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Perfect observers for continuous time linear systems

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W 1999 roku ukończył studia wyższe na Wydziale Elektrycznym Politechniki Warszawskiej, gdzie w 2002 roku uzyskał stopień doktora nauk technicznych w zakresie elektrotechniki. Od 2003 roku pracuje na stanowisku adiunkta w Instytucie Sterowania i Elektroniki Przemysłowej na Wydziale Elektrycznym Politechniki Warszawskiej. Aktualny obszar jego zainteresowań to obserwatory funkcjonalne i doskonałe.

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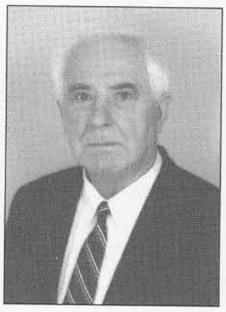

Uzyskał dyplom magistra inżyniera elektryka w roku 1956 na Wydziale Elektrycznym Politechniki Warszawskiej. Na tym samym Wydziale w roku 1962 uzyskał stopień naukowy doktora nauk technicznych, a w roku 1964 - doktora habilitowanego. Tytuł naukowy profesora nadzwyczajnego nadała Mu Rada Państwa w roku 1971, a profesora zwyczajnego w 1974 roku. Członkiem korespondentem PAN został wybrany w 1986 roku, a członkiem rzeczywistym w 1998. Od czerwca 1999 roku jest również członkiem zwyczajnym Akademii Inżynierskiej w Polsce. W latach 1969-1970 był dziekanem Wydziału Elektrycznego, a w latach 1970-1979 prorektorem d/s nauczania Politechniki Warszawskiej. W latach 1970-1981 był dyrektorem Instytutu Sterowania i Elektroniki Przemysłowej Politechniki Warszawskiej. W latach 1988-1991 był dyrektorem Stacji Naukowej PAN w Rzymie. Jest autorem 18 książek, w tym 5-ciu wydanych za granicą oraz ponad 500 artykułów i rozpraw naukowych, opublikowanych w czasopismach krajowych i zagranicznych. Główne kierunki badań naukowych to analiza i synteza układów sterowania i systemów, a w szczególności układy wielowymiarowe, układy singularne i układy dodatnie.

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Abstract

Full-order, reduced-order and polynomial perfect observers are considered. Necessary and sufficient conditions for the existence of the perfect observers are established and procedures for computation of matrices of the perfect observers are derived. Illustrating numerical examples and some simulations results are also presented.

Streszczenie

W pracy rozważana jest teoria obserwatorów doskonałych pełnego rzędu, zredukowanego rzędu oraz doskonałych obserwatorów wielomianowych. Zostały tu przedstawione konieczne i wystarczające warunki istnienia obserwatorów doskonałych oraz procedury pozwalające na ich wyznaczanie poparte przykładami numerycznymi oraz wynikami symulacji.

Keywords: full-order, reduced-order, perfect observers, polynomial perfect observers
Słowa kluczowe: obserwatory doskonałe, pełnego rzędu, zredukowanego rzędu, wielomianowe obserwatory doskonałe

1. Introduction

The design of observers for linear systems has been considered in many papers and books [1, 2, 5-19]. Necessary and sufficient conditions for the existence of the perfect observers for linear standard systems have been established in [5-7]. A new concept of perfect full-order and reduced-order perfect observers for continuous-time linear systems has been proposed in [9, 10].

The subject of this paper is to present new method for designing of full-order, reduced-order and polynomial perfect observers for continuous-time standard and singular linear systems. Necessary and sufficient conditions will be established for the existence of the observers and procedures for computation of the matrices of the observers will be derived. The procedures will be illustrated by numerical examples and some simulation results will be also presented.

2. Problem formulation

Let $R^{n\times m}$ be the set of real matrices of dimension $n\times m$ $\left(R^n := R^{n\times 1}\right)$ and $R^{n\times m}(s)$ be the set of rational matrices. The set of polynomial matrices in s will be denoted by $R^{n\times m}[s]$ and the identity matrix by I_n.

Consider the continuous-time linear system

$$E\dot{x} = Ax + Bu \quad (1a)$$
$$y = Cx + Du \quad (1b)$$

where $\dot{x} = \frac{dx}{dt}$, $x\in R^n$, $u\in R^m$ and $y\in R^p$ are the state, input and output vectors, respectively, and $E, A\in R^{n\times n}$, $B\in R^{n\times m}$, $C\in R^{p\times n}$, $D\in R^{p\times m}$ are known real matrices.

Definition 1 System (1) is called standard continuous-time linear system if and only if $E = I_n$ (in general case $\det E \neq 0$)

$$\dot{x} = Ax + Bu \quad (2a)$$
$$y = Cx + Du \quad (2b)$$

Definition 2 System (1) is called singular (descriptor) continuous-time linear system if and only if $\det E = 0$, ($\operatorname{rank} E = r < n$).

If the system (1) is not singular then using pre-multiplication of the equation (1a) by the inverse matrix E^{-1} we obtain the standard continuous-time linear system.

Definition 3 A singular system [9]

$$E\dot{\hat{x}} = F\hat{x} + Gu + Hy \quad (3)$$

where $E, F\in R^{n\times n}$, $G\in R^{n\times m}$, $H\in R^{n\times p}$ and $\hat{x}(t)$ is the estimate of the vector $x(t)$, is called perfect observer of (1) if and only if

$$\hat{x}(t) = x(t) \text{ for } t > 0 \quad (4)$$

Definition 4 A singular system [9]

$$\begin{aligned} E\dot{\tilde{x}} &= F\tilde{x} + Gu + Hy \\ \hat{x} &= J\tilde{x} + Lu + My \end{aligned} \quad (5)$$

where $E, F\in R^{r\times r}$, $G\in R^{r\times m}$, $H\in R^{r\times p}$, $J\in R^{n\times r}$, $L\in R^{n\times m}$, $M\in R^{n\times p}$ and $\hat{x}(t)$ is estimate of the vector $x(t)$, is called the reduced order perfect observer if and only if the condition (4) satisfied and $\dim\tilde{x}(t)\leq \dim x(t)$.

Consider a linear system described by the polynomial equations

$$x(s) = Pu(s) \quad (6a)$$
$$y(s) = Cx(s) \quad (6b)$$

where $P\in R^{n\times m}(s)$, $C\in R^{p\times n}$ and $x(s)$, $u(s)$ and $y(s)$ are the Laplace transforms of x, u and y, respectively.

Definition 5 Polynomial equation

$$\hat{x}(s) = Lu(s) + Vy(s) \text{ where } L\in R^{n\times m}[s],\ V\in R^{n\times p}[s] \quad (7)$$

is called the polynomial perfect observer if and only if (4) holds.

The aim of this paper is to design the perfect full-order, reduced-order and polynomial perfect observers. Necessary and sufficient conditions for the existence of a solution to the problem will be established and procedures for computation of matrices of the perfect observers will be proposed.

3. Preliminaries

In sequel the following elementary row and column operations will be used [4]:

1) Addition of the j-th row (column) multiplied by any nonzero real number k to the i-th row (column) - denoted by $L[i+j\times(k)]$, $(R[i+j\times(k)])$
2) Multiplication of the i-th row (column) by any nonzero number k - $L[i\times(k)]$, $(R[i\times(k)])$
3) Interchange of the i-th and j-th rows (columns) - $L[i,j]$, $(R[i,j])$

The elementary row operations are equivalent to premultiplication of the matrix by a suitable row operations matrix P and the elementary column operations are equivalent to postmultiplication of the matrix by a suitable elementary column operations matrix Q.

Using the Weierstrass-Kronecker decomposition to (1) and premultiplying equation (1a) by P and defining the new semistate vector $\bar{x}=Q^{-1}x$, the equations (1) can be written as follows.

$$\begin{aligned} PEQ\dot{\bar{x}} &= PAQ\bar{x}+PBu \\ y &= CQ\bar{x} \\ z &= LQ\bar{x} \end{aligned} \tag{8}$$

where $PEQ=\begin{bmatrix} I_d & 0 \\ 0 & E_2 \end{bmatrix}$, $d=\deg(\det[Es-A])$, $E_2=[\tilde{e}_{ij}]$, $\tilde{e}_{ij}=0$ for $i\neq j-1$,

$$PAQ=\begin{bmatrix} A_1 & 0 \\ 0 & I \end{bmatrix},\ PB=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix},\ CQ=[C_1\ \ C_2],\ LQ=[L_1\ \ L_2],\ \bar{x}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Matrices P and Q can be obtained by the use of the procedure given in the appendix of [16].

Using (8) the singular system (1) can be decomposed into the following three subsystems.

$$\dot{x}_1 = A_1x_1+B_1u \tag{9a}$$
$$y_1 = C_1x_1 \tag{9b}$$
$$E_2\dot{x}_2 = x_2+B_2u \tag{9c}$$
$$y_2 = C_2x_2 \tag{9d}$$
$$0 = x_3+B_3u \tag{9e}$$
$$y_3 = C_3x_3 \tag{9f}$$
$$y = y_1+y_2+y_3 \tag{9g}$$

The subsystems are completely independent and the system (9a-9b) is standard, system (9c-9d) is completely singular ($\det[Es-A]=c$, $c\in \mathbf{C}$ the field of complex numbers) and system (9e-9f) is static (algebraic).

4. Perfect observers for standard systems

The standard system (2) or equivalently pair (A,C) is observable if and only if the observability matrix O has full column rank [4]

$$\operatorname{rank}O=\operatorname{rank}\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}=n \tag{10}$$

Lemma 1. There exist matrix $T\in R^{n\times n}$ such that [4, 9]

$$\begin{aligned} \bar{A}=TAT^{-1}&=\begin{bmatrix} \bar{A}_1 & 0^{k\times(n-p)} \\ \bar{A}_2 & I_{n-p} \\ \bar{A}_3 & 0 \end{bmatrix},\ \bar{B}=TB=\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \bar{B}_3 \end{bmatrix}, \\ \bar{C}=CT^{-1}&=[\bar{C}_1\ \ 0] \end{aligned} \tag{11}$$

if and only if the system (2) is observable, where $\bar{A}_1\in R^{k\times p}$, $\bar{A}_2\in R^{(n-p)\times p}$, $\bar{A}_3\in R^{p-k\times p}$, $\bar{B}_1\in R^{k\times m}$, $\bar{B}_2\in R^{(n-p)\times m}$, $\bar{B}_3\in R^{p-k\times m}$, $\bar{C}_1\in R^{p\times p}$, $\det\bar{C}_1\neq 0$ and

$$k=\begin{cases} 2p-\operatorname{rank}\begin{bmatrix} C \\ CA \end{bmatrix} & \text{if } \operatorname{rank}\begin{bmatrix} C \\ CA \end{bmatrix}<\min(n,\ 2p) \\ 0 & \text{otherwise} \end{cases}$$

Proof. It is well-known [4] that there exists a matrix $\tilde{T}$ such that

$$\begin{aligned} \tilde{A}=\tilde{T}A\tilde{T}^{-1}&=\begin{bmatrix} \tilde{A}_{11} & \dots & \tilde{A}_{1p} \\ \vdots & \ddots & \vdots \\ \tilde{A}_{p1} & \dots & \tilde{A}_{pp} \end{bmatrix},\ \tilde{B}=\tilde{T}B, \\ \tilde{C}=C\tilde{T}^{-1}&=[\tilde{C}_1\ \dots\ \tilde{C}_p] \end{aligned} \tag{12}$$

if and only if the system (2) is observable, where $\tilde{A}_{ij}=\begin{bmatrix} 0 & \dots & 0 & a_0^{ij} \\ 0 & \dots & 0 & a_1^{ij} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & a_{d_i-1}^{ij} \end{bmatrix}$,

$$i\neq j,\ \tilde{A}_{ii}=\begin{bmatrix} 0 & 0 & \dots & 0 & a_0^{ii} \\ 1 & 0 & \dots & 0 & a_1^{ii} \\ 0 & 1 & \dots & 0 & a_2^{ii} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & a_{d_i-1}^{ii} \end{bmatrix},\ \tilde{C}_i=\begin{bmatrix} 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 1 \\ 0 & \dots & 0 & c_{1i} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & c_{p-i,i} \end{bmatrix}$$

By the used of the permutation operations we may obtain the canonical form (11). Note that k is equal to the number of first zero rows of the matrix $\bar{A}$ (Kronecker indexes equal 1: $d_q=1$, $1\leq q\leq 2p-\operatorname{rank}\begin{bmatrix} C \\ CA \end{bmatrix}\leq p$, where $\operatorname{rank}\begin{bmatrix} C \\ CA \end{bmatrix}-p$ is number of Kronecker indexes greater than 1).

Theorem 1. The full order perfect observer (3) exists if and only if the standard system (2) is observable.

The following three cases will be considered.

a) In first case when matrix C is of full column rank ($\operatorname{rank}C=n$), the desired estimate is given by $\hat{x}=C^{+}y-C^{+}Du$ where C^{+} is Moore-Penrose inverse matrix.

Procedure 1.

Step 1. Find the Moore-Penrose inverse matrix C^{+} of the matrix C ($C^{+}=C^{-1}$ if $\det C\neq 0$ and then compute $M=C^{+}$ and $L=-C^{+}D$ and $\hat{x}=Lu+My$

b) If then by Lemma 1

$$\dot{\bar{x}}=\begin{bmatrix} \bar{A}_1 & 0^{k\times(n-p)} \\ \bar{A}_2 & I_{n-p} \\ \bar{A}_3 & 0 \end{bmatrix}\bar{x}+\begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \bar{B}_3 \end{bmatrix}u \tag{13a}$$

$$y=[\bar{C}_1\ \ 0]\bar{x}+Du \tag{13b}$$

where $\bar{x}=\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}=Tx$.

Using the internal $n-p$ equations (from $k+1$ to $k+n-p$) of the system (13a) and knowing $\bar{x}_1$ we may find all x. From equation (13b) we obtain $\bar{x}_1=\bar{C}_1^{-1}y-\bar{C}_1^{-1}Du$ and

$$\begin{bmatrix} 0^{(n-p)\times k} & I_{n-p} & 0 \\ & 0 & \end{bmatrix}\dot{\bar{x}}=\begin{bmatrix} 0 & I_{n-p} \\ I_p & 0 \end{bmatrix}\bar{x}+\begin{bmatrix} \bar{B}_2-\bar{A}_2\bar{C}_1^{-1}D \\ \bar{C}_1^{-1}D \end{bmatrix}u+\begin{bmatrix} \bar{A}_2\bar{C}_1^{-1} \\ -\bar{C}_1^{-1} \end{bmatrix}y \tag{14}$$

The system (14) is completely singular ($\det[Es-A]=c$, where $c\in R$ is nonzero real number) and the vector x is known for all $t>0$.

By the use of equation (14) and $\bar{x}=Tx$ we may construct the perfect observer (3).

Estimated error of the vector x (obtained using perfect observer (3)) is equal to the error of the system (14) $e=x-\hat{x}=T^{-1}\bar{e}=T^{-1}(\bar{x}-\hat{\bar{x}})$

The dynamic equation of this error is given by $\begin{bmatrix} 0^{(n-p)\times k} & I_{n-p} & 0 \\ & 0 & \end{bmatrix}\dot{\bar{e}} = \begin{bmatrix} 0 & I_{n-p} \\ I_p & 0 \end{bmatrix}\bar{e}$ where $\bar{e}(t)=0$ is equal $e(t)=0$ for all $t>0$.

Therefore the perfect observer for standard continuous-time linear system can be obtained by the use of the following procedure.

Procedure 2.

Step 1. For known A, B, C find the Brunovsky-Luenberger second canonical form.

Step 2. Using permutation operations find matrix T such that (11) holds.

Step 3. Compute the matrices of the perfect observer (3)

$$E = \begin{bmatrix} 0^{(n-p)\times k} & I_{n-p} & 0 \\ & 0 & \end{bmatrix}T,\quad F = \begin{bmatrix} 0 & I_{n-p} \\ I_p & 0 \end{bmatrix}T,\quad G = \begin{bmatrix} \bar{B}_2 - \bar{A}_2\bar{C}_1^{-1}D \\ \bar{C}_1^{-1}D \end{bmatrix},$$

$$H = \begin{bmatrix} \bar{A}_2\bar{C}_1^{-1} \\ -\bar{C}_1^{-1} \end{bmatrix} \text{ and } E\dot{\hat{x}} = F\hat{x} + Gu + Hy$$

c) If rank$C>1$ then the order of the observer can be reduced

Lemma 2. There exists a matrix $T \in R^{n\times n}$ such that

$$\bar{A} = TAT^{-1} = \begin{bmatrix} \bar{A}_1 \\ 0 \;\; I_{n-p} \\ \bar{a}_2 \end{bmatrix},\quad \bar{B} = TB = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \bar{b}_3 \end{bmatrix}, \tag{15}$$
$$\bar{C} = CT^{-1} = [\bar{C}_1 \;\; 0]$$

if and only if system (2) is observable [4], where $\bar{A}_1 \in R^{(p-1)\times n}$, $\bar{a}_2 \in R^{1\times n}$, $\bar{B}_1 \in R^{(p-1)\times m}$, $\bar{B}_2 \in R^{(n-p)\times m}$, $\bar{b}_3 \in R^{1\times m}$, $\bar{C}_1 \in R^{p\times p}$, $\det \bar{C}_1 \neq 0$ and $\bar{C}_1$ is the permutation matrix (each row and each column consists only one nonzero element equal 1).

For a singular matrix A $(\det A = 0)$ may be l subsystems (the multisystem canonical form)

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & & \bar{A}_{1l} \\ 0 \;\; I_{n_1} & 0 & \cdots & 0 \\ \bar{a}_{11} & \bar{a}_{12} & & \bar{a}_{1l} \\ \bar{A}_{21} & \bar{A}_{22} & & \bar{A}_{2l} \\ 0 & 0 \;\; I_{n_2} & \cdots & 0 \\ \bar{a}_{21} & \bar{a}_{22} & & \bar{a}_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{A}_{l1} & \bar{A}_{l2} & & \bar{A}_{ll} \\ 0 & 0 & \cdots & 0 \;\; I_{n_l} \\ \bar{a}_{l1} & \bar{a}_{l2} & & \bar{a}_{ll} \end{bmatrix},\quad \bar{B} = \begin{bmatrix} \bar{B}_{11} \\ \bar{B}_{12} \\ \bar{b}_{13} \\ \bar{B}_{21} \\ \bar{B}_{22} \\ \bar{b}_{23} \\ \vdots \\ \bar{B}_{l1} \\ \bar{B}_{l2} \\ \bar{b}_{l3} \end{bmatrix}, \tag{15'}$$
$$\bar{C} = \text{diag}([\bar{C}_1 \;\; 0], [\bar{C}_2 \;\; 0], \ldots, [\bar{C}_l \;\; 0])$$

where $1 \le l \le n - \text{rank}A + 1$, $\sum_{i=1}^{l} n_i = n - p$, $\sum_{i=1}^{l} \text{rank}\bar{C}_i = p$.

Proof. Using $\bar{A}$ and $\bar{C}$ in canonical form (15) we compute $\bar{O} = \begin{bmatrix} \bar{C} \\ \bar{C}\bar{A} \\ \vdots \\ \bar{C}\bar{A}^{n-1} \end{bmatrix}$.

Next from $\bar{O}$ we choose n linearly independent rows such that the new matrix $\tilde{\bar{O}} = I_n$ (identity matrix) and $\tilde{\bar{O}} = \tilde{O}T = T$, $\tilde{O}$ where is a matrix constructed from n linearly independent rows of the observability matrix O. There exists nonsingular matrix T such that (15) holds if and only if matrix O has full column rank. The matrix T can be constructed using n linearly independent rows of the matrix O. Analogous results we obtain for the multisystem canonical form (15').

Theorem 2. The reduced order perfect observer (5) exists if and only if the pair (A, C) is observable.

Proof. By the lemma 2. there exists a matrix T such that

$$\dot{\bar{x}} = \begin{bmatrix} \bar{A}_1 \\ 0 \;\; I_{n-p} \\ \bar{a}_1 \end{bmatrix}\bar{x} + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \bar{b}_3 \end{bmatrix}u \tag{16a}$$

$$y = [\bar{C}_1 \;\; 0]\bar{x} + Du \tag{16b}$$

if and only if the standard linear system (2) is observable.

If (15) then $\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = Tx$, where $\bar{x}_1 \in R^p$, $\bar{x}_2 \in R^{n-p}$.

Knowing $\bar{x}_1$ we may find the vector x. From (16b) we obtain

$$\begin{bmatrix} 0\ldots0 & I_{n-p} & 0 \\ & 0 & \end{bmatrix}\dot{\bar{x}} = \begin{bmatrix} 0 & I_{n-p} \\ I_p & 0 \end{bmatrix}\bar{x} + \begin{bmatrix} \bar{B}_2 \\ \bar{C}_1^{-1}D \end{bmatrix}u + \begin{bmatrix} 0 \\ -\bar{C}_1^{-1} \end{bmatrix}y \tag{17}$$

Using (17) we can find the vector x. From (17) and $\bar{x} = Tx$ we obtain the reduced order perfect observer

$$\begin{bmatrix} I_{n-p} & 0 \\ 0 & 0 \end{bmatrix}\dot{\tilde{x}} = \begin{bmatrix} 0 & I_{n-p} \\ 1 & 0 \end{bmatrix}\tilde{x} + \begin{bmatrix} \bar{B}_2 \\ (\bar{C}_1^{-1})_{(p)}D \end{bmatrix}u + \begin{bmatrix} 0 \\ -(\bar{C}_1^{-1})_{(p)} \end{bmatrix}y$$
$$\hat{x} = T^{-1}\begin{bmatrix} 0 & 0 \\ 0 & I_{n-p} \end{bmatrix}\tilde{x} + T^{-1}\begin{bmatrix} -\bar{C}_1^{-1}D \\ 0 \end{bmatrix}u + T^{-1}\begin{bmatrix} \bar{C}_1^{-1} \\ 0 \end{bmatrix}y \tag{18}$$

where $\tilde{x} = \begin{bmatrix} \bar{x}_{1(p)} \\ \bar{x}_2 \end{bmatrix}$ ($\bar{x}_{1(p)}$ p-th row/element of the vector $\bar{x}_1$) and $(\bar{C}_1^{-1})_{(p)}$ (p-th row of the matrix $\bar{C}_1^{-1}$).

The error equation $e = x - \hat{x} = T^{-1}\left(\bar{x} - \begin{bmatrix} 0 & 0 \\ 0 & I_{n-p} \end{bmatrix}\hat{\tilde{x}} - \begin{bmatrix} -\bar{C}_1^{-1}D \\ 0 \end{bmatrix}u - \begin{bmatrix} \bar{C}_1^{-1} \\ 0 \end{bmatrix}y\right)$

where $\bar{e} = \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix}$ and

$$\bar{e}_1 = \bar{x}_1 + \bar{C}_1^{-1}Du - \bar{C}_1^{-1}y \tag{19a}$$
$$\bar{e}_2 = \bar{x}_2 - [0 \;\; I_{n-p}]\tilde{x} \tag{19b}$$

From (19a) and (15b) we obtain $\bar{e}_1 = 0$ and then $\tilde{e} = \begin{bmatrix} \bar{e}_{1(p)} \\ \bar{e}_2 \end{bmatrix} = \begin{bmatrix} \bar{x}_{1(p)} \\ \bar{x}_2 \end{bmatrix} - \tilde{x}$

The dynamic of this error is described by $\begin{bmatrix} I_{n-p} & 0 \\ 0 & 0 \end{bmatrix}\dot{\tilde{e}} = \begin{bmatrix} 0 & I_{n-p} \\ 1 & 0 \end{bmatrix}\tilde{e}$

where $\tilde{e}(t) = 0$ so $\bar{e}_2(t) = 0$ so $\bar{e}(t) = 0$ so $e(t) = 0$ for all $t > 0$.

The reduced order perfect observer can be computed by the use of the following procedure.

Procedure 3.

Step 1. Knowing A and C find T such that (15) holds.

Step 2. Compute the reduced order perfect observer matrices

$$E = \begin{bmatrix} I_{n-p} & 0 \\ & 0 \end{bmatrix},\quad F = \begin{bmatrix} 0 & I_{n-p} \\ 1 & 0 \end{bmatrix},\quad G = \begin{bmatrix} \bar{B}_2 \\ (\bar{C}_1^{-1})_{(p)}D \end{bmatrix},\quad H = \begin{bmatrix} 0 \\ -(\bar{C}_1^{-1})_{(p)} \end{bmatrix},$$

$$J = T^{-1}\begin{bmatrix} 0^{1\times p} & 0 \\ 0 & I_{n-p} \end{bmatrix},\quad L = T^{-1}\begin{bmatrix} -\bar{C}_1^{-1}D \\ 0 \end{bmatrix},\quad M = T^{-1}\begin{bmatrix} \bar{C}_1^{-1} \\ 0 \end{bmatrix}$$ or for the multisystem canonical form (15') $E = \underset{i=1,\ldots,l}{\text{diag}}\left(\begin{bmatrix} I_{n_i-p_i} & 0 \\ & 0 \end{bmatrix}\right)$,

$$F=\operatorname*{diag}_{i=1,\dots,l}\left(\begin{bmatrix}0 & I_{n_i-p_i}\\ 1 & 0\end{bmatrix}\right),\quad G=\begin{bmatrix}\overline{B}_{12}\\ \left(\overline{C}_1^{-1}\right)_{(p_1)}D\\ \overline{B}_{22}\\ \left(\overline{C}_2^{-1}\right)_{(p_2)}D\\ \vdots\\ \overline{B}_{l2}\\ \left(\overline{C}_l^{-1}\right)_{(p_l)}D\end{bmatrix},\quad H=\operatorname*{diag}_{i=1,\dots,l}\left(\begin{bmatrix}0^{(n_i-p_i)\times p_i}\\ -\left(\overline{C}_i^{-1}\right)_{(p_i)}\end{bmatrix}\right),$$

$$J=T^{-1}\operatorname*{diag}_{i=1,\dots,l}\left(\begin{bmatrix}0^{1\times p_i} & 0\\ 0 & I_{n_i-p_i}\end{bmatrix}\right),\quad L=T^{-1}\begin{bmatrix}-\overline{C}_1^{-1}D\\ 0\\ -\overline{C}_2^{-1}D\\ 0\\ \vdots\\ -\overline{C}_l^{-1}D\\ 0\end{bmatrix}$$

The desired observer has the form

$$E\dot{\tilde{x}}=F\tilde{x}+Gu+Hy$$
$$\hat{x}=J\tilde{x}+Lu+My$$

d) Polynomial perfect observer

Our aim is to design a polynomial perfect observer for continuous-time linear system described by the polynomial equations (6). To obtain polynomial perfect observer we compute the matrices $P(s)$, C and $T(s)$, where $T(s)=CP(s)$.

If C is the full row rank then we find $Q\in R^{n\times n}$ such that $CQ=\begin{bmatrix}I_p & 0\end{bmatrix}$,

$$\overline{x}(t)=Q^{-1}x(t) \text{ and } \overline{P}(s)=Q^{-1}P(s)=\begin{bmatrix}\overline{P}_{11}(s) & \dots & \overline{P}_{1m}(s)\\ \vdots & \ddots & \vdots\\ \overline{P}_{n1}(s) & \dots & \overline{P}_{nm}(s)\end{bmatrix}$$

Note that it is enough to find only the part of the vector x. The desired observer has the following form

$$\overline{x}(s)=\overline{L}u(s)+\overline{V}y(s) \tag{20}$$

$$\overline{L}=Q^{-1}L=\begin{bmatrix}0\\ L_{p+1,1}(s) & \dots & L_{p+1,m}(s)\\ \vdots & \ddots & \vdots\\ L_{n1}(s) & \dots & L_{nm}(s)\end{bmatrix},\quad L_{ij}(s)=\sum_{l\ge 0}l_l^{ij}s^l$$

$$\overline{V}=Q^{-1}V=\begin{bmatrix}I_p\\ V_{p+1,1}(s) & \dots & V_{p+1,p}(s)\\ \vdots & \ddots & \vdots\\ V_{n1}(s) & \dots & V_{np}(s)\end{bmatrix},\quad V_{ij}(s)=\sum_{l\ge 0}v_l^{ij}s^l$$

The matrices $\overline{L}$ and $\overline{V}$ can be obtained from the equation

$$\overline{P}_{ij}(s)d(s)=\sum_{k=1}^{p}V_{ik}(s)T_{kj}(s)d(s)+L_{ij}(s)d(s) \tag{21}$$

where $d(s)$ is greatest common divisor of the matrices $\overline{P}(s)$ and $T(s)$.

Elements of the matrices L and V we obtain by comparison of the coefficients at the same power of s in equation (21).

To design the polynomial perfect observer the following procedure can be used.

Procedure 4.

Step 1. Compute system matrices as follows:

Knowing $P(s)$ and C compute $T(s)$ or knowing $P(s)$ and $T(s)$ compute C.

Step 2. Find Q such that $CQ=\begin{bmatrix}I_p & 0\end{bmatrix}$

Step 3. Find greatest common divisor $d(s)$ of the elements of the matrices $P(s)$ and $T(s)$.

Step 4. Using the equation $\overline{P}_{ij}(s)d(s)=\sum_{k=1}^{p}V_{ik}(s)T_{kj}(s)d(s)+L_{ij}(s)d(s)$ for $p+1\le i\le n$, $j=1,\dots,m$ find polynomial matrices $\overline{L}(s)$ and $\overline{V}(s)$

Step 5. Compute $L(s)=Q\overline{L}(s)$ and $V(s)=Q\overline{V}(s)$

The desired polynomial observer is given by $\hat{x}(s)=Lu(s)+Vy(s)$ where $L\in R[s]^{n\times m}$

5. Perfect observers for singular systems

Consider the singular $(\det E=0)$ continuous-time linear system (1)

Theorem 3. The perfect observer for the singular continuous-time linear system exists if and only if the system is observable

$$\operatorname{rank}\begin{bmatrix}Es-A\\ C\end{bmatrix}=n,\qquad \forall s\in C \tag{22}$$

Proof.

a) If matrix C has full row rank: $\operatorname{rank}C=n$.

The estimate of the state vector x is given by $\hat{x}=C^{+}y-C^{+}Du$ and the perfect observer can be obtained by the use of the Procedure 1.

b) If $\operatorname{rank}C<n$

Using the Weierstrass-Kronecker decomposition the regular singular system can be decomposed into the three independent subsystems: standard, completely singular and static one. Using procedure given in the appendix of [16] find P and Q such that

$$PEQ=\begin{bmatrix}I_d & 0 & 0\\ 0 & N & 0\\ 0 & 0 & 0\end{bmatrix},\ PAQ=\begin{bmatrix}A_1 & 0\\ 0 & I_{n-d}\end{bmatrix},\ PB=\begin{bmatrix}B_1\\ B_2\\ B_3\end{bmatrix},\ CQ=\begin{bmatrix}C_1 & C_2 & C_3\end{bmatrix} \tag{23}$$

then

$$\dot{x}_1=A_1x_1+B_1u,\qquad y_1=C_1x_1 \tag{24a}$$

$$N\dot{x}_2=x_2+B_2u,\qquad y_2=C_2x_2 \tag{24b}$$

$$0=x_3+B_3u,\qquad y_3=C_3x_3 \tag{24c}$$

$$y=y_1+y_2+y_3+Du \tag{24d}$$

If the system is observable then all its subsystems are observable and if one of the subsystems is unobservable then the system is unobservable.

If the subsystem (24a) is standard then the perfect observer for this subsystem can be obtained using the method presented in section 4.

$$E_1\dot{\tilde{x}}_1=F_1\tilde{x}_1+G_1u+H_1y_1,\qquad \hat{x}_1=J_1\tilde{x}_1+M_1y_1 \tag{25}$$

where $\det[E_1s-F_1]=c\neq 0$, $c\in R$.

The subsystem (24b) is completely singular and its output signal [4] $x_2(t)=\sum_{j=1}^{\rho}\Phi_{-j}B_2u^{(j-1)}(t)$ is independent of initial conditions x_{20} for all $t>0$.

The perfect observer for this subsystem is described by the same equations.

$$N\dot{\tilde{x}}_2=\tilde{x}_2+B_2u,\qquad \hat{x}_2=\tilde{x}_2 \tag{26}$$

The subsystem (24c) is described by the equation.

$$x_3=-B_3u=L_3u \tag{27}$$

Combining the previous results we obtain

$$\begin{bmatrix}E_1 & 0\\ 0 & N\end{bmatrix}\dot{w}=\begin{bmatrix}F_1 & 0\\ 0 & I\end{bmatrix}w+\begin{bmatrix}G_1\\ B_2\end{bmatrix}u+\begin{bmatrix}H_1\\ 0\end{bmatrix}y_1$$

$$\hat{\bar{x}}=\begin{bmatrix}J_1 & 0\\ 0 & I\\ 0 & 0\end{bmatrix}w+\begin{bmatrix}0\\ 0\\ -B_3\end{bmatrix}u+\begin{bmatrix}M_1\\ 0\\ 0\end{bmatrix}y_1$$

Using the output equation and the perfect observer (26) and (27) we obtain $y_1 = y - C_2 w_2 + C_3 B_3 u - Du$ and

$$\bar{E}\dot{w} = Fw + Gu + Hy$$
$$\hat{x} = Jw + Lu + My$$

where $\bar{E}=\begin{bmatrix}E_1 & 0\\ 0 & N\end{bmatrix}$, $F=\begin{bmatrix}F_1 & -H_1C_2\\ 0 & I\end{bmatrix}$, $G=\begin{bmatrix}G_1+H_1C_3B_3-H_1D\\ B_2\end{bmatrix}$, $H=\begin{bmatrix}H_1\\ 0\end{bmatrix}$,

$J=Q\begin{bmatrix}J_1 & -M_1C_2\\ 0 & I\\ 0 & 0\end{bmatrix}$, $L=Q\begin{bmatrix}M_1(C_3B_3-D)\\ 0\\ -B_3\end{bmatrix}$, $M=Q\begin{bmatrix}M_1\\ 0\\ 0\end{bmatrix}$ and $\hat{x}(t)=x(t)$ for all.

Procedure 5.

Step 1. Find canonical form (23) for singular system (1) - procedure A.1.
Step 2. Compute the matrices E_1, F_1, G_1, H_1, J_1, M_1 of the perfect observer for standard subsystem (24a) (Section 4).
Step 3. Compute N and B_2 of the perfect observer for completely singular subsystem (24b).
Step 4. Compute the matrix L_3 of the perfect observer for the subsystem (24c).
Step 5. Compute the obtained results and compute the final form of the perfect observer

$$\bar{E}\dot{w} = Fw + Gu + Hy$$
$$\hat{x} = Jw + Lu + My$$

where $\bar{E}=\begin{bmatrix}E_1 & 0\\ 0 & N\end{bmatrix}$, $F=\begin{bmatrix}F_1 & -H_1C_2\\ 0 & I\end{bmatrix}$, $G=\begin{bmatrix}G_1+H_1C_3B_3-H_1D\\ B_2\end{bmatrix}$, $H=\begin{bmatrix}H_1\\ 0\end{bmatrix}$,

$J=Q\begin{bmatrix}J_1 & -M_1C_2\\ 0 & I\\ 0 & 0\end{bmatrix}$, $L=Q\begin{bmatrix}M_1(C_3B_3-D)\\ 0\\ L_3\end{bmatrix}$, $M=Q\begin{bmatrix}M_1\\ 0\\ 0\end{bmatrix}$.

6. Examples and simulation results

Example 1.

Find perfect observer for the standard observable linear system with matrices $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -1 & -10\end{bmatrix}$, $B=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$, $C=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$, $D=0$

Using the procedure 3 we obtain

Step 1. $T = I_3$

Step 2. $E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$, $F=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$, $G=\begin{bmatrix}1\\ 2\\ 0\end{bmatrix}$, $H=\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}$,

$J=\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, $L=0$, $M=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$

The desired perfect observer is described by

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}\dot{w}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}w+\begin{bmatrix}1\\ 2\\ 0\end{bmatrix}u+\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}y \quad \hat{z}=\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}w+\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}y$$

The simulation results are shown on fig. 1, 2, 3 and 4.

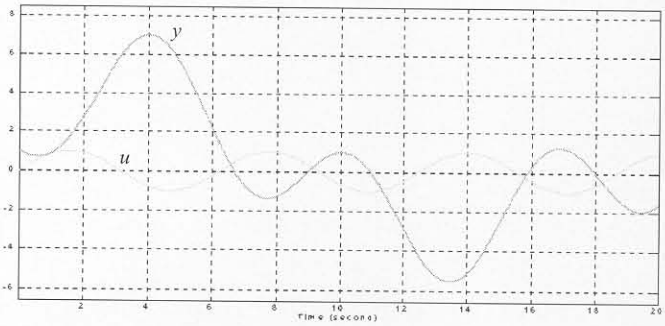

Fig. 1. System input u and output y signals
Rys. 1. Sygnały wejścia u i wyjścia y

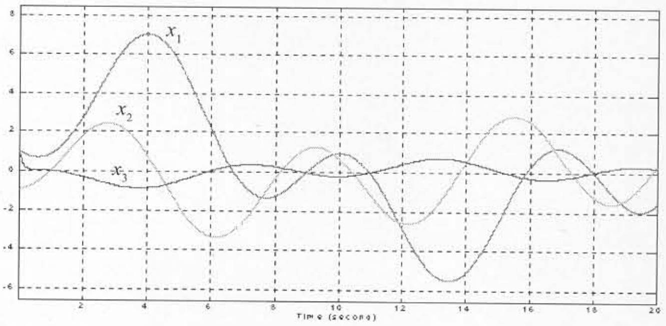

Fig. 2. State vector x (x_1,x_2,x_3)
Rys. 2. Wektor stanu x (x_1,x_2,x_3)

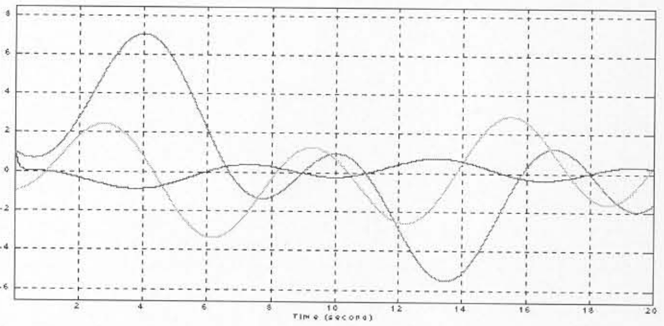

Fig. 3. Observer output signals
Rys. 3. Sygnał wyjściowy obserwatora

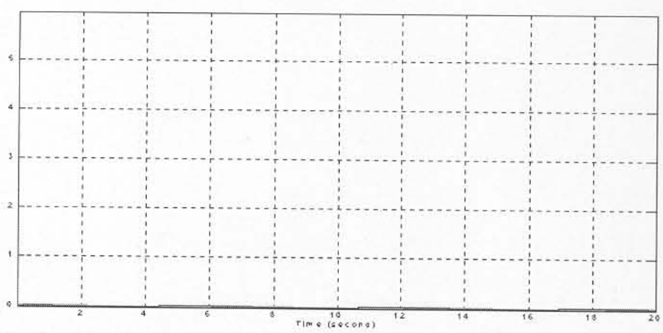

Fig. 4. Observer signals error.
Rys. 4. Błąd obserwatora.

Example 2.

Find the perfect observer for the unstable standard system with

$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -1 & 0{,}01\end{bmatrix}$, $B=\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$, $C=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$, $D=0$

Using the procedure 3 we obtain

Step 1. $T = I_3$

Step 2. $E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$, $F=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$, $G=\begin{bmatrix}1\\2\\0\end{bmatrix}$, $H=\begin{bmatrix}0\\0\\-1\end{bmatrix}$, $J=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}$,

$L=0$, $M=\begin{bmatrix}1\\0\\0\end{bmatrix}$

The desired perfect observer is described by

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\dot{w}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}w+\begin{bmatrix}1\\2\\0\end{bmatrix}u+\begin{bmatrix}0\\0\\-1\end{bmatrix}y \quad \hat{z}=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}w+\begin{bmatrix}1\\0\\0\end{bmatrix}y$$

The simulation results are shown on fig. 5, 6, 7 and 8.

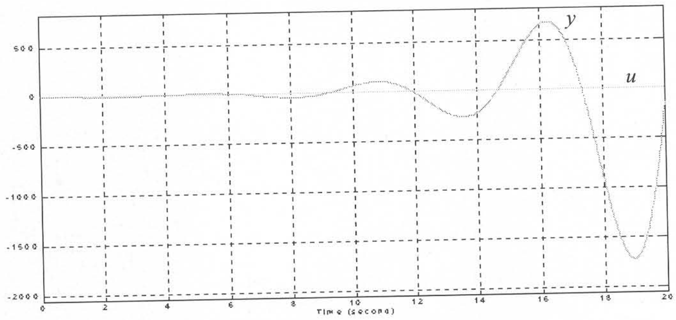

Fig. 5. System input u and output y signals
Rys. 5. Sygnały wejścia u i wyjścia y

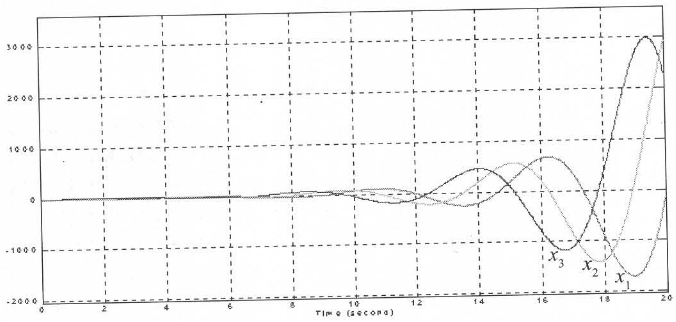

Fig. 6. System state vector x (x_1,x_2,x_3)
Rys. 6. Wektor stanu układu x (x_1,x_2,x_3)

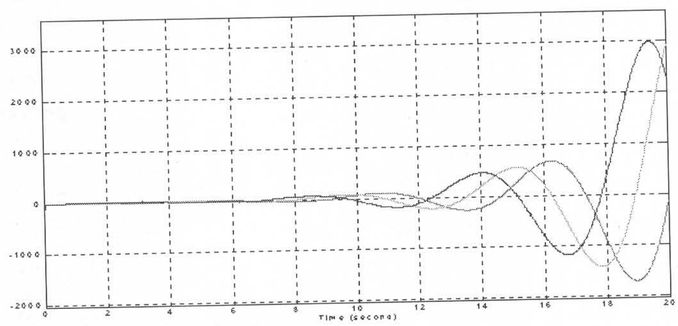

Fig. 7. Observer output signals
Rys. 7. Sygnały wyjściowe obserwatora

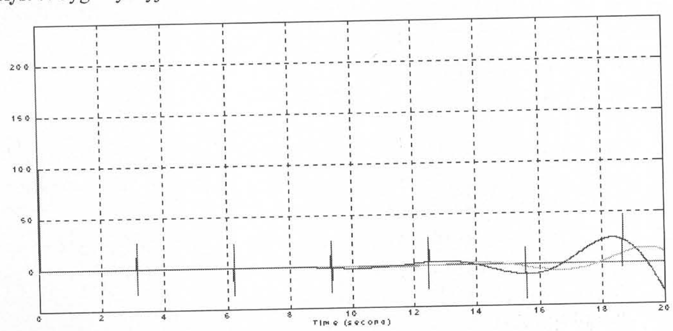

Fig. 8. Observer signals error
Rys. 8. Błąd obserwatora

The perfect observers have one drawback that the output signals can depends on the derivative of the input signals. Up to now we don't know the perfect computation method of the signals derivatives. All errors of the observers output signals depends on the simulation parameters (as of the step size). There is the most important advantage of the perfect obse-rvers that the output signals are completely independent of the input signals for all time greater then zero.

7. Concluding remarks

Three types of perfect observers: full-order, reduced-order and polyno-mial one for continuous-time standard and singular linear systems have been investigated. Necessary and sufficient conditions for the existence of the perfect observers have been established. Procedures for designing of the perfect observers have been derived and illustrated by numerical examples. Some simulations results for the perfect observers have been also presented. The considerations with some modifications can be also extended for standard and singular discrete-time linear systems. An exten-sion of the considerations for two-dimensional [4] linear and nonlinear continuous-time systems is also possible but it is not trivial and it will be a subject of the future research and publications.

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Tytuł: Obserwatory doskonałe liniowych układów ciągłych

Artykuł recenzowany