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Control System Structures and Their Robustness

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He was born in 1936. Since 1963 he has been employed at the Technical University of Szczecin. In 1969 he received his Ph.D. degree, and in 1973 the Habilitation degree. In 1978 he became a Full Professor. He is an author of 5 monographs, about 140 papers and has supervised 13 Ph.D. theses. His research interest include temperature control, process modelling and identification, as well as industrial and adaptive control systems.

Abstract

Linear SISO control structures affected by perturbations and disturbances have been analyzed from the viewpoint of their input and disturbance sensitivity for the case of known and unknown process time delay. It has been shown that disturbance sensitivity is the essential quantity, which differentiates the structures under study in terms of their sensitivity.

Streszczenie

Porównano wrażliwość wejściową oraz zakłóceniową liniowych struktur SISO układów regulacji w obecności perturbacji multiplikatywnych oraz zakłóceń sprawdzonych do wyjścia, dla przypadku nieznanego i znanego opóźnienia procesu. Wykazano, ze wrażliwość zakłóceniowa decyduje o zróżnicowaniu właściwości odpornościowych rozważonych struktur.

Keywords: Closed loop systems, robust control, linear systems
Słowa kluczowe: liniowe układy regulacji, regulacja odporna

1. Introduction

The aim of the paper is to compare basic linear SISO control structures from the viewpoint of their robustness. Disturbances are assumed to be nonmeasurable, and their net effect is represented by a disturbance acting on the output. It is also assumed that the process to be controlled is distinct from its model known, and process perturbations are unknown.

The robustness of a control system being subjected to perturbations and disturbances can be judged from the input sensitivity being a measure of the system ability to follow the reference signal, and from output (disturbance) sensitivity being a measure of the system ability to suppress disturbances.

2. Input and output sensitivity of control system structures

2.1. Classic SISO control structure

The classic SISO control structure is displayed in Fig. 1.

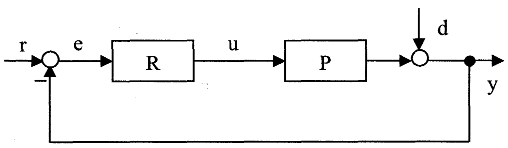

Fig. 1. Classic single loop control system
Rys. 1. Standardowy, jednopętlowy UAR

The following relation may be easily derived from Fig. 1 (here the s argument is omitted for simplicity)

$$y = r\frac{RP}{1+RP} + d\frac{1}{1+RP} \tag{1}$$

For this case the input sensitivity S_r and the output one S_d are defined respectively by

$$S_r = \frac{RP}{1+RP} \tag{2}$$

$$S_d = \frac{1}{1+RP} \tag{3}$$

Taking into account the fact that the controller is usually designed on the basis of the known process model M rather than on the actual process P, the structure of Fig. 1 can be presented alternatively as in Fig. 2.

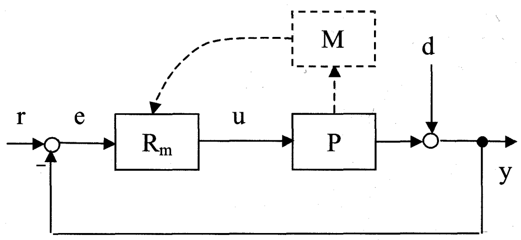

Fig. 2. Classic control system with a controller having been designed from the process model M
Rys. 2. UAR z regulatorem zaprojektowanym na podstawie modelu procesu

The controller R_m has been designed using the process model as the base in full consciousness that the model M may differ from the process P by perturbations Δ

$$P = M(1+\Delta) \tag{4}$$

The perturbations may originate from different sources, such as errors in identification, errors in modelling, deliberate model simplification, process nonlinearities, fluctuations and variations of process parameters, etc.

In general, perturbations can be of random character, being stationary or not, exhibiting specific statistic characteristics [1].

In the following, it will be assumed that perturbations are limited, i.e. they are subject to the condition

$$|\Delta(j\omega)| \le \Delta \quad , \ \omega \in [0,\infty) \tag{5}$$

Putting

$$y_m = r\frac{R_m M}{1+R_m M} \tag{6}$$

as model output and taking eqs. (1), (4) into account, we obtain for the classic control structure

$$y_{CL} = y_m\left[1+\frac{\Delta}{1+R_m M(1+\Delta)}\right] + d\frac{1}{1+R_m M(1+\Delta)} \tag{7}$$

(The s argument is omitted.)

2.2. Control system with Smith predictor

Processes with big and known time delay can be governed by a Smith predictor based control structure shown in Fig. 3.

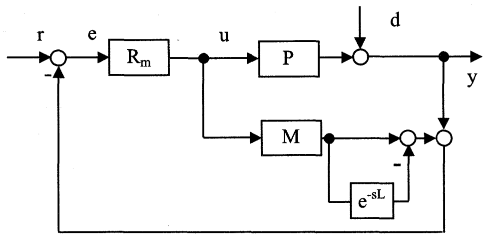

Fig. 3. Control system with Smith predictor
Rys. 3. Układ regulacji z predyktorem Smitha

From Fig. 3 it follows with eq. (6) allowed for that

$$y_{SM} = y_m e^{-sL}\left[1+\Delta\frac{1+R_m M\left(1-e^{-sL}\right)}{1+R_m M\left(1+\Delta e^{-sL}\right)}\right]+ \\ +d\frac{1+R_m M\left(1-e^{-sL}\right)}{1+R_m M\left(1+\Delta e^{-sL}\right)} \tag{8}$$

where

$$P_L = M(1+\Delta)\cdot e^{-sL} \tag{9}$$

2.3. Processes with big and unknown time delay

Processes with big and unknown time delay can be controlled by an open-loop structure employing a delay-free model [7] as shown in Fig. 4a.

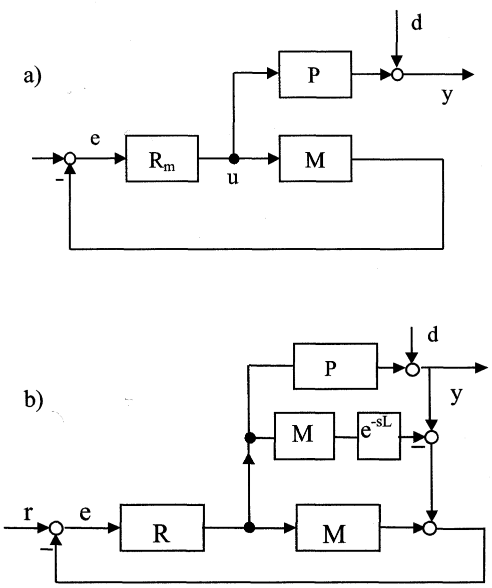

Fig. 4. a) Open-loop control of a process with big unknown time delay, b) Closed-loop control of a process with big known delay
Rys. 4. Sterowanie w układzie otwartym procesem z nieznanym opóźnieniem: a) oraz w układzie zamkniętym b) przy znanym opóźnieniu

However, for the reason of existing perturbations and nonmeasurable disturbances, the solution of Fig. 4a cannot be employed. If the time delay is known, then the structure shown in Fig. 4b is applicable by way of an indirect measurement of disturbances. Such a structure differs slightly from that with Smith predictor, however, the relationship for the output is identical with that of eq. (8).

2.4. The Internal Model Control (IMC) structure

The Internal Model Control (IMC) structure in its standard form is depicted in Fig. 5, from which the following relationship may be derived:

$$y_{IMC} = r\frac{RP}{1+R(P-M)} + d\frac{1-RM}{1+R(P-M)} \tag{10}$$

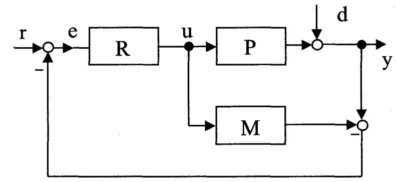

Fig. 5. Standard IMC structure
Rys. 5. Standardowa postać Struktury IMC

Let the controller be exactly equal to the inverse of the process model

$$R = M^{-1} \tag{11}$$

then in view of eq. (4) we get

$$y = r \tag{12}$$

However such an "ideal" system is impracticable. There exist a number of IMC-based solutions widely covered in the literature, e.g. [11, 12, 14].

On the basis of results reported by [11] it is possible to find a single-loop controller via IMC principles. Then

$$R_m^{IMC} = \frac{F}{M_-\left(1+FM_+\right)} \tag{13}$$

where

$$M = M_- M_+ \tag{14}$$

$$M_+ = e^{-sL}\prod_i \frac{1-\beta_i s}{1+\beta_i s}\ ,\quad \mathrm{Re}\beta_i > 0 \tag{15}$$

and F is a filter of nth order with time constant τ and order n to be tuned [5]

$$F(s) = \frac{1}{\left(1+s\tau\right)^n} \tag{16}$$

In this case the model output will be

$$y_m^{IMC} = r\,F\,M_+ \tag{17}$$

In view of eq. (4) we get

$$y_{IMC} = rFM_+\left[1+\Delta\frac{1-FM_+}{1+\Delta FM_+}\right] + d\frac{1-FM_+}{1+\Delta FM_+} \tag{18}$$

2.5. The two-degree-of-freedom system (2 DOF)

The two-degree-of-freedom system (2 DOF) in its standard form is depicted in Fig. 6 from which following relationship may be derived

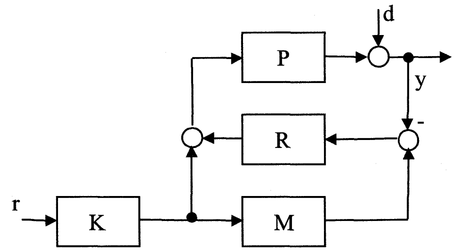

Fig. 6. Two-degree-of-freedom system
Rys. 6. System sterowania z dwoma stopniami swobody

$$y_{2DOF} = y_m^*\left[1+\frac{\Delta}{1+RM(1+\Delta)}\right]+\frac{d}{1+RM(1+\Delta)} \quad (19)$$

where

$$y_m^* = rKM \quad (20)$$

2.6. The Model Following Control (MFC) structure

The Model Following Control (MFC) structure was introduced into control engineering in 1990s. [3] described it as an adaptive system applied to a servomechanism. Earlier, a similar concept was theoretically analyzed by [10] from the viewpoint of robustness. A two-loop MFC structure is displayed in Fig. 7.

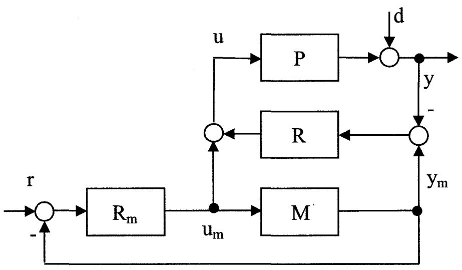

Fig. 7. Model Following Control (MFC) structure
Rys. 7. Struktura Model Following Control

While in [3] the output of the process has been compared with that of the model, in [10] the model state vector has been compared with the process state vector. The latter approach gives an additional control signal, which is added to that stemming from the model containing loop. This results in making the system more "rigid" and less practical, which was pointed out by [9].

From Fig. 7 it follows that

$$y_{MFC} = y_m\left[1+\frac{\Delta}{1+RM(1+\Delta)}\right]+d\frac{1}{1+RM(1+\Delta)} \quad (21)$$

It should be emphasized that the controller R_m is associated with y_m only, according to eq. (6). As for suppressing of perturbations and disturbances, the corrective controller R plays here a crucial role. It may also be noted that MFC offers an effective solution for an unknown delay-affected process control through a delay-free model in case unknown time delay is of limited value (see 2.3).

As was shown in [9], the influence of disturbances and perturbations on the process output is weaker, the greater is the transfer function magnitude of the R controller than that of the R_m controller

$$|R(j\omega)| \rangle |R_m(j\omega)| \ , \ \omega\in[0,\infty) \quad (22)$$

For $R = R_m$ the MFC structure loses its properties and boils down to the classic single-loop feedback system. The inequality (22) is restricted by stability conditions

$$1+RM(1+\Delta)=0 \quad (23)$$

Hence, |R| is bounded above by stability conditions (23) and $|R_m|$ is bounded below by sufficiently good reference tracking conditions (see eq. 6).

Usually, the corrective controller R may be designed by adopting a smaller stability margin than in case of R_m the classic feedback structure allows.

In [8] it was pointed out that for tracking purposes a model being much quicker than the process itself may be employed. This solution entails no impairment of disturbance suppression and makes it possible to modify y_m in a profitable way without abandoning the "strong" controller R.

2.7. If the process and the model are interchanged in the MFC structure

If the process and the model are interchanged in the MFC structure, we get a new structure designated as MFC/IMC [9] depicted in Fig. 8.

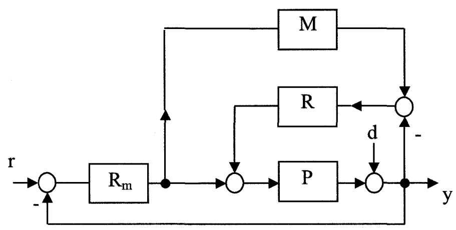

Fig. 8. MFC/IMC structure
Rys. 8. Struktura MFC/IMC

It follows from Fig. 8 that

$$y_{MFC/IMC} = y_m\left[1+\frac{\Delta}{(1+R_mM)(1+RM(1+\Delta))+R_mM\Delta}\right]+ d\frac{1}{(1+R_mM)(1+RM(1+\Delta))+R_mM\Delta} \quad (24)$$

It should be noted that y_m* shown in Fig. 8 is not equivalent to y_m defined by eq. (6). The output sensitivity is here weaker than that in the case of MFC, however, by contrast, the stability conditions are worse.

2.8. If the process delay is known, then another structure called Model Following Control with Time Delay (MFCD)

If the process delay is known, then another structure called Model Following Control with Time Delay (MFCD) may find application (Fig. 9).

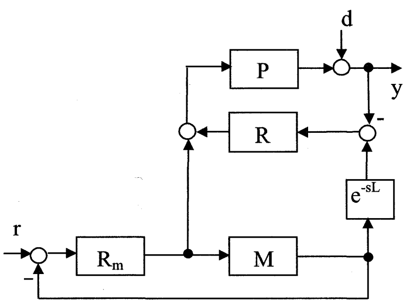

Fig. 9. MFCD structure
Rys. 9. Struktura MFC ze znanym opóźnieniem procesu (MFCD)

As it may be derived from Fig. 9, in view of eqs. (6) and (9), we get

$$y_{MFCD} = y_m e^{-sL}\left[1+\frac{\Delta}{1+RM(1+\Delta)e^{-sL}}\right]+ d\frac{1}{1+RM(1+\Delta)e^{-sL}} \quad (25)$$

Equation (25) differs substantially from eq. (8), which holds for the control system with Smith predictor.

2.9. The Model Feedback Control System (MFCS)

The Model Feedback Control System (MFCS) [13] in its standard form is depicted in Fig. 10.

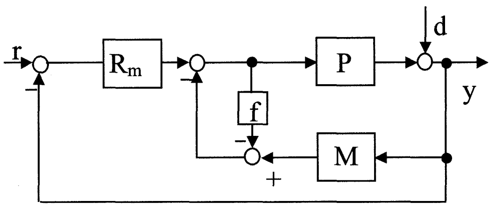

Fig. 10. The model feedback control system (MFCS)
Rys. 10. Układ regulacji ze sprzężeniem przez model (MFCS)

The transfer function from r and d to y are written as

$$y_{MFCS} = y_m \left[1 + \frac{f-1}{1-f+(1+\Delta)(1+R_m M)}\right] + + d\frac{1-f}{1-f+(1+\Delta)(1+R_m M)} \tag{26}$$

When f(s) = 1 is settled then we get from (26)

$$y_{MFCS} = y_m \tag{27}$$

So, exceptional properties this structure offers are distinguished favourably against the background of the others.

2.10. The modified control structure with Smith predictor

[2] have described a modified control structure with Smith predictor with reference to [4]. For a given substantial time delay the structure can be displayed in the way the MFC does, which is depicted in Fig. 11.

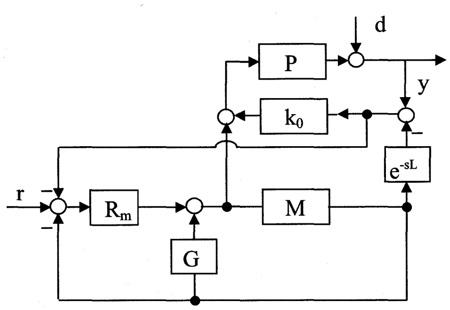

Fig. 11. Modified control system with Smith predictor (SMD)
Rys. 11. Układ regulacji ze zmodyfikowanym predyktorem Smitha (SMD)

In [2] it was assumed that k_0 is a P controller, G_c may be a P or a PD controller, and R_m is a PI one. As was shown by the authors, the structure is characterized by a very good input sensitivity.
From Fig. 11 it may be derived after simple manipulations

$$y_{SMD} = = y_m^{SMD} e^{-sL}\left[1 + \Delta\frac{1+MG+R_m M\left(1-e^{-sL}\right)}{\left[1+k_0 M(1+\Delta)\,e^{-sL}\right]\left[1+(R_m+G)M\right]+\Delta R_m e^{-sL}}\right] + + d\frac{1+MG+R_m M\left(1-e^{-sL}\right)}{\left[1+k_0 M(1+\Delta)\,e^{-sL}\right]\left[1+(R_m+G)M\right]+\Delta R_m e^{-sL}} \tag{28}$$

where

$$y_m^{SMD} = r\frac{R_m M}{1+(R_m+G)M} \tag{29}$$

The authors, however, did not analyze the disturbance sensitivity. Interestingly, they arrived at a structure being akin to MFC, which was obtained by quite other means.

3. Generalized sensitivities of SISO control systems

On the basis of relationships defining the influence of perturbations and disturbances on the output of structures having been considered, a general equation, which holds true for cases under discussion, may be written

$$y = r^* S_m\left[1+\Delta\, S_d\right] + d\, S_d \tag{30}$$

where
$S_m(s)$ is the model input sensitivity
$S_d(s)$ is the output disturbance sensitivity
$r^* = re^{-sL}$ where L is the known time delay to be compensated.

The MFCS structure presents an exception. The model input sensitivity is the same for all cases considered, except for IMC, and is equal to

$$S_m = \frac{R_m M}{1+R_m M} \tag{31}$$

In the case of SMD system, it holds

$$S_m^{SMD} = \frac{R_m M}{1+(R_m+G)M} \tag{32}$$

The basic factor, which differentiates the structures among them, is the disturbance sensitivity $S_d(s)$. This quantity is of decisive importance in suppressing disturbances and modifying the input sensitivity depending on perturbations Δ (s)

$$S_r(s) = S_m(s)\left[1+\Delta(s)S_d(s)\right] \tag{33}$$

On the basis of eq. (30), it may be plotted an equivalent general block diagram of an open-loop control system, which holds for each structure having been considered.

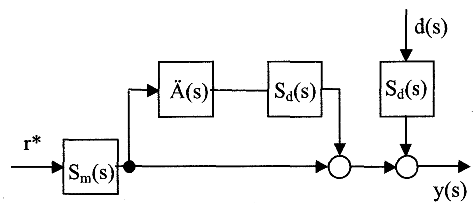

Fig. 12. General control system sensitivity model
Rys. 12. Ogólny schemat modelu wrażliwości UAR

As may be easily inferred from the above given relationships, a well-known relationship holds here for single-loop single-controller structures

$$S_r + S_d = 1 \tag{34}$$

which does not hold true for two-loop structures, such as MFC, MFC/IMC, MFCD and SMD,

$$S_r + S_d \neq 1 \tag{35}$$

However, as follows from eq. (33), the smaller is the disturbance sensitivity S_d, the better is the input sensitivity of the control system in each case under study. It should be emphasized that the disturbance sensitivity S_d depends on perturbations in different ways being peculiar to each structure considered.

4. Effect of time delay perturbations $L_m \neq L_p$

The study of process control with known big time delay in systems with Smith predictor, i.e. SM, SMD, and in MFCD systems, has been carried out under tacit assumption that the process delay is exactly known being equal to the model delay. However, in practice this assumption may be proved wrong. If so, the model is then composed of a delay-free part M and time delay part L_m, while the process allows for perturbations in the delay-free model part and exhibits its own delay $L_m \neq L_p$. So, we have

$$\left.\begin{aligned} P_L &= M(1+\Delta)e^{-sL_p} \\ M_L &= Me^{-sL_m} \end{aligned}\right\} \tag{36}$$

From (36) the counterparts of eq. (8) holding for predictor Smith based control systems can be derived

$$\begin{aligned} y_{SM} &= \\ &= y_m e^{-sL_p}\left(1+\frac{\Delta(1+RM)-RMe^{-sL_m}\left[e^{-s(L_p-L_m)}-1+\Delta e^{-s(L_p-L_m)}\right]}{1+RM\left[1-e^{-sL_m}\left(1-(1+\Delta)e^{-s(L_p-L_m)}\right)\right]}\right)+ \\ &+d\frac{1+RM\left(1-e^{sL_m}\right)}{1+RM\left[1-e^{-sL_m}\left(1-(1+\Delta)e^{-s(L_p-L_m)}\right)\right]} \end{aligned} \tag{37}$$

Similarly, in the case of MFCD the counterpart of eq. (25) will be

$$\begin{aligned} y_{MFCD} &= y_m e^{-sL_m}\left[1+\frac{e^{-s(L_p-L_m)}-1+\Delta e^{-s(L_p-L_m}}{1+RM(1+\Delta)e^{-sL_p}}\right]+ \\ &+d\frac{1}{1+RM(1+\Delta)e^{-sL_p}} \end{aligned} \tag{38}$$

Obviously, if $L_p = L_m$, then eqs. (37) and (38) reduce to eqs. (8) and (25), respectively.

Introducing a new designation δ for cumulative perturbations, we get from (32) by analogy with (4)

$$P_L = M_L(1+\delta) \tag{39}$$

whence

$$\delta = e^{-s(L_p-L_m)} - 1 + \Delta e^{-s(L_p-L_m)} \tag{40}$$

Comparing the general relationship (30) with (38), in view of eq. (40), we get for the MFCD structure

$$y_{MFCD} = y_m e^{sL_m}\left[1+\delta S_d\right] + d \cdot S_d \tag{41}$$

5. Concluding remarks

The presented sensitivity comparison study made for individual SISO control structures leads to a general relationship (30) and a general equivalent block diagram in the form of an open-loop control system (Fig. 12). It has been shown that disturbance sensitivity S_d is the essential quantity, which differentiates the structures under study from the viewpoint of their sensitivity. Exceptional properties and simplicity of the MFCS structure are pointed out in [13]. It should be emphasized that an increase in MFC robustness results in giving up the ideal reference tracking. This is of particular importance for time delay affected systems, if time delay is unknown. In case of delay-free processes this limitation loses in its significance by providing that |R| be much more greater than $|R_m|$. At the same time a sufficiently great value of $|R_m|$ will secure a good reference tracking. It is significant that in case of tracking the model input sensitivity can be substantially improved by employing a model M being much quicker than the process P without impairing the output disturbance sensitivity.

The model following control structure may find wide application in new intelligent controllers to robust control of parameter-time varying plants.

6. References

1. Goodwin G.C., J. H. Braslavsky, M. M. Seron. Non-stationary stochastic embedding for transfer function estimation. Automatica 38 (2002) pp. 47-62.
2. Kaya I., D. P. Atherton. A new PI-PD Smith predictor for control of processes with long dead time. 1999 IFAC 14 th Triennal World Congress, Beijing, P. R. China, C-2a-08-06, pp. 283-288.
3. Li G., K. M. Tsang & S. L. Ho. A novel model following scheme with simple structure for electrical position servo systems. Iternational Journal of Systems Science, 29, 1998, pp. 959-969.
4. Mataušek M. R. and A. D. Micić. A modified Smith predictor for controlling a process with an integrator and long dead-time. IEEE Transactions on Automatic Control. Vol. 41, No. 8, 1996.
5. Morari M. & E Zafiriou. Robust process control. Englewood Clifs. NJ: Prentice-Hall, 1989.
6. Narikiyo T. and Izumi T. On model feedback control for robot manipulators. Journal of Dynamic Systems, Measurement and Ciontrol. 113, 1991. pp. 371-378
7. Skoczowski S. The control of plant containing delay through its model wihout delay (in Polish). Pomiary Automatyka Kontrola, No. 6, 1973, pp. 260-262.
8. Skoczowski S. A robust control system utilizing the plant model (in Polish). Pomiary Automatyka Kontrola No. 9. 1999, pp. 2-4
9. Skoczowski S. Robust model following control with use of a plant model. International Journal of Systems Science, 32, No. 12, 2001, pp. 1413-1427.
10. Sugie T. and K. Osuka. Robust model following control with prescribed accuracy for uncertain nonlinear systems. Inernational Journal of Control. 58, 1993, pp. 991-1009.
11. Wang Q. G., C. C. Hang, X P. Yang. Single-loop controller design via IMC principles. Automatica 37, 2001, pp. 2041-2048.
12. Wang Q. G., Q. Bi and Y. Zhang. Partial Internal Model Control. IEEE Trans. on Industrial Electronics. Vol. 48, No 5, 2001, pp. 976-982.
13. Yamada K., Moki T. Relation between model feedback control systems and parameterization of all stabilizing controllers. 2002 IFAC 15 th Triennial World Congress, Barcelona, Spain, CD.
14. Zhou K. and Z. Ren. A new controller architecture for high performance, robust, and flaut-tolerant control. IEEE Transactions on Automatic Control. Vol. 46, No. 10, 2001, pp. 1613-1618.

Tytuł: Struktury układów automatycznej regulacji oraz ich właściwości odpornościowe

Artykuł recenzowany