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Properties of Analog Low-Pass Filters with Varying Parameters

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Dyplom magistra inżyniera elektryka uzyskał w 1973 roku na Wydziale Elektrycznym Politechniki Szczecińskiej, na którym bezpośrednio po studiach rozpoczął pracę w Zakładzie Teorii Sterowania i Technik Symulacyjnych Instytutu Automatyki Przemysłowej i gdzie pracuje do chwili obecnej. Na macierzystym wydziale uzyskał w roku 1978 stopień naukowy doktora nauk technicznych. Stopień naukowy doktora habilitowanego uzyskał w roku 2002 na Wydziale Elektrycznym Politechniki Śląskiej w Gliwicach. W latach 1990-1999 pełnił obowiązki zastępcy dyrektora Instytutu Automatyki Przemysłowej. W 1994 roku był jednym z inicjatorów zorganizowania konferencji MMAR, którą współorganizuje również obecnie. Jest autorem i współautorem ponad 80 publikacji naukowych. Główne kierunki badań dotyczą analizy i syntezy układów o zmiennych parametrach, przetwarzania sygnałów i miernictwa dynamicznego.

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Abstract

The paper examines properties of solutions to ordinary differential equations from the point of view of their applications as averaging and low-pass filters. The stationarity of solutions to systems with varying parameters was analyzed together with stability conditions of these systems by making use of the theorem on stability of non-homogeneous systems when the corresponding homogeneous systems were stable. The transient state was analyzed and the possibility of its shortening was shown. The performed simulations of the variable parameters Butterworth, Legendre and Chebyshev low-pass filters of the 2-nd up to the 6-th order proved usefulness of the method.

Streszczenie

Przedstawiono właściwości rozwiązań równań różniczkowych zwyczajnych o zmiennych parametrach pod kątem wykorzystania tych układów jako filtrów dolnoprzepustowych. Dokonano analizy stacjonarności rozwiązań układów o zmiennych parametrach oraz warunków stabilności tych układów, wykorzystując twierdzenia o stabilności równań różniczkowych niejednorodnych o zmiennych parametrach wtedy, gdy odpowiednie równania jednorodne są stabilne. Przeprowadzono analizę stanu nieustalonego wykazując możliwość jego skrócenia przez uzmiennienie parametrów. Przeprowadzono badania symulacyjne dolnoprzepustowych filtrów o zmiennych parametrach, zrealizowanych w oparciu o aproksymacje Butterwortha, Legendre'a i Czebyszewa. Badania te potwierdziły korzystne właściwości filtrów o zmiennych parametrach, co pozwala na skrócenie stanu nieustalonego.

Keywords: low-pass filter, varying parameters, stationarity, stability of systems
Słowa kluczowe: filtr dolnoprzepustowy, zmienne parametry, stacjonarność, stabilność systemów

1. Introduction

As it follows from the analysis of spectral and dynamical properties of the constant component filters with time-invariant parameters it is possible to shorten the transient state by introducing time-varying filter parameters during this transient state [1, 2, 5, 6, 10]. Since there are considerable differences between the time of working out the constant component of the filtered signal, which is long, and the time of stopping or passing the variable components, which is significantly (many times) shorter, it is justified to look for the possibility of shortening the transient state by shortening the time of working out the constant component. Such possibilities are given by the introduction of varying filter parameters during the transient state. This may result in lengthening of the time of working out the variable components however as long as this time does not exceed the time of working out the constant component the phenomenon does not need to regarded as undesirable. It is assumed that varying filter parameters during the transient state appears to be an effective way of shortening this transient state.

2. Stationarity of solutions of parametric systems for $t\to\infty$

In order to analyze spectral properties of filters with time-varying parameters one can use methods applicable to time-invariant systems under the conditions that the filter parameters stabilize (with α-accuracy) after passing of the transient state. To show this the theorem on spectral density of the output signal [4] after passing of the transient state in linear systems was used. One proved that this result held also for systems with time-varying parameters if their values stabilize when $t\to\infty$ [7].

In the relations (1), (2) and (3) one introduced the spectral transmittance $K(j\omega)$ of a system with constant parameters which corresponds to a system with time-varying parameters at $t\to\infty$, denoting it by $\overset{var}{K}_{t\to\infty}(j\omega)$ if the condition $\lim\limits_{t\to\infty} a_i(t) = a_i = const$.

Taking into account

$$\int_{-\infty}^{+\infty} k(\tau_1)e^{j\omega\tau_1}d\tau_1 = K(-j\omega) \triangleq \overset{var}{K}_{t\to\infty}(-j\omega)\Big|_{\lim\limits_{t\to\infty} a_i(t)=const} \qquad (1)$$

$$\int_{-\infty}^{+\infty} k(\tau_2)e^{-j\omega\tau_2}d\tau_2 = K(j\omega) \triangleq \overset{var}{K}_{t\to\infty}(j\omega)\Big|_{\lim\limits_{t\to\infty} a_i(t)=const} \qquad (2)$$

one can as follows

$$\begin{aligned} S_y(\omega) &= K(-j\omega)K(j\omega)S_x(\omega) = \left|K(j\omega)\right|^2 S_x(\omega) = \\ &= \left|\overset{var}{K}_{t\to\infty}(j\omega)\right|^2 \cdot S_x(\omega) \end{aligned} \qquad (3)$$

The above presented proof allows to apply the spectral relations which hold in the steady state for linear time-invariant systems to systems with time-varying parameters if these parameters stabilize their values with time.

3. Stability of systems with varying parameters

In order to examine stability of systems with varying parameters one cannot apply the universal methods applicable to time-invariant systems. In this paper one used theorems and their proofs contained in the original literature as well as in works describing ways of making use of them [3, 7, 9]. This allowed to analyze conditions of stability of systems presenting themselves filters with varying parameters.

Every analog filter with time-varying parameters can be described in the form of the following system of differential equations

$$\frac{dy}{dt} = A(t)y + x(t) \qquad (4)$$

where $\mathbf{A}(t) \in C(I^+)$, $\mathbf{x}(t)) \in C(I^+)$.

The quoted theorem and corollaries imply that it is sufficient to obtain stability conditions for the homogeneous linear system, which is a reduced system in comparison to the non-homogeneous linear system. Stability of the linear homogeneous system is equivalent to stability of the corresponding non-homogeneous system described by equation (4) independently of the input $\mathbf{x}(t)$.

To find stability conditions of the system with varying parameters one has used the second Lyapunov method [4, 9]. Stability was examined for the second order system with varying parameters satisfying conditions imposed on the systems described by equation (4). This system can be a part of a structure of higher order filters or be a separate filter of the constant component.

The system which satisfies the relation (4) is the following second order system

$$\frac{d^2y}{dt^2}+2\beta(t)\omega_0(t)\frac{dy}{dt}+\omega_0^2(t)y=0 \tag{5}$$

After some manipulations, the stability condition for the second order system was obtained

$$\left|\dot{\omega}_0(t)\right|<\left|2\beta(t)\omega_0^2(t)\right| \tag{6}$$

The inequality (6) determines stability conditions for the second order system with parameters varying at any time t. It is essential for low-pass and constant component filters that their parameters stabilize after passing of the transient state. For this reason conditions determining asymptotic stability are important and this is described by the inequality

$$\lim_{t\to\infty}\left|\omega_0(t)\right|<\lim_{t\to\infty}\left|2\beta(t)\omega_0(t)\right| \tag{7}$$

Fulfillment of conditions

$$\lim_{t\to\infty}\beta(t)=const\neq 0 \tag{8}$$

and

$$\lim_{t\to\infty}\omega_0(t)=const\neq 0 \tag{9}$$

mean that the time derivative of the characteristic frequency tends to zero

$$\lim_{t\to\infty}\dot{\omega}_0(t)=0 \tag{10}$$

Fulfillment of relations (8), (9) and (10) implies that the homogeneous system described by the parametric equation (5) is asymptotically stable and consequently the non-homogeneous system is asymptotically stable. This means that the necessary and sufficient condition for asymptotic stability of the second order parametric system under the conditions (8), (9) and (10) is convergence of the damping function and the characteristic frequency function to constant values of the same signs when $t\to\infty$. This condition does not exclude temporary local positive feedbacks caused by opposite signs of values of both functions. On this basis one can conclude that if the structure of the filter contains elements with varying parameters so if for $t\to\infty$ value of the function describing the characteristic frequency converges to constant value: $\lim_{t\to\infty}\omega_0(t)=const$, then stability of the filter can examined in the same way as of a system with time-invariant parameters [7]. If relation (10) does not hold, then the condition (6) has to be satisfied which restricts the rate of changes of parameters.

4. Transient states in filters with varying parameters

The simplest system which can act as a low-pass filter or constant component filter is the first order time-lag system. Although it is a system with worst properties if the time constant T is replaced by time-varying function $T(t)$

$$T(t)=T\left[1-c\cdot\exp\left(-tT^{-1}\right)\right] \tag{11}$$

Comparing the step response $h_{var}(t)$ of the filter with varying parameters (parametric) with the step response $h_{const}(t)$ of the corresponding filter with the time constant $T=const$ one obtains and comparing the response $\tilde{y}_{var}(t)$ of the system with varying parameters to the sinusoidal input signal with the response $\tilde{y}_{const}(t)$ of the system with constant parameters to the sinusoidal input signal one obtains, after manipulations,

$$\frac{\tilde{y}_{var}(t)}{\tilde{y}_{const}(t)}=\frac{1}{1-c\cdot\exp\left(-tT^{-1}\right)}=\frac{h_{var}(t)}{h_{const}(t)}=\chi(t,c) \tag{12}$$

From the comparison of the responses to the sinusoidal input one gets the same function (12) as from the comparison of the step responses. To illustrate relation (12) one has plotted function $\chi\left(\frac{t}{T},c\right)$ in Fig. 1.

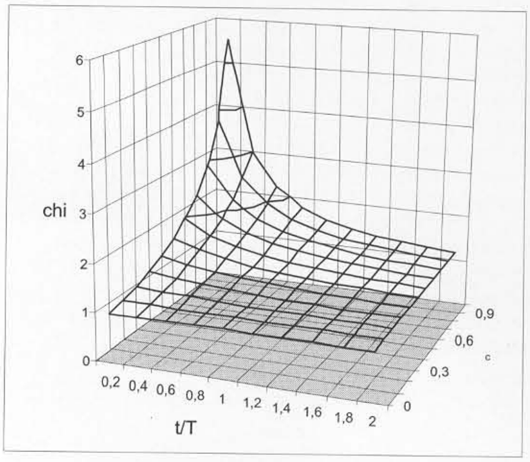

Fig. 1. Function $\chi\left(\frac{t}{T},c\right)$ from relation (12)

Rys. 1. Wykres funkcji $\chi\left(\frac{t}{T},c\right)$ na podstawie zależności (12)

One can see that the surface of function $\chi\left(\frac{t}{T},c\right)$ in the whole range of variables t and c lies above the plane $\chi=1$. Only for $c=0$ the function $\chi=1$. This is obvious since for $c=0$ the time function described by (9) becomes a time constant independent of time. Values of function $\chi\left(\frac{t}{T},c\right)$ in the rest of the range of the variables changes rise with the increase of the range of changes of the time function ($c\to 1$). It can be seen that with the increase of time the value of the function tends asymptotically to unity, independently of the values of the variable c. It means that in the steady state the examined, corresponding to each other time-invariant and time-varying systems have the same responses to the same input signals. The step response of the system with varying parameters achieves the steady state with α-accuracy faster than the analogous system with constant parameters.

Regarding the system with varying parameters as a low-pass filter one can notice that values of function $\chi\left(\frac{t}{T},c\right)$ greater than one result in longer damping time of the components with frequencies $\omega>0$. This means that the variable components will be damped longer in the parametric system than in the system with constant parameters. However, it is known from analysis of systems with constant parameters that damping of the variable components with α-accuracy is many times shorter than working out the constant component. One should then estimate whether the introduction of parameters varying in time can shorten the time of the transient state.

5. Low-pass filters with varying parameters

The Butterworth, Chebyshev and Legendre polynomials are often used for approximation of the frequency characteristic of low-pass filters [10]. For the Chebyshev filters one assumed that the module frequency response had the waviness of 0,5 dB in the pass band and the slope of 3 dB at the band limit. For the Legendre filters one assumed that the module frequency response had the waviness of 3 dB in the pass band and the slope of 3 dB at the band limit.

For the Butterworth filters only the slope of 3dB at the limit of the pass band was assumed since the frequency response does not show any waviness inside the band.

It was decided to introduce time-varying parameters $\beta_i(t)$ as replacement for those which had the lowest values for particular types of filters. It gives the possibility of decreasing the oscillativeness of filters and consequently also shortening of the transient state [4].

Furthermore, one decided to carry out the simulations of the low-pass filters with variable parameters assuming functions $\beta_i(t)$ in the form

$$\beta_i(t) = A\left[\left(1+\frac{t}{T_{01}}\right)exp\left(-\frac{t}{T_{01}}\right)\right]^n + \beta_{i\infty} \qquad (13)$$

where: $A = 1 - \beta_{i\infty}$, $\beta_{i\infty}$ - the final value of the damping factor resulting from the appropriate approximation

The values of parameters of functions (13) were chosen in the way which allowed their values to settle down with α-accuracy faster than the settling times of the examined filters. In the simulations the tolerance band was assumed equal to α=0,05.

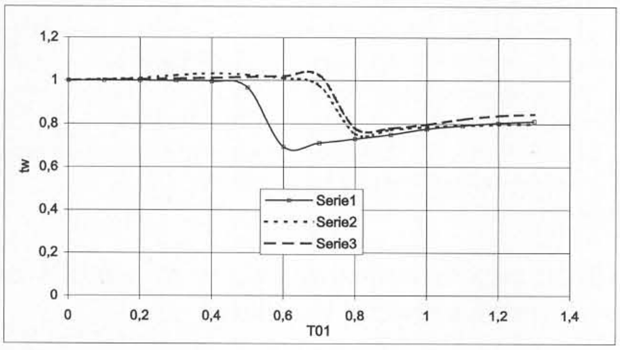

Fig. 2. The exemplary functions $t_w=f(T_{01})$ for the Butterworth filters: of the 3-rd order - series 1, of the 4-th order - series 2, of the 5-th order - series 3 for $\beta_i(t)$ corresponding to (13) and n=1

Rys. 2. Przykładowe wykresy funkcji $t_w=f(T_{01})$ dla parametrycznych filtrów Butterwortha: 3-go rzędu - series 1, 4-go rzędu - series 2, 5-go rzędu - series 3 dla $\beta_i(t)$ opisanej zależnością (13), dla n=1

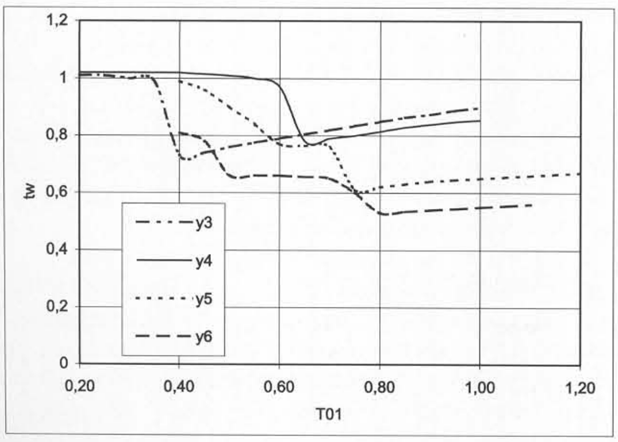

Fig. 3. The exemplary functions $t_w=f(T_{01})$ for the Legendre filters: of the 3-rd order - y3, of the 4-th order - y4, of the 5-th order - y5, of the 6-th order - y6 for $\beta_i(t)$ corresponding to (13) and n=1

Rys. 3. Przykładowe wykresy funkcji $t_w=f(T_{01})$ dla parametrycznych filtrów Legendre'a: 3-go rzędu - y3, 4-go rzędu - y4, 5-go rzędu - y5, 6-go rzędu - y6 dla $\beta_i(t)$ opisanej zależnością (13), dla n=1

During the simulation the oscillativeness of filters was lowered by introducing $\beta_1(t)$ in the 3-rd order filter, $\beta_2(t)$ - in the 4-th and 5-th order filters, and $\beta_3(t)$ - in the 6-th order filter. Examinations were done for the wide range of values T_{01} occurring in the relation (13). At this stage of examination the main impact was put on determining values of parameters in (13), for which the oscillativeness was lowest. Figs 2, 3 and 4 show exemplary values of the relative settling time t_w as function of T_{01} for $\beta_i(t)$ and the assumption n=1, obtained during the simulations. The relative settling time t_w was obtained by dividing the settling time of the parametric filter by the settling time of the analogous filter with constant parameters.

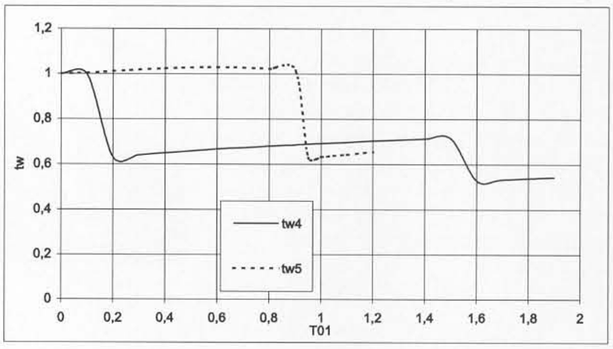

Fig. 4. The exemplary functions $t_{w4}=f(T_{01})$ for the Chebyshev filter of the 4-th order and $t_{w5}=f(T_{01})$ for the Chebyshev filter of the 5-th order for $\beta_2(t)$ corresponding to (13) and n=1

Rys. 4. Przykładowe wykresy funkcji $t_w=f(T_{01})$ dla parametrycznych filtrów Czebyszewa: 4-go rzędu - t_{w4} i, 5-go rzędu - $t_{w5}=f(T_{01})$ dla $\beta_2(t)$ opisanej zależnością (13) i dla n=1

6. Conclusions

Diagrams presented in the Fig. 1 show that. The varying in time of the damping ratio in the Butterworth filters of the 3-rd up to 5-th order allows to shorten the transient state by about 25-30% in comparison to the filters with constant parameters. For the Legendre filters of the 3-rd up to 6-th order the shortening of the settling time by 25 up to 45% was obtained. For the Chebyshev filters of the 4-th and 5-th order one obtained accordingly 45% and 35%. This was achieved mainly by decreasing the oscillativeness of the filters during the transient state.

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Tytuł: Właściwości analogowych dolnoprzepustowych filtrów o zmiennych parametrach

Artykuł recenzowany