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SYNCHRONIZATION DISTURBANCES OF THE VIBRATORY CONVEYER CAUSED BY A NOT TOTAL SYMMETRY OF THE DRIVING SYSTEM

ZABURZENIA SYNCHRONIZACJI WIBRATORÓW PRZENOŚNIKA WIBRACYJNEGO SPOWODOWANE NIEPEŁNĄ SYMETRIĄ UKŁADU NAPĘDOWEGO

The analysis of synchronization disturbances of inertial vibrators driving the vibratory conveyer supported on a system of leaf springs was performed in the paper. Especially an influence of diversification of driving and anti-torque moments on the possibility of obtaining the vibrators synchronous running, its cophasing and the time history of the working motion of the vibratory trough – was discussed.
Keywords: self synchronization, vibratory machine, vibratory conveyor

W pracy przeprowadzono analizę zaburzeń samosynchronizacji wibratorów inercyjnych napędzających przenośnik wibracyjny podparty na układzie resorów płaskich. W szczególności rozważono wpływ zróżnicowania momentów napędowych i oporowych na możliwość uzyskania biegu synchronicznego, jego współfazowość i przebieg ruchu roboczego rynny przenośnika.

Let us discuss the vibratory conveyer placed on a system of parallel leaf springs exited for vibrations by means of two counter-running inertial vibrators. In the currently used devices, driving units are the most often independent and their cophasal synchronization is obtained on the basis of a *self synchronization* [1]. Since such initial system does not allow for obtaining the needed phase angles values[2], the additional elastic support of the vibrator system is usually applied. It increases the degrees of freedom number of the machine by a transverse motion ‘z’ of the driving system – (Fig. 1)

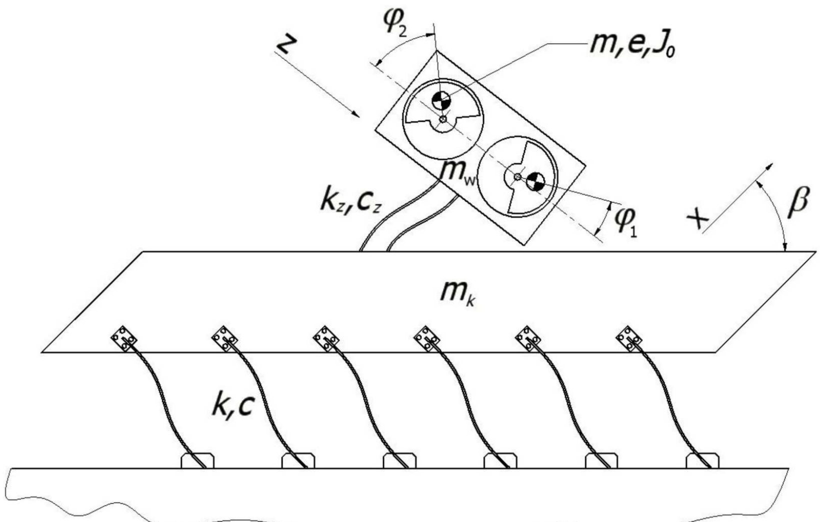

Fig. 1. Computational model of the over-resonance vibratory conveyer; k, k_z, c, c_z – coefficients of elasticity and viscous damping respectively, e- eccentric of unbalanced mass m, J_o – moment of inertia of vibrator, m_k,m_w – mass of conveyor and vibrator body respectively

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A self-synchronization phenomenon occurs in such system in a desired way generating the resulting force in the direction of the working vibrations ‘x’, provided that there is a total symmetry of the driving system and an over-resonance tuning of vibrators system in the ‘z’ axis direction.

However, in practice, a significant diversification of the resistance of bearings or electromagnetic moments of both driving systems occurs - due to operating or assembling reasons.

The determination of the influence of this type of deviation on the cophasal and synchronization ability of vibrators - and in consequence on the conveyer working motion - is the purpose of the presented paper.

Assuming that the system is not operating within the parametric resonance range [4], the dynamic equations of motion can be written as:

$$(m_k + m_w + 2m)\ddot{x} + c\dot{x} + kx = -me[\ddot{\varphi}_1 \cos(\varphi_1) + \ddot{\varphi}_2 \cos(\varphi_2)] + me[\dot{\varphi}_1^2 \sin(\varphi_1) + \dot{\varphi}_2^2 \sin(\varphi_2)] \tag{1}$$

$$(m_w + 2m)\ddot{z} + c_z\dot{z} + k_z z = me[\ddot{\varphi}_1 \sin(\varphi_1) - \ddot{\varphi}_2 \sin(\varphi_2)] + me[\dot{\varphi}_1^2 \cos(\varphi_1) - \dot{\varphi}_2^2 \cos(\varphi_2)] \tag{2}$$

Since the above given asymmetric problem can not be solved by using the most often applied integral criteria [3] *the averaging method* [1], with isolating the fast and slow-variable effects , was used. This allows for assuming the first approximation of velocities and angles in the form:

$$\dot{\varphi}_1 \cong \dot{\varphi}_2 \cong \omega = const, \quad \varphi_2 = \omega t, \quad \varphi_1 = \varphi_2 + \Delta\varphi, \quad \Delta\varphi = const \tag{3}$$

That time the steady state, at omitting a usually negligible energy dissipation in the suspension system (c=c_z=0), is described by the equations of the first approximation:

$$(m_k + m_w + 2m)\ddot{x} + kx = me\omega^2[\sin(\omega t + \Delta\varphi) + \sin(\omega t)] \tag{4}$$

$$(m_w + 2m)\ddot{z} + k_z z = me\omega^2[\cos(\omega t + \Delta\varphi) - \cos(\omega t)] \tag{5}$$

These equations – after rearrangements – can be presented in the following form:

$$(m_k + m_w + 2m)\ddot{x} + kx = me\omega^2 \sqrt{2[1 + \cos(\Delta\varphi)]} \cdot \sin(\omega t + \delta), \text{ where } \quad tg(\delta) = \frac{\sin(\Delta\varphi)}{1 + \cos(\Delta\varphi)} \tag{6}$$

$$(m_w + 2m)\ddot{z} + k_z z = -me\omega^2 \sqrt{2[1 - \cos(\Delta\varphi)]} \cdot \sin(\omega t + \gamma), \text{ where } \quad tg(\gamma) = \frac{1 - \cos(\Delta\varphi)}{\sin(\Delta\varphi)} \tag{7}$$

Second derivatives of the particular integrals of these equations are of the form:

$$\ddot{x}(t) = -\frac{me\omega^4 \sqrt{2[1 + \cos(\Delta\varphi)]}}{k - (m_k + m_w + 2m)\omega^2} \sin(\omega t + \delta) \tag{8}$$

$$\ddot{z}(t) = \frac{me\omega^4 \sqrt{2[1 - \cos(\Delta\varphi)]}}{k_z - (m_w + 2m)\omega^2} \sin(\omega t + \gamma) \tag{9}$$

Presently, we will write equations of motion of vibrators in the uninertial reference system, it means with taking into account vibrations of their axes of rotation resulting from motions along x and z axis.

$$J_o\ddot{\varphi}_1 = M_{z1} + me\ddot{z}\sin\varphi_1 - me\ddot{x}\cos\varphi_1 \tag{10}$$

$$J_o\ddot{\varphi}_2 = M_{z2} - me\ddot{z}\sin\varphi_2 - me\ddot{x}\cos\varphi_2 \tag{11}$$

where:

J_o – moment of inertia of the driving system counted versus the vibrator axis of rotation,

M_{zi} – difference between the motor driving moment M_{ni} and the vibrator resistance to motion moment $M_{oi}, i = 1, 2$.

Let us mark expressions describing in equations (10) and (11) influences of the vibrator axes vibrations as *vibratory moments* $M_{wi}, i = 1, 2$. The time histories of ϕ_1, ϕ_2 angles are substituted according to the first approximation (3):

$$M_{w1} = me[\ddot{z}\sin(\omega t + \Delta\varphi) - \ddot{x}\cos(\omega t + \Delta\varphi)] \tag{12}$$

$$M_{w2} = -me[\ddot{z}\sin(\omega t) + \ddot{x}\cos(\omega t)] \tag{13}$$

Substituting – in these equations – values of accelerations (8) and (9) determined according to the first approximation the following is obtained:

$$M_{w1} = m^2e^2\omega^4[\frac{\sqrt{2[1 - \cos(\Delta\varphi)]}}{k_z - (m_w + 2m)\omega^2} \sin(\omega t + \gamma) \cdot \sin(\omega t + \Delta\varphi) + \frac{\sqrt{2[1 + \cos(\Delta\varphi)]}}{k - (m_k + m_w + 2m)\omega^2} \sin(\omega t + \delta) \cdot \cos(\omega t + \Delta\varphi)] \tag{14}$$

$$M_{w2} = -m^2e^2\omega^4[\frac{\sqrt{2[1 - \cos(\Delta\varphi)]}}{k_z - (m_w + 2m)\omega^2} \sin(\omega t + \gamma) \cdot \sin(\omega t) - \frac{\sqrt{2[1 + \cos(\Delta\varphi)]}}{k - (m_k + m_w + 2m)\omega^2} \sin(\omega t + \delta) \cdot \cos(\omega t)] \tag{15}$$

The vibratory moment values, averaged for the period *T=2π/ω,* are as follows:

$$M_{w1T} = \frac{1}{T}\int_0^T M_{w1}(t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} M_{w1}(t)dt = \\ = \frac{m^2e^2\omega^5}{2\pi}\langle\frac{\sqrt{2[1-\cos(\Delta\varphi)]}}{k_z-(m_w+2m)\omega^2}\int_0^{2\pi/\omega}\sin(\omega t+\gamma)\cdot \\ \sin(\omega t+\Delta\varphi)dt + \frac{\sqrt{2[1+\cos(\Delta\varphi)]}}{k-(m_k+m_w+2m)\omega^2}\cdot \\ \int_0^{2\pi/\omega}\sin(\omega t+\delta)\cdot\cos(\omega t+\Delta\varphi)dt\rangle \tag{16}$$

$$M_{w2T} = \frac{1}{T}\int_0^T M_{w2}(t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} M_{w2}(t)dt = \\ = -\frac{m^2e^2\omega^5}{2\pi}\langle\frac{\sqrt{2[1-\cos(\Delta\varphi)]}}{k_z-(m_w+2m)\omega^2}\int_0^{2\pi/\omega}\sin(\omega t+\gamma)\cdot \\ \sin(\omega t)dt - \frac{\sqrt{2[1+\cos(\Delta\varphi)]}}{k-(m_k+m_w+2m)\omega^2}\cdot \\ \int_0^{2\pi/\omega}\sin(\omega t+\delta)\cdot\cos(\omega t)dt\rangle \tag{17}$$

After the integration we obtain:

$$M_{w1T} = \frac{1}{2}m^2e^2\omega^4\left[\frac{\sqrt{2[1-\cos(\Delta\varphi)]}}{k_z-(m_w+2m)\omega^2}\cos(\gamma-\Delta\varphi)+ \\ \frac{\sqrt{2[1+\cos(\Delta\varphi)]}}{k-(m_k+m_w+2m)\omega^2}\sin(\delta-\Delta\varphi)\right] \tag{18}$$

$$M_{w2T} = -\frac{1}{2}m^2e^2\omega^4\left[\frac{\sqrt{2[1-\cos(\Delta\varphi)]}}{k_z-(m_w+2m)\omega^2}\cos(\gamma)- \\ \frac{\sqrt{2[1+\cos(\Delta\varphi)]}}{k-(m_k+m_w+2m)\omega^2}\sin(\delta)\right] \tag{19}$$

These dependencies were rearranged by using formulas (6),(7) determining phase angles γ and δ, and finally the following formula was obtained:

$$M_{w1T} = \frac{K}{2}\sin(\Delta\varphi) \tag{20}$$

$$M_{w2T} = -\frac{K}{2}\sin(\Delta\varphi) \tag{21}$$

where:

$$K = m^2e^2\omega^4\cdot \\ \left[\frac{1}{k_z-(m_w+2m)\omega^2} - \frac{1}{k-(m_k+m_w+2m)\omega^2}\right] \tag{22}$$

Let us distinguish the driving part M_{eli} and the anti-torque part M_{oi}, i=1,2 of the external moments M_{zi} influencing vibrators and substitute the current values of vibratory moments in equations (10) and (11) by their period averaged values. These arrangements enable writing the equations in the following form:

$$J_o\ddot{\varphi}_1 \cong M_{el1} - M_{o1} + M_{w1T} \tag{23}$$

$$J_o\ddot{\varphi}_2 \cong M_{el2} - M_{o2} + M_{w2T} \tag{24}$$

Additionally we assume, that the driving moment originates from the induction motor of a mechanical characteristic described by the Kloss equation.

This equation can be presented in a linear form [5] for the slip value appropriately smaller than the slip of stall. It can be proved, that in the vicinity of the angular velocity of the steady running ω_{ust} the following occurs (in approximation):

$$M_{el}(\dot{\varphi}) \cong M_{el}(\omega_{ust}) - (\dot{\varphi} - \omega_{ust})\cdot a_{el} \tag{25}$$

where: $a_{el}=2pM_{zn}/(\omega_s - \omega_u)$,

$p-$ motor over-load capacity,

$M_{zn}-$ rated moment,

ω_s- synchronous angular velocity,

ω_u- stall angular velocity.

In a similar fashion it can be shown that the vibrator resistance of bearing moment, assumed as proportional to its excitation force in the vicinity of the steady running velocity ω_{ust}, can be presented as:

$$M_o(\dot{\varphi}) \cong M_o(\omega_{ust}) + (\dot{\varphi} - \omega_{ust})\cdot a_o \tag{26}$$

where: $a_o \approx \mu\mathrm{me}\omega_{ust}d$,

$\mu-$ equivalent coefficient of friction of rolling bearings,

$d-$ journal diameter,

– remaining markings are the same as before.

Substituting in equations (23) and (24) the derived above dependencies for individual components of the moment, we obtain the following:

$$J_o\ddot{\varphi}_1 = [M_{el}(\omega_{ust}) - (\dot{\varphi}_1 - \omega_{ust})a_{el}] - \\ [M_o(\omega_{ust}) + (\dot{\varphi}_1 - \omega_{ust})a_o] + M_{w1T} \tag{27}$$

$$J_o\ddot{\varphi}_2 = [M_{el}(\omega_{ust}) - (\dot{\varphi}_2 - \omega_{ust})a_{el}] - \\ [M_o(\omega_{ust}) + (\dot{\varphi}_2 - \omega_{ust})a_o] + M_{w2T} \tag{28}$$

After subtraction by members of equation:

$$J_o(\ddot{\varphi}_1 - \ddot{\varphi}_2) = -(\dot{\varphi}_1 - \dot{\varphi}_2)(a_{el} + a_o) + (M_{w1T} - M_{w2T}) \tag{29}$$

and using the notation $\Delta\phi = \phi_1 - \phi_2$, it is possible to present the final form:

$$J_o\Delta\ddot{\varphi} + (a_{el} + a_o)\Delta\dot{\varphi} + Ksin(\Delta\phi) = 0 \tag{30}$$

This dependence constitutes the differential equation determining the time variability of disphasing angle $\Delta\phi$ which – due to initial conditions or disturbances -occurred in the system of the self-synchronization ability.

Equation (30) can be investigated numerically, which allows for the determination of the time history $\Delta\phi(t)$ of the vibrator synchronization process. However, we must notice that this equation form corresponds to the equation of motion of the pendulum with viscous damping, which enables the clear qualitative analysis of the effect. If, e.g. both vibrators - in not synchronized state - are running at not much different angular velocities ω_1 and ω_2, then at every time period $T_o = 2\pi/(\omega_1 - \omega_2)$ their angular positions are identical, i.e. $\Delta\phi=0$.

If this moment is assumed as the initial one in the analysis of equation (30) then we can see that the condition to establish synchronization is that the solution satisfies the condition: $\Delta\phi < \pi$. Thus, this means that the pendulum described by equation (30) will not perform the revolution.

When damping is omitted (which improves the safety of calculations) the synchronization condition can be obtained from equating the pendulum kinetic energy for $t=0$ and the potential energy for t $(\Delta\phi = \pi)$. In such case we have:

$$\frac{1}{2} J_o \Delta\dot{\varphi}_o^2 = \int_0^\pi K \sin\Delta\varphi \cdot d(\Delta\varphi) \tag{31}$$

From here, the condition to establish synchronization is:

$$\Delta\dot{\varphi}_o \leqslant \sqrt{\frac{4K}{J_o}} \tag{32}$$

The positive damping occurring in actual conditions, determined in equation (30) by the sum $a_{el} + a_o$, causes that for the arbitrary initial value $\Delta\dot{\varphi}_o$, after a respectively large number of pendulum revolutions the condition (32) is satisfied, the vibrators establish synchronous motion, and the phase angle $\Delta\phi$ will lead to zero in a manner analysed previously.

The case of not total symmetry of both driving systems.

Such case can occur in practice due to differences in workmanship, assembling and conservation of driving motors or bearings of vibrators. That time, the natural velocities (without taking into account vibratory moments) of both driving systems are different: $\omega_{ust1} \neq \omega_{ust2}$. Then it occurs:

$$M_{eli}(\omega_{usti}) - M_{oi}(\omega_{usti}) = 0, i = 1, 2. \tag{33}$$

Making use of equation (33) the system of equations (27) and (28) can be brought to the differential equation (34) determining the time history of phase angle $\Delta\phi$:

$$J_o\Delta\ddot{\varphi} + (a_{el} + a_o)\Delta\dot{\varphi} + K sin\Delta\phi = (\omega_{ust1} - \omega_{ust2})(a_{el} + a_o) \tag{34}$$

A physical interpretation of this equation as the pendulum equation leads to an important conclusion.

It is not possible to obtain the synchronous motion when the following condition occurs:

$$|\omega_{ust1} - \omega_{ust2}|\,(a_{el} + \alpha_o) > K \tag{35}$$

When condition (35) does not occur, then the synchronization depends on the initial conditions and can be investigated by means of the analysis of equation (34).

Analysis of steady states.

Equation (34) supplies very essential dependencies also within the range of the steady state work. Let us assume $\Delta\ddot{\varphi} = \Delta\dot{\varphi} = 0$.

That time, we obtain the dependence determining the vibrator disphasing angle $\Delta\varphi$ as a function of the system asymmetry indicator:

$$\sin(\Delta\varphi) = \frac{(\omega_{ust1} - \omega_{ust2}) \cdot (a_{el} + a_o)}{K} \tag{36}$$

This equation can be also presented in an equivalent form, which is more suitable when instead of data concerning the steady state running of both motors we have data related to the difference of their driving-anti-torque moments in the point of the steady state work ω_{ust}:

$$\sin(\Delta\varphi) = \frac{M_{z1}(\omega_{ust}) - M_{z2}(\omega_{ust})}{K} \tag{37}$$

When the vibrators disphasing angle $\Delta\varphi$ is determined from equations (36) and (37), it is possible to determine (on the basis of the particular integrals of equations (6) and (7)) amplitudes of steady state vibrations of the conveyer A_x and vibrator A_z – under conditions of not total cophasing caused by the asymmetry of the driving system:

$$A_x = \frac{me\omega^2\sqrt{2[1 + \cos(\Delta\varphi)]}}{k - (m_k + m_w + 2m)\omega^2} \tag{38}$$

$$A_z = \frac{me\omega^2\sqrt{2[1 - \cos(\Delta\varphi)]}}{k_z - (m_w + 2m)\omega^2} \tag{39}$$

Conclusions

1. The performed analysis allowed for the derivation of the differential equation describing the time history of the vibrators synchronization process:

a) For the case of the total symmetry of the driving system – (30),

b) For the case of the diversification of the driving and anti-torque moments – (34).

2. The analysis of solutions found in the case (b) allowed for the formulation of condition (35) determining the system asymmetry upper limit, above which the synchronization is not possible.

3. For the case of asymmetric system dependencies (36) and (37) – determining the vibrators disphasing angle, and dependencies (38) and (39) – determining the vibration amplitudes of the body and vibrators system, were given.

4. Analogical dependences for machines supported on helical springs were derived in Ref.[6].

REFERENCES

[1] I.I. B l e c h m a n, Wibracjonnaja Mechanika. Nauka, Moskwa 1994.

[2] J. M i c h a l c z y k, Maszyny wibracyjne WNT, Warszawa,1995.

[3] I.I. B l e k h m a n, N.P. Y a r o s h e v i c h, Extension of the Domain of Applicability of the Integral Stability Criterion in Synchronization Problems, Journal of Applied Mathematics and Mechanics **68**, 6, 938-947 (2004).

[4] J. M i c h a l c z y k, G. C i e p l o k, Ł. B e d n a r s k i, Procesy przejściowe maszyn wibracyjnych i układów wibroizolacji. WNT, Warszawa 2010.

[5] A. P i ą t k i e w i c z, R. S o b o l s k i, Dźwignice, WNT, Warszawa 1969.

[6] J. M i c h a l c z y k, P. C z u b a k, Influence of the Asymmetry of Vibrators Resistance to Motion on the Correctness of the Vibration Distribution on Working Surface of Vibratory Machines. Archives of Metallurgy and Materials **55**, 1 (2010).

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