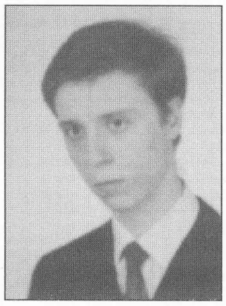
Maciej TWARDY, Tadeusz KACZOREK
POLITECHNIKA WARSZAWSKA, WYDZIAŁ ELEKTRYCZNY

Observer-based fault detection in dynamical systems - part I

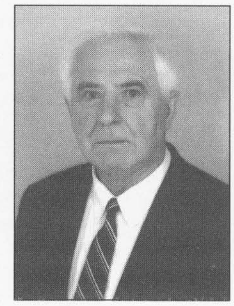
Mgr inż. Maciej TWARDY



Studia ukończył na Wydziale Elektrycznym Politechniki Warszawskiej w roku 2002. W okresie 01.02.2002 - 01.06.2002 przebywał w Aalborg University w Danii w ramach programu Socrates-Erasmus. Obecnie jest słuchaczem studiów doktoranckich dziennych na Wydziale Elektrycznym PW.

mtwardy@ee.pw.edu.pl

Prof. dr hab. inż. Tadeusz KACZOREK



Uzyskał dyplom magistra inżyniera elektryka w roku 1956 na Wydziale Elektrycznym Politechniki Warszawskiej. Na tym samym Wydziale w roku 1962 uzyskał stopień naukowy doktora nauk technicznych, a w roku 1964 - doktora habilitowanego. Tytuł naukowy profesora nadzwyczajnego nadała Mu Rada Państwa w roku 1971, a profesora zwyczajnego w 1974 roku. Członkiem korespondentem PAN został wybrany w 1986 roku, a członkiem rzeczywistym w 1998. Od czerwca 1999 roku jest również członkiem zwyczajnym Akademii Inżynierskiej w Polsce. W latach 1969-1970 był dziekanem Wydziału Elektrycznego, a w latach 1970-1979 prorektorem d/s nauczania Politechniki Warszawskiej. W latach 1970-1981 był dyrektorem Instytutu Sterowania i Elektroniki Przemysłowej Politechniki Warszawskiej. W latach 1988-1991 był dyrektorem Stacji Naukowej PAN w Rzymie. Jest autorem 18 książek, w tym 5-ciu wydanych za granicą oraz ponad 500 artykułów i rozpraw naukowych, opublikowanych w czasopismach krajowych i zagranicznych. Główne kierunki badań naukowych to analiza i synteza układów sterowania i systemów, a w szczególności układy wielowymiarowe, układy singularne i układy dodatnie.

T.Kaczorek@ee.pw.edu.pl

Abstract

The paper is the first part of a survey of observer based fault detection and isolation (FDI) methods. Considerable part of the material is standard control theory, especially observers theory. It is presented for future reference. The paper briefly covers basic ideas and concepts underlying much of model-based FDI. A general introduction to FDI is given. General idea of model based FDI methods is presented. The issue of FDI robustness mentioned and short historical background of FDI is given. The basic concepts of observers, the observability and detectability of LTI systems, Luenberger observer and unknown input observer (UIO), are considered. Some selected results from the theory of UIOs and design procedure for UIO are presented. Finally, a short introduction to singular systems is given and observers for these systems are considered.
Further results concerning observer-based FDI will be presented in the next paper.

Streszczenie

Artykuł jest pierwszą częścią przeglądowej pracy na temat użycia obserwatorów do wykrywania i izolacji uszkodzeń (ang. *FDI*). Część materiału stanowią znane wyniki z teorii sterowania, zwłaszcza z teorii obserwatorów, na które powołujemy się w dalszej części pracy. Artykuł zwięźle prezentuje podstawowe idee i pojęcia opartych na modelu metod wykrywania i izolacji uszkodzeń. Zawiera ogólne wprowadzenie do FDI. Zaprezentowano ogólną ideę metod FDI opartych na modelu. Poruszono również kwestię odporności metod FDI jak również zamieszczono krótki rys historyczny rozwoju tych metod. Rozpatrzono podstawowe pojęcia dotyczące obserwatorów, obserwowalności i wykrywalności liniowych układów stacjonarnych, obserwator Luenbergera oraz obserwatory o nieznanym wejściu. Podano również zwięzłe wprowadzenie w tematykę układów syngularnych syngularnych obserwatorów dla tych układów.
Dalsze wyniki dotyczące opartych na modelu metod wykrywania i izolacji uszkodzeń będą zaprezentowane w części drugiej tej pracy.

1. Introduction

A problem of fault detection and isolation (FDI) in dynamical systems has received considerable attention both theoretically and practically [2, 21, 23]. In Poland, particularly, essential contributions to FDI have been made by professor Korbicz's group from University of Zielona Góra [22, 20, 21, 23]. The early detection of incipient faults can help avoid major plant failures. Similarly, fault detection has become very important issue in the operation of all systems, where high availability is of special importance e.g. airplanes, submarines, off-shore placed wind turbines etc. The range of reactions to the fault situations is wide. It contains immediate emergency actions as well long-term modifications of the maintenance schedule. Automatic diagnostic systems may assist the human operator in process supervision and considerably help to make proper decision in fault situation and to carry out an appropriate remedial action. Generally speaking fault detection and diagnosis consist of the following tasks [15]:

1. *Fault detection*, i.e., the indication that something is going wrong in the system.
2. *Fault isolation*, i.e., the determination of the exact location of the fault.
3. *Fault identification*, i.e., the determination of the size of the fault.

Certainly, relative importance of these three tasks is to some degree subjective. However, detection is necessary in any practical system and isolation is also very important. What concerns identification, it is very helpful, but generally not crucial. Hence, fault diagnosis is most frequently considered as *fault detection and isolation*, abbreviated as FDI.

1.1. Approaches to FDI

The approaches to the problem of fault detection and isolation can be divided into two major groups:

1. Methods making use of a plant model;
2. Methods that do not make use of a plant model.

Observer-based methods belong to the first approach, and are the main object of the paper.

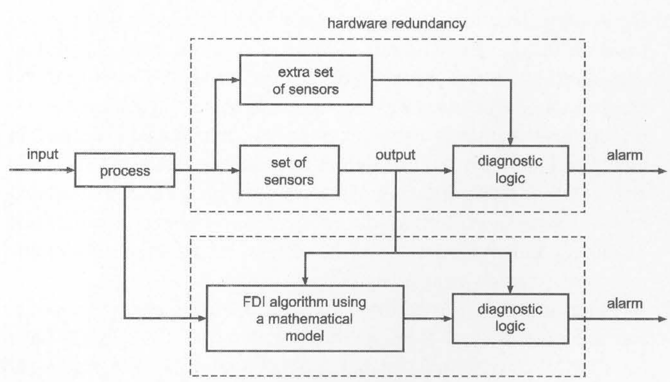


Fig. 1. Hardware versus analytical redundancy

1.2. Model based FDI methods

A broad class of fault detection and isolation methods make explicit use of the mathematical model of the plant. Most model-based fault detection and isolation methods rely on the idea of *analytical redundancy* [4]. In contrast to physical redundancy, where measurements from different sensors are compared, now sensor measurements are compared to analytically obtained values of the respective variable. Such computation uses present and/or previous measurements of other variables and the mathematical model describing their relationship. The resulting differences are called *residuals*. They should be equal or at least close to zero when no fault is present and differ significantly from zero when fault occurs.

In practice, even under fault-free operation conditions the residuals are not zero. Their deviation from zero is the combined result of noise and faults. If the signal/noise ratio is high, residuals can be analyzed directly. When any significant noise is present, one approach is generating disturbance decoupled residuals and this method is adopted in the paper. In any case, a logical pattern is generated, showing which residuals can be considered normal and which ones indicate fault. Such a pattern is called the *signature* of the fault. The final step of the procedure is the analysis of the logical patterns obtained from the residuals, with the aim of isolating the fault(s) that cause them. Such analysis may be performed by comparison to a set of patterns (signatures) known to belong to simple faults or by the use of some more complex logical procedure.

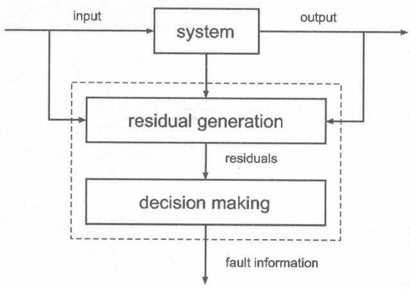


Fig. 2. Conceptual structure of model-based fault diagnosis

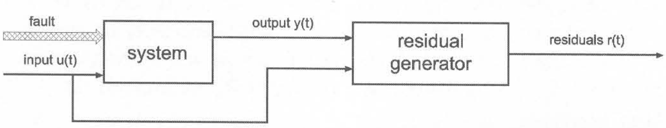


Fig. 3. Block diagram of residual generator

Having disturbance decoupled residuals we set thresholds on them and say the fault has occurred if an appropriate threshold has been exceeded. The particular kind of model-based methods are observer-based techniques and we focus on them throughout the paper. Chow and Willsky [3] proposed two-stage structure for fault diagnosis, which is now widely accepted.

1. **Residual generation.** It is a generation of a fault indicating signal - residual, from available input and output of the system. A residual should reflect a possible fault in the analyzed system. This means, the residual should be normally zero or close to zero when no fault is present and essentially different from zero when a fault occurs. In ideal conditions the residual should be independent of system inputs and outputs. The algorithm or processor generating residuals is called *residual generator*. The residual generation is thus a procedure of extracting fault symptoms from the system, with the fault symptoms represented by the residual signal.
2. **Decision-making.** The residuals are tested for the likelihood of faults, and a decision rule is than applied to determine if any faults have occurred. Well designed residual generator simplifies the process of decision making and therefore most of the effort in the field of quantitative model-based FDI is devoted to the residual-generation problem.

The paper concentrates on the quantitative residual generation stage of fault diagnosis.

1.3. Robustness of FDI

As already mentioned model-based FDI relays on the mathematical model of a supervised process, nevertheless, one must remember that a perfectly accurate and complete model of a real-world system cannot be obtained. There are many reasons for this. Same parameters of a system may vary with time, often disturbance acting on the process are unknown or cannot be modeled precisely. The discrepancy between the real system and its model is the cause of fundamental difficulties in FDI; it causes the false and missed alarms and therefore obscures significantly FDI performance. The FDI must be made robust against disturbances while in the same time sensitive to faults. An FDI scheme designed to provide satisfactory sensitivity to faults, associated with the necessary robustness with respect to modeling uncertainty and disturbances is called a *robust FDI scheme*. [13, 26]. The development of robust model-based FDI methods has been very important research topic during last decade. A number of methods has been proposed that address this problem, e.g. the unknown input observer or eigenstructure assignment.

1.4. A brief historical background

More details on history of model-based FDI methods can be found in i.e. [2]. Hire we will only mention that observer-based methods were first introduced by Clark and co-workers [8] and [5, 6, 7]. Frank's comprehensive survey paper [12] established the position of observer-based methods in model-based FDI. In this paper, many different schemes using both linear and non-linear observers were reviewed and some application examples were given. Some more recent results can be found e.g. in [20].

As already mentioned, Chow and Willsky [3, 4] first defined the model-based FDI as a two-stages process: (1) residual generation, (2) decision-making (including residual evaluation). This two-stages process is accepted as a standard procedure for model-based FDI nowadays. The first attempt of improving robustness of observer-based FDI approaches is attributed to Frank and Keller [14]. To solve the robust FDI problem, Watanabe and Himmelblau [29] introduced a robust sensor fault detection method using an unknown input observer (UIO). A comprehensive treatment on robust FDI using UIOs is presented in [2].

Patton at al. [27] proposed an FDI method based on observer eigenstructure assignment and this approach has been studied extensively by Patton's group. A comprehensive treatment on robust FDI via eigenstructure assignment is presented in [2].

To solve robust FDI problems, a mathematical representation for describing modeling uncertainty is needed. Patton and Chen [25, 1] proposed several schemes to represent modeling uncertainties from various sources as additive disturbances with an estimated distribution matrix. Robust FDI is thus achieved using disturbance decoupling approaches. This is an important contribution to robust FDI. So far, most robust residual generation methods are based on the assumption that disturbance distribution matrices are known, however, in many cases this assumption is not valid.

2. Basic principles of observers

2.1. Notation and preliminaries

In the paper capital, italic letters as X, U and Y denote real vector spaces, with typical elements being denoted by x, u and y, respectively. The fields of real and complex numbers are denoted by R and C, respectively. We use C^+ to denote the open right half-plane formed from the complex numbers with positive real part; $\overline{C}^+$ is the corresponding closed half-plane, and the left half-planes C^- and $\overline{C}^-$ are analogously defined. R^n and C^n denote n-dimensional real and complex vector space, respectively. $R^{m\times n}$ and $C^{m\times n}$ denote space of real and complex $m\times n$ matrices, respectively. The dimension of a vector space, e.g. X, is described by $\dim X$. The zero vector, zero space, etc. are denoted by . Matrices and linear maps are both denoted by capital italic letters, e.g. A, B and C. Matrices as well vector spaces are denoted by capital, italic letters but since the meaning is clear from context, there is no reason for ambiguity. $ImB = B$ denotes the *image* of B. $KerC$ denotes the *kernel* of C. The spectrum (eigenvalues) of a matrix A is denoted by $\sigma(A)$. A set of complex numbers $\Lambda \in C$ is self-conjugate if $\lambda \in \Lambda$ implies $\lambda^* \in \Lambda$ where $*$ denotes the complex conjugate. $B^\dagger$ denotes Moore-Penrose's pseudo-inverse of matrix B. Symbol $:=$ means equal by definition.

2.2. Observability and detectability of LTI systems

Let us consider the LTI system

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(0) = x_0, \tag{1a}$$

$$y(t) = Cx(t), \tag{1b}$$

where $x(t) \in R^n, u(t) \in R^m$ and $y(t) \in R^p$ are the state, input and output vectors respectively and A, B, and C are real matrices of appropriate dimensions. The system given above is a first order linear equation with an initial condition, and therefore has a unique solution which is given by

$$y(t) = Ce^{At}x_0 + C\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau, \quad t \geq 0. \tag{2}$$

The transient part of this solution depends entirely on the initial condition $x(0) = x_0$ $y_{tr}(t) = Ce^{At}x_0, \qquad t \geq 0.$

Suppose, we are able to measure (observe) the output function $y(t)$ over a finite interval $[0,T]$, but want to find a value of x_0. The expression for y_{tr} can be regarded as a map Ψ from the state space R^n to the vector space $F\left(R,R^p\right)$ of functions that are R^p -valued; i.e., $\Psi: R^n \to F\left(R,R^p\right)$ and is defined by $x_0 \to Ce^{At}x_0$.

Here we consider only the case where t is in the interval $[0,T]$. So we have that $y_{tr} = \Psi x_0$.

One can see clearly that Ψ is linear map, namely $\Psi(\alpha x_0 + \beta z_0) = \alpha\Psi(x_0) + \beta\Psi(z_0)$ and it is possible to determine x_0 given the function y exactly when there is only one solution to the above equation. When y is given than there exists the unique solution, if and only if the kernel of Ψ is zero, i.e. $Ker\Psi = 0$.

Thus if the above condition holds we are able to determine x_0, and if it is violated then the initial conditions cannot be uniquely determined by observing y. According to this, the matrix pair (C, A) is called *observable* if $Ker\Psi = 0$. The subspace $Ker\Psi$ is usually denoted N_{CA} and is called *unobservable subspace* of the pair (C, A). Thus the system is observable when $N_{CA} = 0$.

Lemma 1. The kernel of Ψ is given by

$$Ker\Psi = Ker\,C \cap \cdots \cap Ker(CA^{n-1}) = Ker\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}.$$

The matrix on the right in the theorem is named the *observability matrix*; from the theorem, observability is equivalent to this matrix having full column rank.

Proof. [11] We first show that $Ker\Psi \subset Ker C \cap \cdots \cap Ker(CA^{n-1})$. Let $x_0 \in Ker\Psi$ and therefore by definition we know that $Ce^{At}x_0 = 0, \quad t \geq 0.$ It is straightforward to verify that for each $k \geq 0$ the following holds

$$\left.\frac{d^k}{dt^k}Ce^{At}x_0\right|_{t=0} = CA^k x_0.$$

But by the equation that precedes the above we see that the left-hand side must be zero, and so x_0 must be in the kernel of any matrix CA^k where k is non-negative integer.

To complete the proof we must show that $KerC \cap \cdots \cap Ker(CA^{n-1}) \subset Ker\Psi$. From the Cayley-Hamilton theorem the function e^{At} can be written as $e^{At} = \phi_0(t)I + \cdots + \phi_{n-1}(t)A^{n-1}, \quad t \geq 0$ where $\phi_k(t)$ $k = 1,2,\ldots,n-1$ are scalar functions. From this it is clear that if $x_0 \in KerC \cap \cdots \cap Ker(CA^{n-1})$, then $Ce^{At}x_0 = 0$. Q.E.D.

Theorem 1. [11, 18]

a) The pair (C,A) is observable if and only if

$$rank\begin{bmatrix} A-\lambda \\ C \end{bmatrix} = n, \quad \text{for all} \quad \lambda \in C;$$

b) the eigenvalues of $A + LC$ are freely assignable by choosing L if and only if (C, A) is observable;

c) the unobservable subspace N_{CA} is A-invariant;

d) there exist a state transformation matrix T such that

$$TAT^{-1} = \begin{bmatrix} \tilde{A}_{11} & 0 \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \quad CT^{-1} = [\tilde{C}_1 \;\; 0], \text{ where } (\tilde{C}_1, \tilde{A}_{11}) \text{ is observable;}$$

e) the matrix $\tilde{A}_{22}$ in part (d) is Hurwitz, if and only if, the condition

$$rank\begin{bmatrix} A-\lambda \\ C \end{bmatrix} = n, \quad \text{for all} \quad \lambda \in \bar{C}^+ \text{ holds;}$$

Notice that in part (d) the following holds $TN_{CA} = Im\begin{bmatrix} 0 \\ I \end{bmatrix}$.

We will call the decomposition of part (d) an *observability form*, because it explicitly isolates the invariant subspace N_{CA}. Writing out the state equations in this form gives us particular insight:

$$\begin{aligned} \dot{\tilde{x}}_1(t) &= \tilde{A}_{11}\tilde{x}_1(t), \\ \dot{\tilde{x}}_2(t) &= \tilde{A}_{21}\tilde{x}_1(t) + \tilde{A}_{22}\tilde{x}_2(t), \\ \tilde{y}(t) &= \tilde{C}_1\tilde{x}_1(t), \end{aligned} \quad \text{with the initial condition} \begin{bmatrix} \tilde{x}_1(0) \\ \tilde{x}_2(0) \end{bmatrix} = Tx(0).$$

From here it is clear that $\tilde{x}_1$ only depends on the initial condition $\tilde{x}_1(0)$, and is completely unaffected by $\tilde{x}_2$; therefore y is entirely independent of $\tilde{x}_2$. For this reason we say the vector $\tilde{x}_1$ contains the observable state and the vector $\tilde{x}_2$ contains the unobservable state.

If $\tilde{x}_1(0) = 0$ then $\tilde{x}_1(t)$ is zero for all time, $\dot{\tilde{x}}_2(t) = \tilde{A}_{22}\tilde{x}_2(t)$, and $y = 0$. If $\tilde{A}_{22}$ is not Hurwitz then there exist some initial state $\tilde{x}_2(0)$, such that y is zero for all time yet $\tilde{x}_2(t)$ does not tend to zero. When $\tilde{A}_{22}$ is Hurwitz, i.e., $\sigma(\tilde{A}_{22}) \subset C^-$ we say that (C, A) is *detectable* matrix pair. The detectability is a weaker condition than observability. A pair (C, A) is detectable when all unobservable eigenvalues of matrix A are stable.

2.3. Luenberger full-order observer

Let us consider the state space system (1). Our objective is to find an asymptotic approximation of $x(t)$ for the given input u and the output y, without knowledge of the initial condition $x(0)$.

Definition 1. The system

$$\dot{w}(t) = Mw(t) + Ny(t) + Pu(t), \quad w(0) = w_0, \tag{3a}$$

$$\hat{x}(t) = Qw(t) + Ry(t) + Su(t), \tag{3b}$$

is called a full-order observer of system (1) if $\lim_{t\to\infty}\left[x(t) - \hat{x}(t)\right] = 0$ for all initial conditions $x(0)$ and $w(0)$, and all system inputs $u(t)$.

Defining *estimation error* $e(t) := x(t) - \hat{x}(t)$, we can say in words that system (3) is observer of (1) if his estimation error $e(t)$ tends to zero regardless of initial conditions and system input. We additionally require, that observer is itself stable system, i.e., M in (3) is Hurwitz. The inputs to the observer are $u(t)$ and $y(t)$, and the output is $\hat{x}(t)$.

Theorem 2. An observer exist if and only if the pair (C, A) is detectable. Furthermore, in that case one such observer is given by $\dot{\hat{x}} = \left(A - KC\right)\hat{x} + Ky(t) + Bu(t)$, where the matrix K is chosen such that $\sigma(A - KC) \subset C^-$.

An observer with this structure is called a full order Luenberger observer. Matrix K is usually called observer gain matrix.

Proof. Proof can be found in many standard books on control theory, for instance [11, 17].

The above conciderations can be extended for reduced-order observers [17, 18].

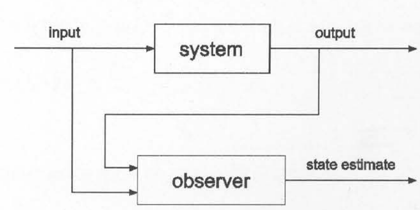


Fig. 4. Block diagram of a system with observer

2.4. Unknown input observers

Many results presented in this section can be found in [2]. We consider the observer design for a class of systems described by the following equations:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t), \tag{4a}$$

$$y(t) = Cx(t), \quad (4b)$$

where $x(t) \in R^n$, $y(t) \in R^m$, $u(t) \in R^r$ are the state, output and the known input vector, respectively, and $d(t) \in R^q$ is the unknown input (disturbance) vector. A, B, C and E are known real matrices with appropriate dimensions. As we see, the system uncertainty is summarized as an additive unknown disturbance term in the state equation. Without loss of generality it can be assumed that the disturbance distribution matrix E is full column rank matrix. One should note that the number of independent row of the matrix C must not be less than the number of the independent columns of the matrix E i.e. the maximum number of disturbances which can be decoupled cannot be larger than the number of independent measurements.

$$rankC = m \geq rankE = q \quad (5)$$

The term $Ed(t)$ describes an additive disturbance as well as a number of other different kinds of modeling uncertainties as e.g. noise or non-linear terms in system dynamics. More details on dealing with disturbances appearing in a system in other ways can be found in [2].

Definition 2. The system (3) is called an unknown input observer (UIO) of system (4), if $\lim_{t\to\infty}[x(t) - \hat{x}(t)] = 0$ for all initial conditions $x(0)$ and $w(0)$, and all system inputs $u(t)$, regardless of the presence of the unknown input (disturbance) in the system.

2.4.1. Theory of UIOs

Let us consider the structure of a full-order observer

$$\dot{z}(t) = Fz(t) + TBu(t) + Ky(t), \quad (6a)$$

$$\hat{x}(t) = z(t) + Hy(t), \quad (6b)$$

where $\hat{x} \in R^n$ and $z \in R^n$ are the estimated state vector and the state of this full-order observer, respectively, and F, T, K and H are matrices to be designed for achieving unknown input decoupling and other design requirements. One can see that (6) is the special case of (3). When the observer (6) is applied to the system (4), the estimation error $e(t) := x(t) - \hat{x}(t)$ is governed by the equation

$$\begin{aligned}\dot{e}(t) = (A - HCA - K_1C)e(t) + \left[(A - HCA - K_1C) - F\right]z(t) \\ + \left[(A - HCA - K_1C)H - K_2\right]y(t) + \left[(I - HC) - T\right]Bu(t) \\ + (I - HC)Ed(t) \quad (7)\end{aligned}$$

where $\quad K = K_1 + K_2. \quad (8)$

If one can make the following relations hold true

$$(HC - I)E = 0, \quad (9a)$$

$$T = I - HC, \quad (9b)$$

$$F = A - HCA - K_1C, \quad (9c)$$

$$K_2 = FH, \quad (9d)$$

the estimation error is governed by

$$\dot{e}(t) = Fe(t). \quad (10)$$

If F is Hurwitz, $e(t)$ will approach zero asymptotically. This means that the observer (6) is an unknown input observer for the system (4) according to Definition 2. The design of this UIO is to solve equations (8) and (9) and making all eigenvalues of the system matrix F be stable, i.e. $\sigma(F) \subset C^-$. Before we give the necessary and sufficient conditions for the existence of a UIO, two lemmas are introduced.

Lemma 2. Equation (9a) is solvable if and only if

$$rank(CE) = rankE, \quad (11)$$

and its solution is given by

$$\tilde{H} = E\left[(CE)^T CE\right]^{-1}(CE)^T. \quad (12)$$

Proof. Proof follows immediately from Kronecker-Capelli theorem and (5)

Lemma 3. [2] Let $C_1 = \begin{bmatrix} C \\ CA \end{bmatrix}$, then the pair (C_1, A) is detectable if and only of the pair (C, A) is detectable.

Proof. According to Theorem 1 if $s_1 \in C$ is an unobservable mode of the pair (C_1, A), we have $rank\begin{bmatrix} s_1I - A \\ C_1 \end{bmatrix} = rank\begin{bmatrix} s_1I - A \\ C \\ CA \end{bmatrix} < n$, i.e. s_1 is also an unobservable mode of the pair (C, A). If $s_2 \in C$ is an unobservable mode of the pair (C, A) we have $rank\begin{bmatrix} s_2I - A \\ C \end{bmatrix} < n.$

This means that a vector $\beta \in C^n$ can always be found, such that $\begin{bmatrix} s_2I - A \\ C \end{bmatrix}\beta = 0.$

This leads to $(s_2I - A)\beta = 0, \quad C\beta = 0, \ CA\beta = Cs_2\beta = s_2C\beta = 0.$

Hence $\begin{bmatrix} s_2I - A \\ C \\ CA \end{bmatrix}\beta = \begin{bmatrix} s_2I - A \\ C_1 \end{bmatrix}\beta = 0 \ rank\begin{bmatrix} s_2I - A \\ C_1 \end{bmatrix} < n$, i.e., s_2 is also an unobservable mode of the pair (C_1, A). As the pairs (C_1, A) and (C, A) have the same unobservable modes, their detectability is formally equivalent. Q.E.D.

Theorem 3. Necessary and sufficient conditions for (6) to be an UIO for the system defined by (4) are:

a) $rank(CE) = rankE$.

b) (C, A_1) is detectable pair, where

$$A_1 = A - E(CE)^{\dagger}CA. \quad (13)$$

Proof. Sufficiency. According to Lemma 2 the equation (9a) is solvable when condition a) holds. A special solution for H is $\tilde{H} = E(CE)^{\dagger}$.

In this case, the system dynamics matrix is $F = A - HCA - K_1C = A_1 - K_1C$, and can be stabilized by appropriate choice of the gain matrix K_1 due to the condition b). Finally, the remaining UIO matrices described in (6) can be calculated using equations (8) and (9). Thus the observer (6) is a UIO for the system (4).

Necessity. Since (6) is a UIO for (4), equation (9a) is solvable. This leads to the fact that the condition a) hold true according to Lemma 2. A general solution of the matrix H for equation (9a) can be calculated as $H = E(CE)^L + H_0\left[I_m - CE(CE)^{\dagger}\right]$, where $H_0 \in R^{n\times m}$ is an arbitrary matrix.

Substituting the solution for H into equation (9c) yields the following formula for the system dynamics matrix F

$$F = A - HCA - K_1C = \left[I_n - E(CE)^{\dagger}C\right]A - \left[K_1 \ \ H_0\right]\begin{bmatrix} C \\ [I_m - CE(CE)^{\dagger}]CA \end{bmatrix}$$

$$= A_1 - \left[K_1 \ \ H_0\right]\begin{bmatrix} C \\ CA_1 \end{bmatrix} = A_1 - \tilde{K}_1\tilde{C}_1, \text{ where } \tilde{K}_1 = \left[K_1 \ \ H_0\right] \text{ and } \tilde{C}_1 = \begin{bmatrix} C \\ CA_1 \end{bmatrix}.$$

Since the matrix F is stable, the pair $(\tilde{C}_1, A)$ is detectable and the pair (C, A_1) is also detectable according to Lemmas 2 and 3.

By setting $T = I$, $H = 0$ and $E = 0$ (no unknown inputs in the system) the observer (6) will become a classical full-order Luenberger observer. In this situation, condition a) in Theorem 3 clearly holds and condition b) is simply changed to that of (C, A) being detectable.

Condition b) can be verified in terms of the structural properties of the original system. In fact, this condition is equivalent to the requirement that the transmission zeros from the unknown inputs to the measurements must be stable, i.e. matrix $\hat{M} := \begin{bmatrix} sI_n - A & E \\ C & 0 \end{bmatrix}$ is of full column rank for all $s \in \overline{C}^+$. This can be proved as follows [2]. It can be verified that

$$\begin{bmatrix} I_n - E(CE)^{\dagger}C & sE(CE)^{\dagger} \\ 0 & I_m \\ E(CE)^{\dagger}C & -sE(CE)^{\dagger} \end{bmatrix}\hat{M} = \begin{bmatrix} sI_n - A_1 & 0 \\ C & 0 \\ -E(CE)^{\dagger}CA & E \end{bmatrix}$$

As the first matrix in the left side of the above equation is a full column rank matrix, we have $rank\hat{M} = rank\begin{bmatrix} sI_n - A_1 & 0 \\ C & 0 \\ -E(CE)^{\dagger}CA & E \end{bmatrix}$.

We have assumed that E is a full column rank matrix. Hence, condition b) is equivalent to the case when the matrix in the left side of the above equation is full column rank for all $s \in \overline{C}^+$. This is because the condition for the pair (C, A_1) to be detectable is equivalent to the following matrix $\begin{bmatrix} sI - A_1 \\ C \end{bmatrix}$ having full column rank for all $s \in \bar{C}^+$.

From the above analysis, it can be seen that K_1 is a free matrix of parameters in the design of UIO. After K_1 is determined, other parameter matrices in the UIO can be computed by equations (8) and (9). The only restriction on the matrix K_1 is that it must stabilize the system dynamics matrix F. The matrix K_1 which stabilizes the matrix F is in general not unique; therefore there is still some design freedom left in the choice of K_1, after unknown input decoupling conditions have been satisfied.

2.4.2. Design procedure for UIOs

A crucial issue in designing a UIO is to stabilize $F = A_1 - K_1C$ by choosing the matrix K_1, when the pair (C, A_1) is detectable. If (C, A_1) is observable, the task can be accomplished easily by using a pole placement procedure from any control system design packages. If (C, A_1) is only detectable an observable canonical decomposition procedure should be applied to (C, A_1), namely

$$PA_1P^{-1} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}, \qquad A_{11} \in R^{n_1 \times n_1},$$

$$CP^{-1} = \begin{bmatrix} \tilde{C} & 0 \end{bmatrix}, \qquad \tilde{C} \in R^{m \times n_1},$$

where n_1 is the of the observability matrix for (C, A_1), and $(\tilde{C}, A_{11})$ is observable. The details on computing the transformation matrix P are given in [2]. If all eigenvalues of A_{22} are stable, (C, A_1) is detectable and the matrix F can be stabilized.

$$F = A_1 - K_1C = P^{-1}\left[PAP^{-1} - PK_1CP^{-1}\right]P$$

$$= P^{-1}\left\{\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} - \begin{bmatrix} K_p^1 \\ K_p^2 \end{bmatrix}\begin{bmatrix} \tilde{C} & 0 \end{bmatrix}\right\}P \qquad = P^{-1}\begin{bmatrix} A_{11} - K_p^1\tilde{C} & 0 \\ A_{21} - K_p^2\tilde{C} & A_{22} \end{bmatrix}P,$$

where: $K_p = PK_1 = \begin{bmatrix} K_p^1 \\ K_p^2 \end{bmatrix}$, $\sigma(F) = \sigma(A_{22}) \cup \sigma(A_{11} - K_p^1\tilde{C})$.

As $(\tilde{C}, A_{11})$ is observable, K_p^1 can be determined via the pole placement. The matrix K_p^2 can be any matrix, because it does not affect the eigenvalues of matrix F. The design procedure of a UIO is thus given below.

Procedure 1.

1. Check the rank condition for E and CE; if $rank(CE) \neq rank E$, an UIO does not exist, go to 10.
2. Compute H, T and A_1 using $H = E(CE)^{\dagger}$, $T = I - HC$, $A_1 = TA$.
3. Check the observability; if (C, A_1) observable, a UIO exists and K_1 can be computed using pole placement, go to 9.
4. Construct a transformation matrix P for the observable canonical decomposition.
5. Perform an observable canonical decomposition on (C, A_1) $PA_1P^{-1} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$, $CP^{-1} = \begin{bmatrix} \tilde{C} & 0 \end{bmatrix}$.
6. Check the detectability of (C, A_1); if any one of the eigenvalues of A_{22} is unstable, a UIO does not exist, go to 10.
7. Select n_1 desirable eigenvalues and assign them to $A_{11} - K_p^1\tilde{C}$ using pole placement.
8. Compute $K_1 = P^{-1}K_p = P^{-1}\left[(K_p^1)^T (K_p^2)^T\right]^T$, where K_p^2 is arbitrary $(n - n_1) \times m$ matrix.
9. Compute F and K; $F = A_1 - K_1C$, $K = K_1 + K_2 = K_1 + FH$.
10. STOP.

Remarks. There exist simpler methods of determining if pair (C, A_1) is detectable than one using observable canonical decomposition e.g. Popov-Belevitch-Hautus (BPH) test. However, state transformation matrix P is used to compute observer gain matrix K i.e. assuming that to be designed observer exist we have to compute P and decompose pair (C, A_1) in order to compute K. Having already observable canonical form of pair (C, A_1) we use it in step 6 to check detectability of this pair.

Example. Consider the example used in [28, 24, 30, 6, 2] with the following parameter matrices: $A = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

1. It can easily be checked that $rank(CE) = rank E = 1$.
2. The matrices H, T and A_1 are calculated as:

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; A_1 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}.$$

3. The pair (C, A_1) is observable, a UIO exists, and the matrix K_1 can be determined via the pole placement procedure. We choose $\sigma(A_1 - K_1C) = \{-2, -2-i, -2+i\}$. Using standard pole placement routine we obtain $K_1 = \begin{bmatrix} 1 & 0 \\ -1 & -5 \\ 0 & 3 \end{bmatrix}$.

9. The matrices F and K are calculated as:

$$F = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & -1 & -4 \end{bmatrix}, \; K = \begin{bmatrix} 0 & 0 \\ -1 & -5 \\ 0 & 3 \end{bmatrix}.$$

Remark. Usually the choice of the gain matrix $K_1 \in R^{3\times 2}$ is not unique.

2.5. Observers for singular systems

2.5.1. Introduction and preliminaries

Singular systems are found in many fields of engineering, e.g. electrical circuits, aerospace engineering, chemical engineering etc.; they are endowed with many special features as for instance impulse terms and input derivatives in the state response, consistent initial conditions nonproperness of transfer matrix etc., these are not found in standard systems and make the singular systems theory more sophisticated than of standard systems.

We provide in this section basic results concerning singular systems and their observability and observers in order to discuss later FDI problem for these systems. Observers for singular systems can be themselves either standard or singular systems. The results provided in this section relay mostly on [18, 19].

Let us consider the singular continuous-time linear system

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \tag{14a}$$

$$y(t) = Cx(t), \tag{14b}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$, are the state, input and output vectors, respectively and E, A, B and C are real matrices of appropriate dimensions. In general case matrix E is singular, $\det E = 0$. It can be shown [18] that there exist nonsingular matrices P and Q such that (14) can be decomposed as follows

$$\dot{x}_1(t) = A_1x_1(t) + B_1u(t), \tag{15a}$$

$$y_1(t) = C_1x_1(t), \tag{15b}$$

and

$$N\dot{x}_2(t) = x_2(t) + B_2u(t), \tag{16a}$$

$$y_2(t) = C_2x_2(t), \tag{16b}$$

where $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} := Q^{-1}x$, $PEQ = \begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix}$, $PAQ = \begin{bmatrix} A_1 & 0 \\ 0 & I \end{bmatrix}$, $PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} := Q^{-1}x$,

$x_1 \in R^{n_1}$, $n_1 =$ order $\det[Es - A]$, $x_2 \in R^{n_2}$, $n_1 + n_2 = n$, $y = y_1 + y_2$, $CQ = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$, N is nilpotent matrix with index q, i.e., $N^{q-1} \neq 0$ and $N^q = 0$.

Definition 3. The system (14) is called completely controllable (c-controllable), if for any initial state $x_0 \in R^n$ and every $x_f \in R^n$ there exist

$t_f > 0$ and q - times piecewise continuously differentiable input function $u(t)$ on the $[0, t_f]$ such that, $x(t_f) = x_f$.

Lemma 4. System (14) is c-controllable if and only if

$$rank\left[B_1, A_1B_1, \cdots, A^{n_1-1}B_1\right] = n_1, \quad (17a)$$
$$rank\left[B_2, NB_2, \cdots, N^{q-1}B_2\right] = n_2. \quad (17b)$$

Proof. See e.g. [18].

Definition 4. The system (14) is called completely observable (c-observable), if the following conditions are satisfied.

1. $rank\begin{bmatrix} Es-A \\ C \end{bmatrix} = n$ for all finite $s \in C$;

2. $rank\begin{bmatrix} E \\ C \end{bmatrix} = n$.

2.5.2. Standard observers for singular systems

We assume that system (14) is c-controllable and c-observable and $rankC = p$. Let us consider the following system

$$\dot{z}(t) = Fz(t) + Gu(t) + Ky(t), \quad (18)$$

where $z(t) \in R^{n-p}$ is a state vector, $u(t)$, $y(t)$ are defined as for (14), F, G, K are real matrices of appropriate dimensions. Defining $\tilde{e}(t) := Tx(t) - z(t)$ where $T := XE$, , and taking into consideration (14) and (18) we obtain

$$\dot{\tilde{e}}(t) = T\dot{x}(t) - \dot{z}(t) = XAx(t) + XBu(t) - F\left[Tx(t) - \tilde{e}(t)\right]$$
$$- Gu(t) - KCx(t) = (XA - FT - KC)x(t) + Fe(t)$$
$$+ (XB - G)u(t). \quad (19)$$
$$XA = FT + KC, \qquad G = XB, \quad (20)$$

then (19) yields

$$\dot{\tilde{e}}(t) = F\tilde{e}(t). \quad (21)$$

Clearly if matrix F is Hurwitz than $\tilde{e}(t)$ tends to zero. We choose matrix $T(X)$ such that

$$\det\begin{bmatrix} T \\ C \end{bmatrix} = \det\begin{bmatrix} XE \\ C \end{bmatrix} \neq 0. \quad (22)$$

We define state estimation $\hat{x}$ by the following relationship

$$\begin{bmatrix} T \\ C \end{bmatrix}\hat{x} = \begin{bmatrix} z \\ y \end{bmatrix}. \quad (23)$$

From the above we have

$$\hat{x} = \begin{bmatrix} T \\ C \end{bmatrix}^{-1}\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} M & N \end{bmatrix}\begin{bmatrix} z \\ y \end{bmatrix}, \quad (24)$$

where

$$\begin{bmatrix} M & N \end{bmatrix} := \hat{x} = \begin{bmatrix} T \\ C \end{bmatrix}^{-1} \quad (25)$$

It is not difficult to show that

$$e(t) := x(t) - \hat{x}(t) = Me^{Ft}e(0). \quad (26)$$

Thus, if F is Hurwitz $e(t)$ tends to zero as required and

$$\dot{z}(t) = Fz(t) + Gu(t) + Ky(t), \quad (27a)$$
$$\hat{x} = Mz + Ny, \quad (27b)$$

is standard observer for (14). According to Definition 4 system (14) is c-observable if and only if

$$rank\begin{bmatrix} Es-A \\ C \end{bmatrix} = n \text{ forallfinite } S \in C \quad (28a)$$

and

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = n. \quad (28b)$$

From the relationship $rank\begin{bmatrix} XE \\ C \end{bmatrix} = rank\left\{\begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} E \\ C \end{bmatrix}\right\}$

follows that (22) implies (28b) and $rankX = n - p$. It is easy to show that if conditions (28) are satisfied, the equation $XA = FXE + KC$ has solution X satisfying condition (22). From (20) and (27) we have

$$XA = \begin{bmatrix} F & K \end{bmatrix}\begin{bmatrix} T \\ C \end{bmatrix}$$

and

$$\begin{bmatrix} F & K \end{bmatrix} = XA\begin{bmatrix} T \\ C \end{bmatrix}^{-1} = XA\begin{bmatrix} M & N \end{bmatrix}. \quad (29)$$

Given matrices A, B, C, E we choose matrix X subject to (22) and matrix F having desired eigenvalues with negative real parts. Than we compute matrices $K = XAN$ and $G = XB$.

2.6. Full-order perfect observers

The observer problem for singular continuous-time and discrete time systems has been considered by many researchers [9, 10]. The results provided in this section relay on [19].

Consider the singular continuous-time linear system

$$Ex(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \quad (30a)$$
$$y(t) = Cx(t), \quad (30b)$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$, are the state, input and output vectors, respectively and E, A, B and C are real matrices of appropriate dimensions. We assume that:

$$\det E = 0, \quad rank\begin{bmatrix} E \\ C \end{bmatrix} = n, \quad (31a)$$
$$\det\left[Es - A\right] \neq 0 \quad \text{for some} \quad s \in C, \quad (31b)$$
$$rankC = p. \quad (31c)$$

Consider the singular system

$$E\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K\left[C\hat{x}(t) - y(t)\right], \hat{x}(0) = \hat{x}_0 \quad (32)$$

where $\hat{x}(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$, are the observer state, observed system input and output vectors, respectively and E, A, B, C and K are real matrices of appropriate dimensions.

Definition 5. The system (32), is called a full-order perfect observer of the system (30) if and only if $\hat{x}(t) = x(t)$ for $t > 0$ and any initial conditions x_0 and $\hat{x}_0$ of (30) and (32).

Theorem 4. There exist a full-order perfect observer of the system (30) if the conditions (31) and

$$\text{rank}\begin{bmatrix} Es-A \\ C \end{bmatrix} = n \quad \text{forallfinite} \quad s \in C \quad (33)$$

are satisfied.

Proof. See [19].

The proof presented in [19] yields the following procedure for computing of the full-order perfect observer (32).

Procedure 2.

1. Check the conditions (31) and (33). If they are satisfied than go to the next step.
2. Choose a matrix K so that $\det\left[Es - (A + KC)\right] = \alpha$ forall $s \in C$ holds.
3. Compute the desired observer (32).

3. Concluding remarks

A general introduction to FDI methods has been given. General idea of model based FDI methods has been presented. The issue of FDI robustness has been mentioned and short historical background of FDI has been given. Further, the basic concepts of observers have been considered, such as observability and detectability of LTI systems, Luenberger observer, unknown input observer (UIO). Same celected results from the thory of UIOs and design procedure for UIO has been given. Further results concerning observer-based FDI will be presented in the next paper. Finally, a short introduction to singular systems has been given and observers for these systems have been considered.

References

[1] J. Chen. Robust residual generation for model-based fault diagnosis of dynamic systems. PhD thesis, Univeristy of York, York, UK, 1995.
[2] J. Chen and R. J. Patton. Robust model-based fault diagnosis for dynamic

systems. Kluwer Academic Publishers, 1999.
[3] E. Y. Chow and A. S. Willsky. Issues in the development of a general algorithm for reliable failure detection. In Proceedings of the 19th IEEE Conference on Decision and Control, Albuquerque, NM., 1980.
[4] E. Y. Chow and A. S. Willsky. Analytical redundancy and the design of robust detection systems. IEEE Transactions on Automatic Control, AC-29(7):603-614, 1984.
[5] R. N. Clark. Instrument fault detection. IEEE Trans. Aero. & Electron. Syst., AES-14:456-465, 1978.
[6] R. N. Clark. A simplied instrument failure detection scheme. IEEE Trans. Aero. & Electron. Syst., AES-14:558-563, 1978.
[7] R. N. Clark. The dedicated observer approach to instrument failure detection. In Proceedings of the 18th IEEE Conference on Decision and Control, pages 237-241, Fort Lauderdale, Fla., 1979.
[8] R. N. Clark, D. C. Fosth, and V. M. Walton. Detecting instrument malfunctions in control systems. IEEE Trans. Aero. & Electron. Syst., AES-11:465-473, 1975.
[9] L. Dai. Observers for discrete singular systems. IEEE Transactions on Automatic Control, AC- 33(2):187-191, 1988.
[10] L. Dai. Singular control systems. Springer-Verlag, Berlin, Germany, 1989.
[11] G. E. Dullerud and F. Paganini. A course in robust control theory: a convex approach. Springer-Verlag, New York, 2000.
[12] P. M. Frank. Fault diagnosis in dynamic system via state estimation - - a survey. In S. Tzafestas, Singh, editor, System Fault Diagnostics, Reliability & Related Knowledge-based Approaches, volume 1, pages 35-98, Dordrecht, 1987. D. Reidel Press.
[13] P. M. Frank. Enhancement of robustness in observer-based fault detection. In Preprints of IFAC/IMACS Sympo. SAFEPROCESS'91, volume 1, pages 275-287, Baden-Baden, 1991.
[14] P. M. Frank and L. Keller. Sensitivity discriminating observer design for instrument failure detection. IEEE Trans. Aero. & Electron. Syst., AES-16:460-467, 1981.
[15] J. J. Gertler. Survey of model-based failure detection and isolation in complex plants. Control Systems Magazine, 8(6):3-11, 1988.
[16] M. Hou and P. C. Mueller. Design of observers for linear systems with unknown inputs.
IEEE Transactions on Automatic Control, AC-37(6):871-875, 1992.
[17] T. Kaczorek. Linear control systems. Research Studies Press and J. Wiley, New York, 1992.
[18] T. Kaczorek. Teoria sterowania i systemów. Wydawnictwo Naukowe PWN, Warszawa, 1999. (in Polish).
[19] T. Kaczorek. Reduced-order and standard observers for singular continuous-time linear systems. Machine Intelligence & Robotic Control, 2(3):93-98, 2000.
[20] J. Korbicz. Obserwatory stanu w układach diagnostyki procesów. In XIV Krajowa Konferencja Automatyki, Zielona Góra, 2002. (in Polish).
[21] J. Korbicz, J. M. Kościelny, Z. Kowalczuk, and W. Cholewa. Diagnostyka procesów. WNT, Warszawa, 2002. (in Polish).
[22] J. Korbicz, K. Patan, and A. Obuchowicz. Neural network fault detection systems for dynamic processes. Bulletin of the Polish Academy of Sciences, Technical Sciences, 49(2):301-321, 2001.
[23] J. M. Kościelny. Diagnostyka zautomatyzowanych procesów przemysłowych. Exit,
Warszawa, 2000. (in Polish).
[24] R. J. Miller and R. Mukundan. On designing reduced-order observers for linear time-invariant systems subject to unknown inputs. International Journal of Control, 35(1):183-188, 1982.
[25] R. J. Patton and J. Chen. A re-examination of the relationship between parity space and
observer-based approaches in fault diagnosis. European J. of Diagnosis and Safety in Automation, 1(2):183-200, 1991.
[26] R. J. Patton and J. Chen. Observer-based fault detection and isolation: robustness and applications. Control Engineering Practice, 5(5):671-682, 1997.
[27] R. J. Patton, S. W. Willcox, and S. J. Winter. A parameter insensitive technique for aircraft sensor fault analysis. In Proc. of the AIAA Conf. on Guidance, Navigation & Control, Williamsburg, Va., 1986.
[28] S. H. Wang, E. L. Davison, and P. Dorato. Observing the states of systems with unmeasureable disturbance. IEEE Transactions on Automatic Control, AC-20:716-717, 1975.
[29] K. Watanabe and D. M. Himmelblau. Instrument fault detection in systems with uncertainties. Int. J. Sys. Sci., 13(2):137-158, 1982.
[30] F. Yang and W. W. Richard. Observers for linear systems with unknown inputs. IEEE Transactions on Automatic Control, AC-33(7):677-681, 1988.

Tytuł: Wykrywanie uszkodzeń w układach dynamicznych przy użyciu obserwatorów - część I

Artykuł recenzowany

Maciej TWARDY, Tadeusz KACZOREK
POLITECHNIKA WARSZAWSKA, WYDZIAŁ ELEKTRYCZNY

Observer-based fault detection in dynamical systems - part II

Abstract

The paper is the second part of a survey of observer-based fault detection and isolation (FDI) methods. Observer-based FDI schemes are presented, both for standard and singular systems. The applications of unknown input observer (UIO) as well perfect observers to FDI are shown. Basic ideas and concepts underlying much of model-based FDI, as well a general introduction to FDI have been given in the first part of this survey.

Streszczenie

Artykuł jest drugą częścią przeglądowej pracy nt. użycia obserwatorów do wykrywania uszkodzeń izolacji uszkodzeń (ang. *FDI*) w układach dynamicznych. Zaprezentowano schematy wykrywania i izolacji uszkodzeń przy użyciu obserwatorów zarówno dla układów standardowych jak i singularnych. Przedstawiono zastosowanie obserwatorów o nieznanym wejściu jak również obserwatorów doskonałych dla celów FDI. Podstawowe idee i pojęcia metod FDI opartych na modelu, jak również ogólne wprowadzenie w tematykę FDI zawiera pierwsza część pracy.

1. Observer-based FDI schemes

1.1. FDI schemes based on UIOs

The main requirement for fault detection is to generate a residual signal which is robust to the system uncertainty. On the other hand to detect a particular fault, the residual has to be sensitive to this fault. In this section we present several FDI observer-based schemes.

The results concerning FDI observer-based schemes for standard systems have been given in [1]. We present some generalizations of theses results for singular systems. We also try to apply perfect observers (they have been discussed in the first part of this survey [5]) to observer-based FDI.

According to [1], a system with possible sensor and actuator faults can be described as

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) + Bf_a(t), \quad (1a)$$
$$y(t) = Cx(t) + f_s(t), \quad (1b)$$

where $f_a \in R^r$ and $f_s \in R^m$ are vectors of actuator and sensor faults, respectively. Our task is to generate a disturbance decoupled residual, using the following UIO

$$\dot{z}(t) = Fz(t) + TBu(t) + Ky(t),$$
$$\hat{x}(t) = z(t) + Hy(t),$$

where $\hat{x} \in R^n$, $z \in R^n$, F, T, K and H are real matrices of appropriate dimensions. These matrices must satisfy additional conditions guaranteeing the convergence to zero of estimation error. More details can be