics (phase or gain) which guarantees that the frequency specifications are satisfied without taking into consideration the character of the transient state. The possibility of improvement of the filter properties is provided by varying in time their parameters. Analysis of parametric systems is much more complicated and the number of works on this subject is rather low. Only very specific types of parametric differential equations can be solved analytically. However the development of modern simulation techniques makes examination of parametric systems possible.

The paper shows that it is possible to shorten the transient state in low-pass analog filters by varying in time selected parameters. A methodology of varying in time the values of $\omega_{0i}=\omega_{0i}(t)$ of the 2-nd order elements of the structure for low-pass Bessel filters. The function $\omega_{0i}(t)$ is described by the relation:

$$\omega_{0i}(t)=d\omega_{0i}\left[1-\frac{d-1}{d}h(t)\right] \tag{3}$$

where: ω_{0i} – limit value following from the Bessel approximation to which $\omega_{0i}(t)$ is convergent,
$h(t)$ – step respond of the 2-nd order system described by the relation:

$$h(t)=L^{-1}\left[\frac{1}{s}\cdot\frac{1}{\frac{1}{\omega_{0f}^{2}}s^{2}+\frac{2\beta_{f}}{\omega_{0f}}s+1}\right] \tag{4}$$

for $\beta_f < 1$

$$(t)=1-\left[\cos\left(\omega_{0f}t\sqrt{1-\beta_f^{2}}\right)+\frac{1}{\sqrt{1-\beta_f^{2}}}\sin\left(\omega_{0f}t\sqrt{1-\beta_f^{2}}\right)\right]\cdot\exp\left(-\beta_f\omega_{0f}t\right) \tag{5}$$

This guarantees short settling time of the function $h(t)$ and consequently fast settling of the function $\omega_{0i}(t)$ described by the relation (3). Values of coefficient d defining the range of changes of the function $\omega_{0i}(t)$ are calculated from the following relation:

$$d=\frac{\omega_{0i}(0)}{\omega_{0i}(\infty)} \tag{6}$$

For simulations it is assumed d = 2; 5; 10 which gives appropriate range of changes of the function $\omega_{0i}(t)$. This character of changes of the function describing variations of filter parameters follows from previous examinations of filters with varying parameters [4].

The introduction of time-varying parameters requires examination of stability of the systems with element containing varying parameters. Papers [5] present the proof of a theorem saying that if for the 2-nd order system the functions $\omega_0(t)$ and $\beta(t)$ have the same sign and

$$\lim_{t\to\infty}\frac{d\omega_{0i}(t)}{dt}\to 0 \tag{7}$$

then stability can be determined in the same manner as for time-invariant systems. It allows us to claim that if the filter structure contains elements with varying parameters, then for the time going to infinity the values of varying parameters converge to limit values following from the Bessel approximation and the filter stability can be examined as in the time-invariant case [5, 6]. Since the Bessel approximation guarantees stability of time-invariant filter one can skip the stability issue for the parametric filter.

Using the results of the previous investigations [4, 6] the following concept of examinations of low-pass Bessel filters with varying parameter based on simulation has been assumed:

- simulation and examination of the 2-nd order element by introducing variable characteristic frequency $\omega_{0i}(t)$ in order to minimize the settling time under the condition $\lim_{t\to\infty}\omega_{0i}(t)=\omega_{0i}$,
- simulation and examination of the complete structure of the Bessel low-pass filter after varying parameters $\omega_{0i}(t)$ in all elements,
- examination of a Bessel filter with varying parameters and repetitive variations of the characteristic frequency $\omega_{0i}(t)$.

3. Results

Examinations have been carried out using Matlab-Simulink package and modeling elements as 2-nd order systems with varying parameters. Typical element of the 2-nd order with variable parameters is shown in Fig. 1. Examinations of a single 2-nd order element with the function $\omega_{0i}(t)$ described by (3) and (5) revealed the possibility of significant even multiple shortening of the settling time of such an element. It does not mean that the inclusion of this element into the structure of the Bessel filter will result only in useful changes of its properties. Variation of filter parameters also causes ìtime variationî of the filter frequency properties. This may lead e. g. to larger oscillations or change of the value the critical frequency.

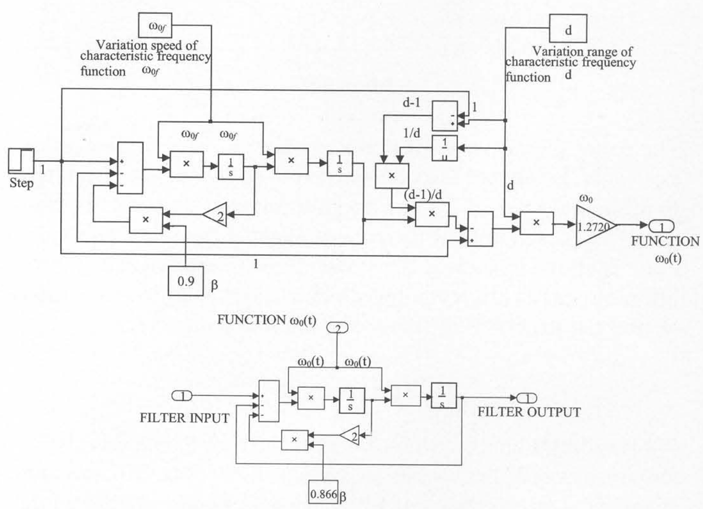

Fig. 1. Block diagram of the 2-nd order element with varying parameters.

For this reason the examination of the whole filter with elements containing time-varying parameters has been carried out. It follows from the obtained results that the least perturbations of the frequency properties occur when the parameters in all elements vary according to the same procedure. An example of filtration of highly noised rectangular impulses is presented. The filtered signal is shown in Fig. 2.

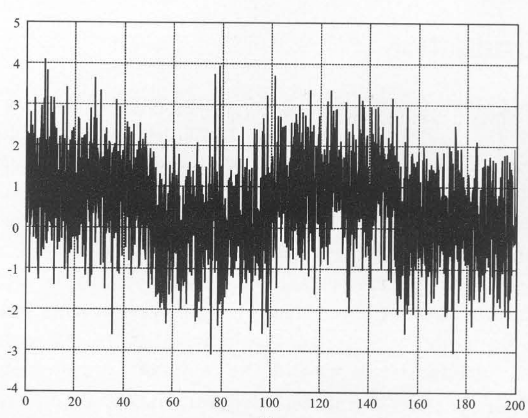

Fig. 2. Highly noised rectangular impulse under filtration.

The filtration has been performed by means of the 4-th order Bessel filter with varying parameters, where in the both 2-nd order elements the parameters are varied according to the same function

$\omega_{0i}(t)$ described by expressions (3) and (5). Fig. 3 and 4 shows the initial phase of the growth of the impulse.

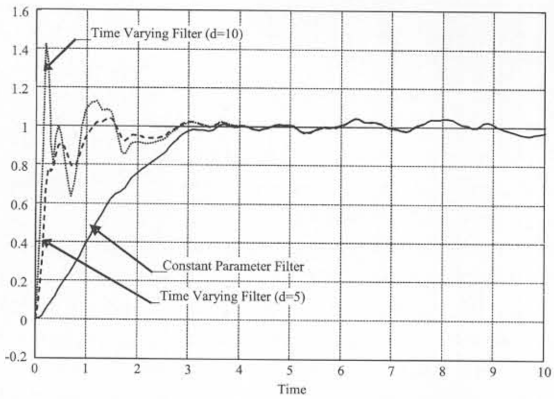

Fig. 3. The initial phase of the noised rectangular impulse filtration by using the 2-nd order Bessel filter.

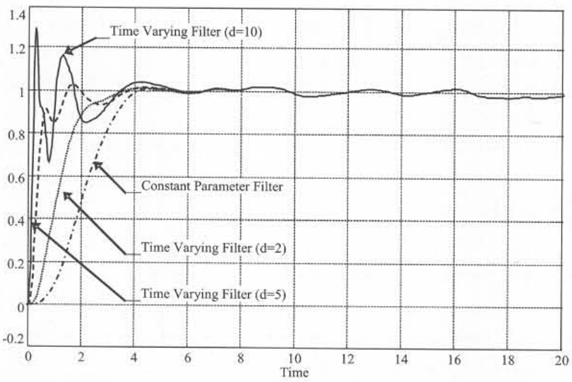

Fig. 4. The initial phase of the noised rectangular impulse filtration by using the 4-th order Bessel filter.

During filtration of rectangular impulses the characteristic frequency $\omega_{0i}(t)$ of the Bessel filter is varied repetitively according to the filtered signal frequency. The same procedure is applied to the filtration of signals with step changes. Fig. 5 and 7 shows filtration of a highly noised signal with step changes by using a Bessel filter with constant parameters and Fig. 6 and 8 shows filtration of the same signal by using a Bessel filter with varying parameters.

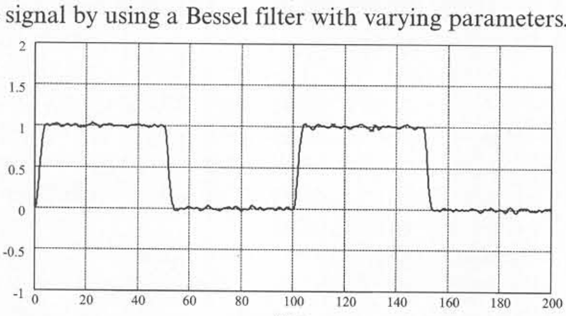

Fig. 5. Typical graphs of the noised rectangular impulses filtration by using the 4-th order Bessel filter with time-invariant parameters.

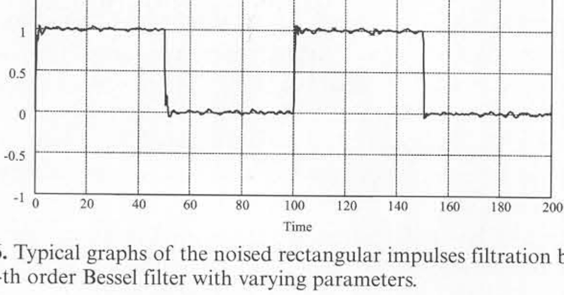

Fig. 6. Typical graphs of the noised rectangular impulses filtration by using the 4-th order Bessel filter with varying parameters.

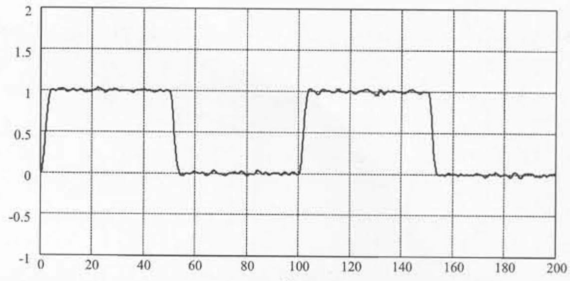

Fig. 7. Typical graphs of the noised signal with step changes filtration by using the 4-th order Bessel filter with time-invariant parameters.

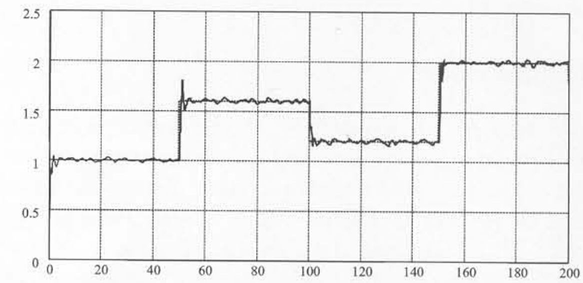

Fig. 8. Typical graphs of the noised signal with step changes filtration by using the 4-th order Bessel filter with varying parameters.

It follows from the presented graphs that filters with time-varying parameters are much faster than the time-invariant filters. Some oscillations appearing during the initial stage of the graphs are caused by variations in the spectral properties following from parameter variations. At the same time one can noticed that filter with varying parameters for d=5 gives the best (the fastest and only slightly perturbed) growth of the impulse. It seems that the proposed method varying the parameters in Bessel filters is useful and allows to achieve better filtration results for rectangular signals.

Very interesting results were obtained during examinations of odd filter orders. In this case the first step of examinations consisted in varying in time only the characteristic frequencies. The time constant T remained unchanged. For variation range d of the function $\omega_{0i}(t)$ equal to 1,5 and 2 the dependence of the settling time $\tilde{t}_{u\alpha}$ on the variation speed ω_{0f} of the function $\omega_{0i}(t)$ is approximately linear, what for d=2 is shown in Fig. 9.

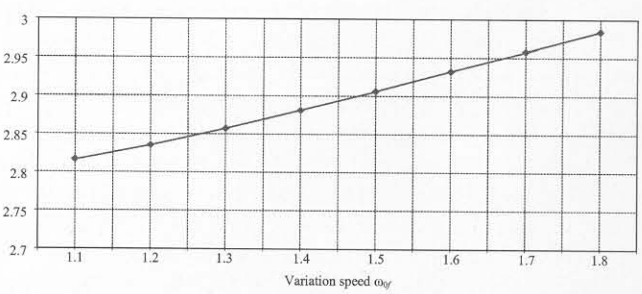

Fig. 9. The dependence of the settling time of 5-th order time varying Bessel filter on the variation speed ω_{0f} of function $\omega_{0i}(t)$ for d=2 (T=const).

The shortest settling time $\tilde{t}_{u\alpha}$ was obtained for such value of ω_{0f}, for which the settling time with an α-accuracy of $\omega_{0i}(t)$ was approximately equal to the settling time of the analogous constant parameter filter. The situation underwent a change for variation range d>2. From Fig. 10 and Fig. 11 we see, that for larger variation ranges d of the function $\omega_{0i}(t)$, the local minimum of the function $\tilde{t}_{u\alpha} = f(\omega_{0f})$ exists. On the basis of simulation research one can determine this minimum for arbitrary variation ranges of the function $\omega_{0i}(t)$.

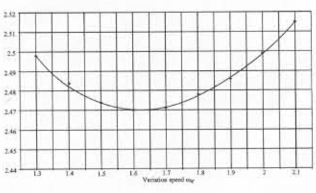

Fig. 10

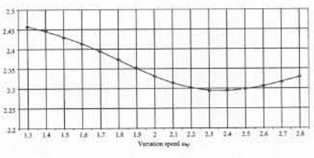

Fig. 11

Fig. 10. The dependence of the settling time of 5-th order time varying Bessel filter on the variation speed ω_{0f} of the function $\omega_{0i}(t)$ for d=3 (T=const).

Fig. 11. The dependence of the settling time of 5-th order time varying Bessel filter on the variation speed ω_{0f} of the function $\omega_{0i}(t)$ for d=4 (T=const).

4. Conclusions

As it has been proven, application of time varying coefficients in low-pass Bessel filters causes considerable shortening of the settling time. The best results of shortening of the settling time were obtained by varying in time the characteristic frequency? 0i and the inverse of the time constant T according to the same function. It seems that further examinations of time varying Bessel filters are needed. Especially, the problems of optimal selection of variation range and the speed range of functions are open. Nevertheless already this paper proves possibilities and practical usefulness of the proposed filter concept as a signal processing instrument.

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Tytuł: Filtry Bessela o zmiennych parametrach

Artykuł recenzowany

Ciąg dalszy ze strony 3

Stowarzyszenie PolSPAR, pomimo, że jest instytucjonalnie głównym twórcą i patronem cyklu ogólnopolskich konferencji metrologicznych, z których wywodzi się Kongres, zostało wymienione w informatorach i materiałach Kongresu jako ostatnie, a przedstawiciel tego Stowarzyszenia nie został nawet poproszony do stołu prezydialnego w sesji plenarnej rozpoczynającej obrady. To Prezydium PKPAiR, a potem tego Stowarzyszenia decydowało poprzednio o powierzeniu organizacji i składzie organów kolegialnych Konferencji lub Kongresu. Stowarzyszenie dba do dziś o regulowanie opłat członkowskich w IMEKO, które organizuje międzynarodowe sympozja i Kongresy o tematyce metrologicznej. Na ostatnim Kongresie Przewodniczący Podkomitetu Pomiarów POLSPAR-u był członkiem Komitetu Naukowego, ale tylko jako profesor Wyższej Szkoły Morskiej. Jest to brak uszanowania tradycji i niedocenianie dorobku społecznego bardzo wielu pokoleń metrologów, członków PKPiA, a następnie tego Stowarzyszenia, którzy w przeszłości byli twórcami, organizatorami i uczestnikami tego cyklu konferencji. To oni doprowadzili do podniesienia rangi konferencji i nadali im na tyle wysoką markę, że uzasadniło to w Gdańsku zmianę nazwy na Kongres.

Redakcje głównych czasopism naukowo-technicznych związanych ze Stowarzyszeniem PolSPAR też nie były reprezentowane w składzie żadnego z Komitetów ostatniego Kongresu. Dotyczyło to też wielce zasłużonego czasopisma PAK, dzięki któremu zdobyło ostrogi zawodowe wiele pokoleń metrologów i które od zarania było nie tylko organem prasowym pomiarowców ale i towarzyszyło różnym sympozjom i konferencjom pomiarowym. Było także inicjatorem i współorganizatorem Krajowych Konferencji, z których wywodzą się Kongresy Metrologii.

Przedstawiciel Polskiego Towarzystwa Metrologicznego też nie został zaproszony na Kongres, chociaż czynili to organizatorzy obu poprzednich jego edycji. Dlatego piszący te słowa, postanowił z własnej inicjatywy zostać aktywnym obserwatorem obrad. Otrzymał jednak, w ostatnim dniu obrad, materiały kongresowe, za co niniejszym składa **Przewodniczącemu osobiste serdeczne podziękowanie**.

A oto kilka różnej rangi innych niedociągnięć organizacyjnych będących w sprzeczności z zazwyczaj przyjętymi zasadami.

Kongres rozpoczął się hymnem Unii Europejskiej, a zabrakło w piastowskim Wrocławiu kilku minut czasu na hymn polski!? Jest też utwór o nazwie metrologicznej, skomponowany specjalnie z okazji Kongresu w Gdańsku. Warto było by go przypomnieć.

Migawki z otwarcia niewielkiej wystawy aparatury towarzyszącej Kongresowi ukazały się w lokalnej telewizji. Dzięki komisarzowi wystawy były one bardzo pomysłowo ujęte (ważenie pisma), ale nie zadbano o to, by uczestnicy Kongresu zostali poinformowani o porannej godzinie emisji następnego dnia.

Wystąpienia zaledwie dziewięciu firm ustawiono w programie trzech sekcji komercyjnych równolegle, a nie szeregowo, co umożliwiało skorzystanie tylko z trzech z nich.

Obrady plenarne odbywały się w zwykłym audytorium, gdyż termin Kongresu był w kolizji z nie zakończonym remontem reprezentacyjnej sali Politechniki Wrocławskiej. Przebiegające równolegle inne, chyba nawet nieco mniejsze Sympozjum miało lepsze warunki lokalowe w jednym z nowych budynków Uczelni.

Termin Kongresu nie został skorelowany z innymi, bardzo licznymi w tym roku, węższymi tematycznie sympozjami i konferencjami metrologicznymi. Powinien być w kolejności ostatni, aby ich tematykę można było na Kongresie podsumować. Przy ograniczonych środkach uczelni zaważyło to też istotnie na liczbie uczestników i nawet kilku najwybitniejszych metrologów krajowych nie wzięło niestety udziału w Kongresie.

Tych parę konstruktywnych uwag krytycznych nie ma w zamierzeniu ich autora charakteru osobistego. Ma służyć następnym organizatorom jako swoiste, informacyjne sprzężenie zwrotne.

Zygmunt Warsza