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Pairwise control principle in large-scale systems

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The purpose of the paper is present an algorithm of partially decentralized control design for one type of large-scale linear dynamical system. The pairwise autonomous principle is preferred where design conditions are derived in the bounded real lemma form, and global system stability is reproven to formulate potential application principle in fault tolerant control. The validity of the proposed method is demonstrated by the numerical example.

Key words: large-scale systems, decentralized control, state feedback, linear matrix inequality, asymptotic stability

1. Introduction

Control structure implementation in large-scale systems is computationally pretentious due to high complexity and large dimensionality. Thus, generally all approaches to large-scale system control (see e.g. [6], [7], [8], [13], [14]) are based on some sort of model alteration (minimization of interconnection, model reduction through aggregation methods, etc.), and a certain type of decentralization. For well-defined system inputs in large-scale interconnected dynamical systems where each subsystem input is decoupled from others, a partially decentralized design approach can be used. This approach provides some advantage in parallel processing and interconnection utilization. Application of mentioned above ideas for the purpose of decentralization and reduction of computational requirements in LQ control, and Kalman filtering was subjects of the papers [12], [2], [3].

A number of problems that arise in the state feedback control can be reduced to a handful of standard convex and quasi-convex problems that involve matrix inequalities. It is known that optimal solution can be computed by using interior point method [15] which converge in polynomial time with respect to the problem size, and efficient interior point algorithms have recently been developed for and further development of algorithms of these standard problems is the area of active research. In such approaches, the stability conditions may be expressed in terms of linear matrix inequalities (LMI), which have a

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notable practical interest due to the existence of numerical solvers. Some progress review in this field can be found in [1], [5], [17], and the references therein.

In this paper the pairwise partially decentralized approach to continuous-time large-scale linear systems control design is formulated in the bounded real lemma form, and conditions for the global system stability are reproved. The resulting controller structures are pairwise autonomous (implying parallel processing) and partially decentralized (implying interconnection utilization). The potential application is in fault tolerant control systems where such control reconfiguration principle like [10, 11] can be relatively simple introduced in the pairwise partially decentralized structures with respect to sensors fault in a subsystem.

2. Problem formulation

Considering the system model of the form

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{2}$$

reordered in such way that

$$\boldsymbol{A} = \left[\ \boldsymbol{A}_{i,l}\ \right],\ \boldsymbol{C} = \left[\ \boldsymbol{C}_{i,l}\ \right],\ \boldsymbol{B} = \mathrm{diag}\left[\ \boldsymbol{B}_{i}\ \right],\ \boldsymbol{D} = \boldsymbol{0} \tag{3}$$

where $i,l = 1,2,\ldots,p$, $\boldsymbol{q}(t) \in \Re^{n}$ stands up for the system state, $\boldsymbol{u}(t) \in \Re^{r}$ denotes the control input, $\boldsymbol{y}(t) \in \Re^{m}$ is the reference output, and the matrices $\boldsymbol{A} \in \Re^{n\times n}$, $\boldsymbol{B} \in \Re^{n\times r}$, $\boldsymbol{C} \in \Re^{m\times n}$, and $\boldsymbol{D} \in \Re^{m\times r}$ are real finite valued.

Thus, respecting the above give matrix structures it yields

$$\dot{\boldsymbol{q}}_{h}(t) = \boldsymbol{A}_{hh}\boldsymbol{q}_{h}(t) + \sum_{\substack{l=1\\ l\neq h}}^{p}(\boldsymbol{A}_{hl}\boldsymbol{q}_{l}(t) + \boldsymbol{B}_{h}\boldsymbol{u}_{h}(t)) \tag{4}$$

$$\boldsymbol{y}_{h}(t) = \boldsymbol{C}_{hh}\boldsymbol{q}_{h}(t) + \sum_{\substack{l=1\\ l\neq h}}^{p}\boldsymbol{C}_{hl}\boldsymbol{q}_{l}(t) \tag{5}$$

where $\boldsymbol{q}_{h}(t) \in \Re^{n_h}$, $\boldsymbol{u}_{h}(t) \in \Re^{r_h}$, $\boldsymbol{y}_{h}(t) \in \Re^{m_h}$, $\boldsymbol{A}_{hl} \in \Re^{n_h\times n_l}$, $\boldsymbol{B}_{h} \in \Re^{r_h\times n_h}$, and $\boldsymbol{C}_{hl} \in \Re^{m_h\times n_h}$, respectively, and $n = \sum_{l=1}^{p} n_l$, $r = \sum_{l=1}^{p} r_l$, $m = \sum_{l=1}^{p} m_l$.

Problem of the interest is to design a closed-loop structure based on the pairwise control in such way that the large-scale system be pairwise asymptotically stable, having properties of an equivalent state feedback control of the form

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{q}(t) \tag{6}$$

where

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{11} & \boldsymbol{K}_{12} & \dots & \boldsymbol{K}_{1p} \\ \boldsymbol{K}_{21} & \boldsymbol{K}_{22} & \dots & \boldsymbol{K}_{2p} \\ & & \vdots & \\ \boldsymbol{K}_{p1} & \boldsymbol{K}_{p2} & \dots & \boldsymbol{K}_{pp} \end{bmatrix}, \ \boldsymbol{K}_{hh} = \sum_{\substack{l=1 \\ l \neq h}}^{p} \boldsymbol{K}_h^l \tag{7}$$

A matrix on the main diagonal of the block matrix $\boldsymbol{K}$ is considered as a summation of $p-1$ matrices, where p is the number of all subsystems. Structurally, $\boldsymbol{K}_h^l$ represents a partial gain matrix through the h-th subsystem input is affected by the h-th subsystem state if the h-th subsystem is paired with the l-th subsystem.

3. Preliminaries

Proposition 1 (*Quadratic performance*) *Given a stable system* (1), (2)*, then there exists such* $\gamma > 0$, $\gamma \in \Re$ *that*

$$\int_0^t (\boldsymbol{y}^T(r)\boldsymbol{y}(r) - \gamma \boldsymbol{u}^T(r)\boldsymbol{u}(r))dr > 0 \tag{8}$$

Proof. Let $\widetilde{\boldsymbol{y}}(s)$, $\widetilde{\boldsymbol{u}}(s)$ stand for Laplace transform of m dimensional output vector $\boldsymbol{y}(t)$ and r dimensional input vector $\boldsymbol{u}(t)$, respectively, which are identically zero for $t < 0$. Then

$$\widetilde{\boldsymbol{y}}(s) = \boldsymbol{G}(s)\widetilde{\boldsymbol{u}}(s) \tag{9}$$

is the operator relation, where $\boldsymbol{G}(s)$ is noted generally as the transfer function matrix (short referred as the transfer matrix). Its elements are fraction of scalar polynomials of s, and its dimension is $m \times r$, and subsequently (9) implies

$$\|\widetilde{\boldsymbol{y}}(s)\| \leqslant \|\boldsymbol{G}(s)\|\|\widetilde{\boldsymbol{u}}(s)\| \tag{10}$$

where $\|\cdot\|$ represents the Euclidean norm for vectors and the spectral norm for matrices. Since the infinity norm property implies

$$\frac{1}{\sqrt{m}}\|\boldsymbol{G}(s)\|_\infty \leqslant \|\boldsymbol{G}(s)\| \leqslant \sqrt{r}\|\boldsymbol{G}(s)\|_\infty \tag{11}$$

with the notation

$$\|\boldsymbol{G}(s)\|_\infty = \sqrt{\gamma} \tag{12}$$

where $\sqrt{\gamma}$ is the value of the infinity norm of the transfer matrix $\boldsymbol{G}(s)$, then (11) can be rewritten as

$$0 < \frac{1}{\sqrt{m}} \leqslant 1 < \frac{\|\widetilde{\boldsymbol{y}}(s)\|}{\sqrt{\gamma}\|\widetilde{\boldsymbol{u}}(s)\|} \leqslant \|\boldsymbol{G}(s)\| \leqslant \sqrt{r} \tag{13}$$

Now (13) results in the inequality

$$\|\widetilde{\boldsymbol{y}}(s)\| - \sqrt{\gamma}\|\widetilde{\boldsymbol{u}}(s)\| > 0 \tag{14}$$

$$\|\widetilde{\boldsymbol{y}}(s)\|^2 - \gamma\|\widetilde{\boldsymbol{u}}(s)\|^2 > 0 \tag{15}$$

respectively, and using the Parceval's theorem property then (15) gives

$$\int_0^\infty (\boldsymbol{y}^T(r)\boldsymbol{y}(r) - \gamma\boldsymbol{u}^T(r)\boldsymbol{u}(r))\mathrm{d}r > 0 \tag{16}$$

which implies that with such defined $\gamma > 0$ (16) is satisfied.

Thus, if the system is stable (16) implies (8). This concludes the proof. □

Proposition 2 (*Bounded real lemma*) *System* (1), (2) *is stable with quadratic performance* $\|\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}\|_\infty \leqslant \sqrt{\gamma}$ *if there exist a symmetric positive definite matrix* $\boldsymbol{P} \in \Re^{n\times n}$ *and a positive scalar* $\gamma \in \Re$ *such that*

$$\boldsymbol{P} = \boldsymbol{P}^T > 0, \quad \gamma > 0 \tag{17}$$

$$\begin{bmatrix} \boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A} & \boldsymbol{P}\boldsymbol{B} & \boldsymbol{C}^T \\ * & -\gamma\boldsymbol{I}_r & \boldsymbol{D}^T \\ * & * & -\boldsymbol{I}_m \end{bmatrix} < 0 \tag{18}$$

where $\boldsymbol{I}_r \in \Re^{r\times r}$, $\boldsymbol{I}_m \in \Re^{m\times m}$ *are identity matrices, respectively,*

Hereafter, $*$ *denotes the symmetric item in a symmetric matrix.*

Proof. In the sense of the quadratic performance there exists an enough large $\gamma > 0$ such that Lyapunov function of the form

$$v(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{q}(t) + \int_0^t \big(\boldsymbol{y}^T(r)\boldsymbol{y}(r) - \gamma\boldsymbol{u}^T(r)\boldsymbol{u}(r)\big)\mathrm{d}r > 0 \tag{19}$$

be positive definite, and the time derivative of (19) be negative definite. Thus, evaluating derivative of $v(\boldsymbol{q}(t))$ with respect to t along a trajectory of the system (1), (2) it yields

$$\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t) + \boldsymbol{q}^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t) + \boldsymbol{y}^T(t)\boldsymbol{y}(t) - \gamma\boldsymbol{u}^T(t)\boldsymbol{u}(t) < 0 \tag{20}$$

$$\begin{aligned} \dot{v}(\boldsymbol{q}(t)) = (\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{u}(t))^T\boldsymbol{P}\boldsymbol{q}(t) + \boldsymbol{q}^T(t)\boldsymbol{P}(\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{u}(t)) + \\ +(\boldsymbol{C}\boldsymbol{q}(t)+\boldsymbol{D}\boldsymbol{u}(t))^T(\boldsymbol{C}\boldsymbol{q}(t)+\boldsymbol{D}\boldsymbol{u}(t)) < 0 \end{aligned} \tag{21}$$

respectively. Introducing the next notation

$$\boldsymbol{q}_c^T(t) = \begin{bmatrix} \boldsymbol{q}^T(t) & \boldsymbol{u}^T(t) \end{bmatrix} \tag{22}$$

then $\dot{v}(\boldsymbol{q}(t))$ can be written as follows

$$\dot{v}(\boldsymbol{q}(t)) = \boldsymbol{q}_c^T(t)\boldsymbol{P}_c\boldsymbol{q}_c(t) < 0 \tag{23}$$

where

$$\boldsymbol{P}_c = \begin{bmatrix} \boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A} & \boldsymbol{P}\boldsymbol{B}_u \\ * & -\gamma\boldsymbol{I}_r \end{bmatrix} + \begin{bmatrix} \boldsymbol{C}^T\boldsymbol{C} & \boldsymbol{C}^T\boldsymbol{D} \\ * & \boldsymbol{D}^T\boldsymbol{D} \end{bmatrix} < 0 \tag{24}$$

Since

$$\begin{bmatrix} \boldsymbol{C}^T\boldsymbol{C} & \boldsymbol{C}^T\boldsymbol{D} \\ * & \boldsymbol{D}^T\boldsymbol{D} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}^T \\ \boldsymbol{D}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \geqslant 0 \tag{25}$$

applying the Schur complement formula to the inequality (25) it can be obtained

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{C}^T \\ * & \boldsymbol{0} & \boldsymbol{D}^T \\ * & * & -\boldsymbol{I}_m \end{bmatrix} \geqslant 0 \tag{26}$$

and using (26) the LMI condition (24) can be written compactly as (18). This concludes the proof. □

Proposition 3 *Autonomous system* (1)-(3) *is stable if there exists a set of symmetric matrices,* $h = 1,2\ldots,p-1$, $k = h+1, h+2\ldots,p$, $h \neq k$

$$\boldsymbol{P}_{hk}^{\circ} = \begin{bmatrix} \boldsymbol{P}_h^k & \boldsymbol{P}_{hk} \\ \boldsymbol{P}_{kh} & \boldsymbol{P}_k^h \end{bmatrix} > 0 \tag{27}$$

such that

$$\sum_{h=1}^{p-1}\sum_{k=h+1}^{p} \left\{ \begin{array}{c} \dot{\boldsymbol{q}}_{hk}^T(t) \begin{bmatrix} \boldsymbol{P}_h^k & \boldsymbol{P}_{hk} \\ \boldsymbol{P}_{kh} & \boldsymbol{P}_k^h \end{bmatrix} \boldsymbol{q}_{hk}(t) + \\ +\boldsymbol{q}_{hk}^T(t) \begin{bmatrix} \boldsymbol{P}_h^k & \boldsymbol{P}_{hk} \\ \boldsymbol{P}_{kh} & \boldsymbol{P}_k^h \end{bmatrix} \dot{\boldsymbol{q}}_{hk}(t) \end{array} \right\} < 0 \tag{28}$$

where

$$\boldsymbol{q}_{hk}^T(t) = \begin{bmatrix} \boldsymbol{q}_h^T(t) & \boldsymbol{q}_k^T(t) \end{bmatrix} \tag{29}$$

Proof. Defining Lyapunov function as follows

$$v(\boldsymbol{q}(i)) = \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) > 0 \tag{30}$$

where $\boldsymbol{P} = \boldsymbol{P}^T > 0$, $\boldsymbol{P} \in \Re^{n\times n}$, then the time rate of change of $v(\boldsymbol{q}(i))$ along a solution of the system (1)-(3) is

$$\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t) + \boldsymbol{q}^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t) < 0 \tag{31}$$

Considering the same form of $\boldsymbol{P}$ with respect to $\boldsymbol{K}$, i.e.

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} & \dots & \boldsymbol{P}_{1p} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} & \dots & \boldsymbol{P}_{2p} \\ & & \vdots & \\ \boldsymbol{P}_{p1} & \boldsymbol{P}_{p2} & \dots & \boldsymbol{P}_{pp} \end{bmatrix}, \ \boldsymbol{P}_{hh} = \sum_{\substack{l=1 \\ l \neq h}}^{p} \boldsymbol{P}_h^l \tag{32}$$

then the next separation is possible

$$\boldsymbol{P} = \left(\left(\begin{bmatrix} \boldsymbol{P}_1^2 & \boldsymbol{P}_{12} & \mathbf{0} & \dots & \mathbf{0} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_2^1 & \mathbf{0} & \dots & \mathbf{0} \\ & & & \vdots & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix} + \dots + \begin{bmatrix} \boldsymbol{P}_1^p & \mathbf{0} & \dots & \mathbf{0} & \boldsymbol{P}_{1p} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ & & \vdots & & \\ \boldsymbol{P}_{p1} & \mathbf{0} & \dots & \mathbf{0} & \boldsymbol{P}_p^1 \end{bmatrix} \right) + \right.$$
$$\left. + \dots + \begin{bmatrix} \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \vdots & & & \\ \mathbf{0} & \dots & \mathbf{0} & \boldsymbol{P}_{p-1}^p & \boldsymbol{P}_{p-1,p} \\ \mathbf{0} & \dots & \mathbf{0} & \boldsymbol{P}_{p,p-1} & \boldsymbol{P}_p^{p-1} \end{bmatrix} \right) \tag{33}$$

Writing

$$\dot{\boldsymbol{q}}_{hk}(t) = \begin{bmatrix} \boldsymbol{A}_{hh} & \boldsymbol{A}_{hk} \\ \boldsymbol{A}_{kh} & \boldsymbol{A}_{kk} \end{bmatrix} \boldsymbol{q}_{hk}(t) + \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \begin{bmatrix} \boldsymbol{A}_{hl} \\ \boldsymbol{A}_{kl} \end{bmatrix} \boldsymbol{q}_l(t) + \begin{bmatrix} \boldsymbol{B}_h & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B}_k \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_h(t) \\ \boldsymbol{u}_k(t) \end{bmatrix} \tag{34}$$

then with (33), (34) the inequality (31) implies (28), owing to that for unforced system $\boldsymbol{u}_l(t) = \mathbf{0},\ l = 1, \dots p$. This concludes the proof. □

4. Pairwise decentralized control

The subsystem interaction implies that the control law $\boldsymbol{u}_h(t)$ generally takes form

$$\boldsymbol{u}_h(t) = -\boldsymbol{K}_{hh}\boldsymbol{q}_h(t) - \sum_{\substack{l=1 \\ l \neq h}}^{p} \boldsymbol{K}_{hl}\boldsymbol{q}_l(t) \tag{35}$$

where $\boldsymbol{K}_{hl},\ h,l = 1,2,\dots,p$ are non-zero gain matrices. Considering the structure of $\boldsymbol{K}$ as is defined in (7) then it yields

$$\boldsymbol{u}_h(t) = -\sum_{\substack{l=1 \\ l \neq h}}^{p} \begin{bmatrix} \boldsymbol{K}_h^l & \boldsymbol{K}_{hl} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_l(t) \end{bmatrix} =$$
$$= -\begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_k(t) \end{bmatrix} - \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \begin{bmatrix} \boldsymbol{K}_h^l & \boldsymbol{K}_{hl} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_l(t) \end{bmatrix} = \boldsymbol{u}_h^k(t) + \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \boldsymbol{u}_h^l(t) \tag{36}$$

where

$$\boldsymbol{u}_h^l(t) = -\begin{bmatrix} \boldsymbol{K}_h^l & \boldsymbol{K}_{hl} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_l(t) \end{bmatrix} \tag{37}$$

$i = 1, 2, \ldots, p$, $i \neq h$. Defining

$$-\begin{bmatrix} \boldsymbol{u}_h^k(t) \\ \boldsymbol{u}_k^h(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \\ \boldsymbol{K}_{kh} & \boldsymbol{K}_k^h \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_k(t) \end{bmatrix} = \boldsymbol{K}_{hk}^{\circ} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_k(t) \end{bmatrix} \tag{38}$$

$$\boldsymbol{K}_{hk}^{\circ} = \begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \\ \boldsymbol{K}_{kh} & \boldsymbol{K}_k^h \end{bmatrix} \tag{39}$$

$h = 1, 2 \ldots, p-1$, $k = h+1, h+2 \ldots, p$, and combining (36) for h and k it is obtained

$$\begin{bmatrix} \boldsymbol{u}_h(t) \\ \boldsymbol{u}_k(t) \end{bmatrix} = -\begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{kh} \\ \boldsymbol{K}_{hk} & \boldsymbol{K}_k^h \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_k(t) \end{bmatrix} - \begin{bmatrix} \sum\limits_{\substack{l=1 \\ l \neq h,k}}^{p} \begin{bmatrix} \boldsymbol{K}_h^l & \boldsymbol{K}_{hl} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_h(t) \\ \boldsymbol{q}_l(t) \end{bmatrix} \\ \sum\limits_{\substack{l=1 \\ l \neq h,k}}^{p} \begin{bmatrix} \boldsymbol{K}_k^l & \boldsymbol{K}_{kl} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_k(t) \\ \boldsymbol{q}_l(t) \end{bmatrix} \end{bmatrix} \tag{40}$$

$$\begin{bmatrix} \boldsymbol{u}_h(t) \\ \boldsymbol{u}_k(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_h^k(t) \\ \boldsymbol{u}_k^h(t) \end{bmatrix} + \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \begin{bmatrix} \boldsymbol{u}_h^l(t) \\ \boldsymbol{u}_k^l(t) \end{bmatrix} \tag{41}$$

respectively. Then substituting (41) in (34)

$$\begin{gathered} \dot{\boldsymbol{q}}_{hk}(t) = \\ = \left[\begin{bmatrix} \boldsymbol{A}_{hh} & \boldsymbol{A}_{hk} \\ \boldsymbol{A}_{kh} & \boldsymbol{A}_{kk} \end{bmatrix} - \begin{bmatrix} \boldsymbol{B}_h & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_k \end{bmatrix} \begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \\ \boldsymbol{K}_{kh} & \boldsymbol{K}_k^h \end{bmatrix} \right] \boldsymbol{q}_{hk}(t) + \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \begin{bmatrix} \boldsymbol{B}_h \boldsymbol{u}_h^l(t) + \boldsymbol{A}_{hl} \boldsymbol{q}_l(t) \\ \boldsymbol{B}_k \boldsymbol{u}_k^l(t) + \boldsymbol{A}_{kl} \boldsymbol{q}_l(t) \end{bmatrix} \end{gathered} \tag{42}$$

Using the next notations

$$\boldsymbol{A}_{hkc}^{\circ} = \begin{bmatrix} \boldsymbol{A}_{hh} & \boldsymbol{A}_{hk} \\ \boldsymbol{A}_{kh} & \boldsymbol{A}_{kk} \end{bmatrix} - \begin{bmatrix} \boldsymbol{B}_h & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_k \end{bmatrix} \begin{bmatrix} \boldsymbol{K}_h^k & \boldsymbol{K}_{hk} \\ \boldsymbol{K}_{kh} & \boldsymbol{K}_k^h \end{bmatrix} = \boldsymbol{A}_{hk}^{\circ} - \boldsymbol{B}_{hk}^{\circ} \boldsymbol{K}_{hk}^{\circ} \tag{43}$$

$$\begin{gathered} \boldsymbol{\omega}_{hk}^{\circ}(t) = \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \begin{bmatrix} \boldsymbol{B}_h \boldsymbol{u}_h^l(t) + \boldsymbol{A}_{hl} \boldsymbol{q}_l(t) \\ \boldsymbol{B}_k \boldsymbol{u}_k^l(t) + \boldsymbol{A}_{kl} \boldsymbol{q}_l(t) \end{bmatrix} = \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \left(\boldsymbol{B}_{hk}^{\circ} \begin{bmatrix} \boldsymbol{u}_h^l(t) \\ \boldsymbol{u}_k^l(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{A}_{hl} \\ \boldsymbol{A}_{kl} \end{bmatrix} \boldsymbol{q}_l(t) \right) = \\ = \boldsymbol{B}_{hk}^{\circ} \boldsymbol{\omega}_{hk}(t) + \sum_{\substack{l=1 \\ l \neq h,k}}^{p} \boldsymbol{A}_{hk}^{l\circ} \boldsymbol{q}_l(t) \end{gathered} \tag{44}$$

where

$$\boldsymbol{\omega}_{hk}(t) = \sum_{\substack{l=1\\ l\neq h,k}}^{p} \begin{bmatrix} \boldsymbol{u}_h^l(t) \\ \boldsymbol{u}_k^l(t) \end{bmatrix}, \quad \boldsymbol{A}_{hk}^{\circ l} = \begin{bmatrix} \boldsymbol{A}_{hl} \\ \boldsymbol{A}_{kl} \end{bmatrix} \tag{45}$$

(42) can be written as

$$\dot{\boldsymbol{q}}_{hk}(t) = \boldsymbol{A}_{hk}^{\circ C}\boldsymbol{q}_{hk}(t) + \sum_{\substack{l=1\\ l\neq h,k}}^{p} \boldsymbol{A}_{hk}^{l\circ}\boldsymbol{q}_l(t) + \boldsymbol{B}_{hk}^{\circ}\boldsymbol{\omega}_{hk}(t) \tag{46}$$

On the other hand, if

$$\boldsymbol{C}_{hh} = \sum_{\substack{l=1\\ l\neq h}}^{p} \boldsymbol{C}_h^l, \quad \boldsymbol{C}_{hk}^{\circ} = \begin{bmatrix} \boldsymbol{C}_h^k & \boldsymbol{C}_{hk} \\ \boldsymbol{C}_{kh} & \boldsymbol{C}_k^h \end{bmatrix}, \quad \boldsymbol{C}_{hk}^{l\circ} = \begin{bmatrix} \boldsymbol{C}_{hl} \\ \boldsymbol{C}_{kl} \end{bmatrix} \tag{47}$$

then

$$\boldsymbol{y}(t) = \sum_{h=1}^{p-1}\sum_{k=h+1}^{p} \Big(\boldsymbol{C}_{hk}^{\circ}\boldsymbol{q}_{hk}(t) + \sum_{\substack{l=1\\ l\neq h}}^{p} \boldsymbol{C}_h^l \boldsymbol{q}_l(t) \Big) \tag{48}$$

$$\boldsymbol{y}_{hk}(t) = \boldsymbol{C}_{hk}^{\circ}\boldsymbol{q}_{hk}(t) + \sum_{\substack{l=1\\ l\neq h}}^{p} \boldsymbol{C}_h^l \boldsymbol{q}_l(t) + \boldsymbol{0}\,\boldsymbol{\omega}_{hk}(t) \tag{49}$$

Now, taking (46), (49) considered pair of controlled subsystems is fully described.

5. Controller parameter design

Theorem 4 *Subsystem pair* (34) *in system* (1), (3), *controlled by control law* (41) *is stable with the quadratic performances* $\|\boldsymbol{C}_{hk}^{\circ}(s\boldsymbol{I} - \boldsymbol{A}_{hkc}^{\circ})^{-1}\boldsymbol{B}_{hk}^{\circ}\|_{\infty}^{2} \leqslant \gamma_{hk}$, *and* $\|\boldsymbol{C}_{hk}^{l\circ}(s\boldsymbol{I} - \boldsymbol{A}_{hkc}^{\circ})^{-1}\boldsymbol{B}_{hk}^{l\circ}\|_{\infty}^{2} \leqslant \varepsilon_{hkl}$, $h = 1,2\dots,p-1$, $k = h+1,h+2\dots,p$, $l = 1,2\dots,p$, $l \neq h,k$, *if there exist a symmetric positive definite matrix* $\boldsymbol{Y}_{hk}^{\circ} \in \Re^{(n_h+n_k)\times(n_h+n_k)}$, *a matrix* $\boldsymbol{Z}_{hk}^{\circ} \in \Re^{(r_h+r_k)\times(n_h+n_k)}$, *and positive scalars* γ_{hk}, $\varepsilon_{hkl} \in \Re$ *such that*

$$\boldsymbol{Y}_{hk}^{\circ} = \boldsymbol{Y}_{hk}^{\circ T} > 0, \quad \varepsilon_{hkl} > 0, \quad \gamma_{hk} > 0, \quad h,l = 1,\dots,p,\ l \neq h,k,\ h < k \leqslant p \tag{50}$$

$$\begin{bmatrix} \boldsymbol{\Phi}_{hk}^{\circ} & \boldsymbol{A}_{hk}^{1\circ} & \cdots & \boldsymbol{A}_{hk}^{p\circ} & \boldsymbol{B}_{hk}^{\circ} & \boldsymbol{Y}_{hk}^{\circ}\boldsymbol{C}_{hk}^{\circ T} \\ * & -\varepsilon_{hk1}\boldsymbol{I}_{n_1} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{C}_{hk}^{1\circ T} \\ \vdots & & \ddots & \vdots & \vdots & \vdots \\ * & * & \cdots & -\varepsilon_{hkp}\boldsymbol{I}_{n_p} & \boldsymbol{0} & \boldsymbol{C}_{hk}^{p\circ T} \\ * & * & \cdots & * & -\gamma_{hk}\boldsymbol{I}_{(r_h+r_k)} & \boldsymbol{0} \\ * & * & \cdots & * & * & -\boldsymbol{I}_{(m_h+m_k)} \end{bmatrix} < 0$$

(51)

where $\boldsymbol{A}_{hk}^{\circ}$, $\boldsymbol{B}_{hk}^{\circ}$, $\boldsymbol{A}_{hk}^{l\circ}$, $\boldsymbol{C}_{hk}^{\circ}$, $\boldsymbol{C}_{hk}^{l\circ}$ *are defined in* (43), (45), (47), *respectively,*

$$\boldsymbol{\Phi}_{hk}^{\circ} = \boldsymbol{Y}_{hk}^{\circ}\boldsymbol{A}_{hk}^{\circ T} + \boldsymbol{A}_{hk}^{\circ}\boldsymbol{Y}_{hk}^{\circ} - \boldsymbol{B}_{hk}^{\circ}\boldsymbol{Z}_{hk}^{\circ} - \boldsymbol{Z}_{hk}^{\circ T}\boldsymbol{B}_{hk}^{\circ T} \tag{52}$$

and where $\boldsymbol{A}_{hk}^{h\circ T}$, $\boldsymbol{A}_{hk}^{k\circ T}$, *as well as* $\boldsymbol{C}_{hk}^{h\circ}$, $\boldsymbol{C}_{hk}^{k\circ}$ *are not included into the structure of* (51). *Then* $\boldsymbol{K}_{hk}^{\circ}$ *is given as*

$$\boldsymbol{K}_{hk}^{\circ} = \boldsymbol{Z}_{hk}^{\circ}\boldsymbol{Y}_{hk}^{\circ -1} \tag{53}$$

Proof. Considering $\boldsymbol{\omega}_{hk}^{\circ}(t)$ given in (44) as an generalized input into the subsystem pair (46), (49) then using (18), (23) it can be written

$$\sum_{h=1}^{p-1}\sum_{k=h+1}^{p}\begin{bmatrix} \boldsymbol{q}_{hk}^{T}(t) & \boldsymbol{\omega}_{hk}^{\circ T}(t) \end{bmatrix}\boldsymbol{P}_{hk}^{\bullet}\begin{bmatrix} \boldsymbol{q}_{hk}(t) \\ \boldsymbol{\omega}_{hk}^{\circ}(t) \end{bmatrix} < 0 \tag{54}$$

where

$$\boldsymbol{P}_{hk}^{\bullet} = \begin{bmatrix} \boldsymbol{\Phi}_{hk}^{\bullet} & \boldsymbol{\Psi}_{hk}^{\bullet} & \boldsymbol{C}_{hk}^{\circ T} \\ * & -\boldsymbol{\Upsilon}_{hk}^{\circ} & \boldsymbol{D}_{hk}^{l\circ T} \\ * & * & -\boldsymbol{I}_{m_h+m_k} \end{bmatrix} < 0 \tag{55}$$

$$\boldsymbol{\Phi}_{hk}^{\bullet} = \boldsymbol{A}_{hkc}^{\circ T}\boldsymbol{P}_{hk}^{\circ} + \boldsymbol{P}_{hk}^{\circ}\boldsymbol{A}_{hkc}^{\circ} \tag{56}$$

$$\boldsymbol{\Psi}_{hk}^{\bullet} = \boldsymbol{P}_{hk}^{\circ}\begin{bmatrix} \left\{\boldsymbol{A}_{hk}^{l\circ}\right\}_{l=1,\ l\neq h,k}^{p} & \boldsymbol{B}_{hk}^{\circ} \end{bmatrix} = \boldsymbol{P}_{hk}^{\circ}\boldsymbol{B}_{hk}^{l\circ} \tag{57}$$

$$\boldsymbol{\Upsilon}_{hk}^{\circ} = \operatorname{diag}\begin{bmatrix} \{\varepsilon_{hkl}\boldsymbol{I}_{n_l}\}_{l=1,l\neq h,k}^{p} & \gamma_{hk}\boldsymbol{I}_{(r_h+r_k)} \end{bmatrix} \tag{58}$$

$$\boldsymbol{\omega}_{hk}^{l\circ T} = \begin{bmatrix} \{\boldsymbol{q}_{l}^{T}\}_{l=1,\ l\neq h,k}^{p} & \boldsymbol{\omega}_{hk}^{T}(t) \end{bmatrix} \tag{59}$$

$$\boldsymbol{D}_{hk}^{l\circ} = \begin{bmatrix} \{\boldsymbol{C}_{hk}\}_{l=1,\ l\neq h,k}^{p} & \boldsymbol{0} \end{bmatrix} \tag{60}$$

Defining the congruence transform matrix

$$\boldsymbol{T}_{hk} = \operatorname{diag}\begin{bmatrix} \boldsymbol{P}_{hk}^{\circ -1} & \boldsymbol{I}_{hk}^{\circ} \end{bmatrix} \tag{61}$$

where $\boldsymbol{I}_{hk}^{\circ}$ is the identity matrix of appropriate dimension, and multiplying left-hand as well as right-hand side of (55) by (61) results in

$$\begin{bmatrix} \boldsymbol{\Phi}_{hk}^{\circ} & \boldsymbol{\Psi}_{hk}^{\circ} & \boldsymbol{P}_{hk}^{\circ -1}\boldsymbol{C}_{hk}^{\circ T} \\ * & -\boldsymbol{\Upsilon}_{hk}^{\circ} & \boldsymbol{D}_{hk}^{l\circ T} \\ * & * & -\boldsymbol{I}_{(m_h+m_k)} \end{bmatrix} < 0 \tag{62}$$

$$\mathbf{\Phi}_{hk}^{\diamond} = \boldsymbol{P}_{hk}^{\circ-1}\boldsymbol{A}_{hk}^{\circ T} + \boldsymbol{A}_{hk}^{\circ}\boldsymbol{P}_{hk}^{\circ-1} - \boldsymbol{B}_{hk}^{\circ}\boldsymbol{K}_{hk}^{\circ}\boldsymbol{P}_{hk}^{\circ-1} - \boldsymbol{P}_{hk}^{\circ-1}\boldsymbol{K}_{hk}^{\circ T}\boldsymbol{B}_{hk}^{\circ T} \tag{63}$$

$$\mathbf{\Psi}_{hk}^{\circ} = \left[\begin{array}{cc} \left\{\boldsymbol{A}_{hk}^{\circ l}\right\}_{l=1,\ l\neq h,k}^{p} & \boldsymbol{B}_{hk}^{\circ} \end{array}\right] \tag{64}$$

Thus, with the substitutions

$$\boldsymbol{P}_{hk}^{\circ-1} = \boldsymbol{Y}_{hk}^{\circ}, \quad \boldsymbol{Z}_{hk}^{\circ} = \boldsymbol{K}_{hk}^{\circ}\boldsymbol{P}_{hk}^{\circ-1} = \boldsymbol{K}_{hk}^{\circ}\boldsymbol{Y}_{hk}^{\circ} \tag{65}$$

(62) implies (51). □

Note, this formulation includes in every inequality (51) all state variables of the large-scale systems (reordering in a prescribed form) and solving these in a symmetrical structure conditioned by LMIs structure definitions.

Theorem 5 (*Control expansion*) *If a system comprising $p-1$ interconnected pairwise controlled linear subsystems is stable then adding the pairwise control of the p-th subsystem the resulting structure be stable.*

Proof. Providing the base of mathematical induction principle the number of subsystem is chosen as $j = 2$. Thus,

$$\boldsymbol{P}_{12}^{\circ} = \boldsymbol{P}_{12}^{\circ T} = \left[\begin{array}{cc} \boldsymbol{P}_1^2 & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_2^1 \end{array}\right] > 0 \tag{66}$$

and this condition is satisfied solving it in the sense of Proposition 3. Moreover, Schur complement property implies

$$\boldsymbol{P}_1^2 > 0, \quad \boldsymbol{P}_2^1 - \boldsymbol{P}_{21}(\boldsymbol{P}_1^2)^{-1}\boldsymbol{P}_{12} > 0 \tag{67}$$

Since the statement holds true for at least one value, it is assumed that it will hold true for an arbitrary fixed value $j-1$, i.e. $\boldsymbol{P}_{j-1} = \boldsymbol{P}_{j-1}^T > 0$ be a positive definite Lyapunov weighting matrix of a stable system comprising $j-1$ interconnected pairwise controlled linear subsystems.

To prove that the induction hypothesis holds trues for all values of p let the j-th subsystem is connected into control in such way that

$$\boldsymbol{P}_{hj}^{\circ} = \left[\begin{array}{cc} \boldsymbol{P}_h^j & \boldsymbol{P}_{hj} \\ \boldsymbol{P}_{jh} & \boldsymbol{P}_j^h \end{array}\right] > 0 \tag{68}$$

for $h = 1, 2, \ldots, j-1$. Considering (33) a matrix $\boldsymbol{P}_j = \boldsymbol{P}_j^T$ associated with the system comprising j interconnected pairwise controlled linear subsystems takes the form

$$\boldsymbol{P}_j = \begin{bmatrix} \left[\boldsymbol{P}_{j-1} + \boldsymbol{P}_{j-1}^{\diamond}\right] & \begin{bmatrix} \boldsymbol{P}_{1j} \\ \boldsymbol{P}_{2j} \\ \vdots \\ \boldsymbol{P}_{j-1,j} \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{P}_{j1} & \boldsymbol{P}_{j2} & \cdots & \boldsymbol{P}_{j,j-1} \end{bmatrix} & \boldsymbol{P}_j^j \end{bmatrix} \tag{69}$$

where

$$\boldsymbol{P}_{j-1}^{\diamond} = \operatorname{diag}\begin{bmatrix} \boldsymbol{P}_1^j & \boldsymbol{P}_2^j & \ldots & \boldsymbol{P}_{j-1}^j \end{bmatrix} > 0 \tag{70}$$

$$\boldsymbol{P}_j^j = \sum_{\substack{l=1 \\ l \neq j}}^{j} \boldsymbol{P}_j^l > 0 \tag{71}$$

Since $\boldsymbol{P}_{j-1}^{\diamond}$, $\boldsymbol{P}_j^j$ are positive definite, with respect to Schur complement property then $\boldsymbol{P}_j$ be positive definite if

$$\boldsymbol{P}_j^j - \begin{bmatrix} \boldsymbol{P}_{j1} & \boldsymbol{P}_{j2} & \cdots & \boldsymbol{P}_{j,j-1} \end{bmatrix} (\boldsymbol{P}_{j-1} + \boldsymbol{P}_{j-1}^{\diamond})^{-1} \begin{bmatrix} \boldsymbol{P}_{1p} \\ \boldsymbol{P}_{2p} \\ \vdots \\ \boldsymbol{P}_{j-1,j} \end{bmatrix} \tag{72}$$

be positive definite.

Using Shermann–Morrison–Woodbury equality it yields

$$(\boldsymbol{P}_{j-1} + \boldsymbol{P}_{j-1}^{\diamond})^{-1} = (\boldsymbol{P}_{j-1}^{\diamond})^{-1} - (\boldsymbol{P}_{j-1}^{\diamond})^{-1}((\boldsymbol{P}_{j-1})^{-1} + (\boldsymbol{P}_{j-1}^{\diamond})^{-1})^{-1}(\boldsymbol{P}_{j-1}^{\diamond})^{-1} \tag{73}$$

and (72) can be rewritten as

$$\boldsymbol{P}_j^j - \begin{bmatrix} \boldsymbol{P}_{j1} & \boldsymbol{P}_{j2} & \cdots & \boldsymbol{P}_{j,j-1} \end{bmatrix} (\boldsymbol{P}_{j-1}^{\diamond})^{-1} \begin{bmatrix} \boldsymbol{P}_{1p} \\ \boldsymbol{P}_{2p} \\ \vdots \\ \boldsymbol{P}_{j-1,j} \end{bmatrix} + \boldsymbol{P}^{\#} \tag{74}$$

Since

$$\boldsymbol{P}_{j-1}^{\bullet} = (\boldsymbol{P}_{j-1}^{\diamond})^{-1} + (\boldsymbol{P}_{j-1})^{-1} > 0 \tag{75}$$

$\boldsymbol{P}^{\#}$ is positive definite, i.e.

$$\boldsymbol{P}^{\#}=\left[\begin{array}{cccc}\boldsymbol{P}_{j1} & \boldsymbol{P}_{j2} & \cdots & \boldsymbol{P}_{j,j-1}\end{array}\right](\boldsymbol{P}^{\diamond}_{j-1}\boldsymbol{P}^{\bullet}_{j-1}\boldsymbol{P}^{\diamond}_{j-1})^{-1}\left[\begin{array}{c}\boldsymbol{P}_{1p}\\ \boldsymbol{P}_{2p}\\ \vdots\\ \boldsymbol{P}_{j-1,j}\end{array}\right]>0 \tag{76}$$

It is evident that (74) can now be written as follows

$$\sum_{h=1}^{j-1}\left(\boldsymbol{P}^{h}_{j}-\boldsymbol{P}_{jh}(\boldsymbol{P}^{j}_{h})^{-1}\boldsymbol{P}_{hj}\right)+\boldsymbol{P}^{\#} \tag{77}$$

Since Schur complement of (68) implies, $h=1,2,\ldots,j-1$

$$\boldsymbol{P}^{h}_{j}-\boldsymbol{P}_{jh}(\boldsymbol{P}^{j}_{h})^{-1}\boldsymbol{P}_{hj}>0, \qquad \boldsymbol{P}^{j}_{h}>0 \tag{78}$$

such positive definiteness of (77) implies positive definiteness of $\boldsymbol{P}_j$. □

Remark 1 Inequality (69) implies that if faulty sensors occur in a subsystem sensor structure for $\boldsymbol{q}_h$, the rest pairwise autonomous systems stay stable if loops with $\boldsymbol{q}_h$ are blocked, and reconfiguration is necessary only with respect to the h-th subsystem sensors.

6. Illustrative example

To demonstrate properties of the proposed approach a simple system with four-inputs and four-outputs is used in the example. The parameters of this system are

$$\boldsymbol{A}=\left[\begin{array}{rrrr}3 & 1 & 2 & -1\\ -1 & 2 & 0 & 1\\ 1 & -1 & 1 & 3\\ 1 & -2 & -2 & 2\end{array}\right],\ \boldsymbol{C}=\left[\begin{array}{rrrr}3 & 1 & 2 & 1\\ 0 & 6 & 1 & 0\\ 2 & -1 & 3 & 0\\ 0 & 0 & 1 & 3\end{array}\right]$$

$$\boldsymbol{B}=\mathrm{diag}\left[\begin{array}{cccc}1 & 1 & 1 & 1\end{array}\right],\ \boldsymbol{B}_{hk}=\left[\begin{array}{cc}1 & 0\\ 0 & 1\end{array}\right],\ h=1,2,3,\ k=2,3,4,\ h<k$$

$$\boldsymbol{A}^{\circ}_{12}=\left[\begin{array}{rr}3 & 1\\ -1 & 2\end{array}\right],\boldsymbol{A}^{3\circ}_{12}=\left[\begin{array}{r}2\\ 0\end{array}\right],\boldsymbol{A}^{4\circ}_{12}=\left[\begin{array}{r}-1\\ 1\end{array}\right],\boldsymbol{C}^{\circ}_{12}=\left[\begin{array}{rr}1 & 1\\ 0 & 2\end{array}\right],\boldsymbol{C}^{3\circ}_{12}=\left[\begin{array}{r}2\\ 1\end{array}\right],\boldsymbol{C}^{4\circ}_{12}=\left[\begin{array}{r}1\\ 0\end{array}\right]$$

$$\boldsymbol{A}^{\circ}_{13}=\left[\begin{array}{rr}3 & 2\\ 1 & 1\end{array}\right],\boldsymbol{A}^{2\circ}_{13}=\left[\begin{array}{r}1\\ -1\end{array}\right],\boldsymbol{A}^{4\circ}_{13}=\left[\begin{array}{r}-1\\ 3\end{array}\right],\boldsymbol{C}^{\circ}_{13}=\left[\begin{array}{rr}1 & 2\\ 2 & 1\end{array}\right],\boldsymbol{C}^{2\circ}_{13}=\left[\begin{array}{r}1\\ -1\end{array}\right],\boldsymbol{C}^{4\circ}_{13}=\left[\begin{array}{r}1\\ 0\end{array}\right]$$

$$\boldsymbol{A}_{14}^{\circ}=\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}, \boldsymbol{A}_{14}^{2\circ}=\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \boldsymbol{A}_{14}^{3\circ}=\begin{bmatrix} 2 \\ -2 \end{bmatrix}, \boldsymbol{C}_{14}^{\circ}=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \boldsymbol{C}_{14}^{2\circ}=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \boldsymbol{C}_{14}^{3\circ}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\boldsymbol{A}_{23}^{\circ}=\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \boldsymbol{A}_{23}^{1\circ}=\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \boldsymbol{A}_{23}^{4\circ}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \boldsymbol{C}_{23}^{\circ}=\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \boldsymbol{C}_{23}^{1\circ}=\begin{bmatrix} 0 \\ 2 \end{bmatrix}, \boldsymbol{C}_{23}^{4\circ}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{A}_{24}^{\circ}=\begin{bmatrix} 2 & 1 \\ -2 & 2 \end{bmatrix}, \boldsymbol{A}_{24}^{1\circ}=\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \boldsymbol{A}_{24}^{3\circ}=\begin{bmatrix} 0 \\ -2 \end{bmatrix}, \boldsymbol{C}_{24}^{\circ}=\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \boldsymbol{C}_{24}^{1\circ}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{C}_{24}^{3\circ}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boldsymbol{A}_{34}^{\circ}=\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix}, \boldsymbol{A}_{34}^{1\circ}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \boldsymbol{A}_{34}^{2\circ}=\begin{bmatrix} -1 \\ -2 \end{bmatrix}, \boldsymbol{C}_{34}^{\circ}=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \boldsymbol{C}_{34}^{1\circ}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \boldsymbol{C}_{34}^{2\circ}=\begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Considering e.g. $h = 2$, $k = 3$ then (51) implies

$$\begin{bmatrix} \boldsymbol{\Phi}_{23}^{\circ} & \boldsymbol{A}_{23}^{1\circ} & \boldsymbol{A}_{23}^{4\circ} & \boldsymbol{B}_{23}^{\circ} & \boldsymbol{Y}_{23}^{\circ}\boldsymbol{C}_{23}^{\circ T} \\ * & -\varepsilon_{231} & 0 & \boldsymbol{0} & \boldsymbol{C}_{23}^{\circ 1T} \\ * & * & -\varepsilon_{232} & \boldsymbol{0} & \boldsymbol{C}_{23}^{\circ 4T} \\ * & * & * & -\gamma_{23}\boldsymbol{I}_2 & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{I}_2 \end{bmatrix} < 0$$

$$\boldsymbol{\Phi}_{23}^{\circ} = \boldsymbol{A}_{23}^{\circ}\boldsymbol{Y}_{23}^{\circ} + \boldsymbol{Y}_{23}^{\circ}\boldsymbol{A}_{23}^{\circ T} - \boldsymbol{B}_{23}^{\circ}\boldsymbol{Z}_{23}^{\circ} - \boldsymbol{Y}_{23}^{\circ T}\boldsymbol{B}_{23}^{\circ T}$$

and solving this inequality with respect to the LMI matrix variables $\boldsymbol{Y}_{23}^{\circ}$, $\boldsymbol{Z}_{23}^{\circ}$, ε_{231}, ε_{232}, and γ_{23} the problem was feasible giving the next solutions

$$\boldsymbol{Y}_{23}^{\circ} = \begin{bmatrix} 0.6109 & 0.0196 \\ 0.0196 & 0.8891 \end{bmatrix}, \ \boldsymbol{Z}_{23}^{\circ} = \begin{bmatrix} 5.4325 & -0.1854 \\ -0.1896 & 5.7929 \end{bmatrix}$$

$$\varepsilon_{231} = 8.9610, \quad \varepsilon_{234} = 6.1122, \quad \gamma_{23} = 5.6881$$

which results in

$$\boldsymbol{K}_{23}^{\circ} = \begin{bmatrix} 8.9063 & -0.4047 \\ -0.5197 & 6.5267 \end{bmatrix}$$

By the same way the gain matrix set was computed as follows

$$\boldsymbol{K}_{12}^{\circ} = \begin{bmatrix} 7.2425 & 2.5305 \\ 2.3707 & 10.5833 \end{bmatrix}, \ \boldsymbol{K}_{13}^{\circ} = \begin{bmatrix} 8.1138 & 4.3414 \\ 4.1602 & 9.0512 \end{bmatrix}$$

$$\boldsymbol{K}_{14}^{\circ} = \begin{bmatrix} 8.1794 & 2.0529 \\ 1.1264 & 5.3293 \end{bmatrix}, \ \boldsymbol{K}_{23}^{\circ} = \begin{bmatrix} 8.9063 & -0.4047 \\ -0.5197 & 6.5267 \end{bmatrix}$$

$$\boldsymbol{K}_{24}^{\circ} = \begin{bmatrix} 7.3799 & -0.5361 \\ -0.7325 & 4.3683 \end{bmatrix}, \ \boldsymbol{K}_{34}^{\circ} = \begin{bmatrix} 5.7969 & 2.3816 \\ 3.7088 & 6.0249 \end{bmatrix}$$

Note, the control laws are realized in the partly-autonomous structure (38), (39), where every subsystem pair is stable, and the large-scale system be stable, too.

To compare the results, an equivalent gain matrix (7) to centralized control can be constructed

$$\boldsymbol{K} = \begin{bmatrix} 23.5358 & 2.5305 & 4.3414 & 2.0529 \\ 2.3707 & 26.8694 & -0.4047 & -0.5361 \\ 4.1602 & -0.5197 & 21.3748 & 2.3816 \\ 1.1264 & -0.7325 & 3.7088 & 15.7225 \end{bmatrix}$$

where the closed-loop eigenvalue spectrum is

$$\rho(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}) = \left\{ \begin{array}{ccc} -22.7536 & -26.2085 & -15.2702 \pm 2.5280\mathrm{i} \end{array} \right\}$$

Matrix $\boldsymbol{K}$ structure implies evidently that the control is diagonally dominant.

It is possible to verify - routinely - that using (18) directly to compute $\boldsymbol{K}$ gives

$$\boldsymbol{K}_{ce} = \begin{bmatrix} 8.7328 & -0.6035 & 5.3992 & 1.5464 \\ -0.4476 & 12.6053 & 0.7168 & 0.1421 \\ 4.1167 & 0.6944 & 6.3277 & 0.8524 \\ 1.2344 & 0.2404 & 1.5967 & 7.8333 \end{bmatrix}$$

and the resulting closed-loop eigenvalue spectrum be

$$\rho(\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K}_{ce}) = \left\{ \begin{array}{ccc} -8.8985 & -10.2202 & -4.1902 \pm 1.9046\mathrm{i} \end{array} \right\}$$

However, the purpose of this example is only to illustrate this method and does not address issues of numerical stability.

7. Concluding remarks

The pairwise autonomous partially decentralized approach to the large-scale linear systems control design is considered in this paper, where the design conditions are derived as the set of inequalities in the bounded real lemma form. Including subsystem interconnections the obtained inequalities are separable. Afterwards the resulting controller structures are pairwise autonomous (implying parallel processing) and partially decentralized (implying interconnection utilization).

Reproving the global system stability condition more informal approach to system design is outlined. If the interconnected subsystem outputs are assumed to be completely measurable but through certain corrupted sensors in one subsystem only, the presented principle seems to be suitable for control reconfiguration in the frame of the partly decentralized fault tolerant control structure.

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