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**JOINT THEORY OF THERMAL WORK FOR HEATING ZONE OF COKE, COKE-GAS AND GAS-FIRED CUPOLAS  
(based on theory of heat exchangers of H. Le Chatelier)**

**WSPÓLNA TEORIA PRACY CIEPLNEJ DLA STREFY PODGRZANIA ŻELIWIĄKÓW KOKSOWYCH, KOKSOWO-GAZOWYCH  
ORAZ GAZOWYCH  
(zbudowana na podstawach teorii wymienników ciepła H. Le Chateliera)**

The analytical theory of thermal work in the heating zone of coke, coke-gas and gas-fired cupolas has been presented in the work. The conception of elaboration of joint (one) thermal theory for the mentioned groups of cupolas has not been considered in the literature so far due to the following reasons:

- lack of consistent theory for coke cupolas,
- total lack of theoretical basis for thermal work of gas-fired and coke-gas-fired cupolas.

The presented paper contains a set of equations for the calculation of characteristic parameters of the heating zone in the coke-, coke-gas- and gas-fired cupolas such as: efficiency, zone heights, temperatures of combustion gas at zone inlets and outlets as well as at height of zones, heating time of metal pieces up to the melting temperature, rate of movement of charge materials down the furnace shaft, losses of physical heat transferred in combustion gases to the ambience. The contribution has derived a basis for the energy analysis of thermal work of the cupolas as well as examples of calculations of characteristic parameters connected with their thermal process.

A new approach to selected controversial aspects of the theory of coke cupola performance has been formulated.

*Keywords:* coke cupolas, coke-gas cupolas, gas cupolas, heating zone height of coke cupolas, heating zone of coke-gas cupolas, heating zone of gas cupolas

W pracy przedstawia się analityczną teorię pracy cieplnej strefy podgrzania żeliwiaków koksowych, gazowych oraz koksowo-gazowych. Koncepcja opracowania wspólnej (jednej) teorii cieplnej, dla wymienionych trzech grup żeliwiaków, nie była dotąd rozważana w literaturze, co wyjaśnić można następującymi faktami:

- brak spójnej teorii dla żeliwiaków koksowych,
- zupełny brak teoretycznych podstaw pracy cieplnej żeliwiaków gazowych oraz koksowo-gazowych.

Niniejsza praca zawiera układ równań do obliczania charakterystycznych wielkości strefy podgrzania żeliwiaków koksowych, gazowych oraz koksowo-gazowych, przy czym wielkości charakterystyczne obejmują: wydajność; wysokość stref; temperaturę spalin na wlocie do stref, na wylocie ze stref oraz na wysokości stref; czas podgrzewania kawałków metalu do temperatury topienia; szybkość przemieszczania się materiałów wsadowych w szybie pieca, straty ciepła fizycznego unoszonego do otoczenia w spalinach. Praca daje podstawy do energetycznej analizy pracy cieplnej wymienionych trzech grup żeliwiaków; zawiera przykłady obliczania charakterystycznych wielkości, związanych z ich pracą cieplną.

W pracy podaje się nowe wyjaśnienie wybranych spornych problemów istniejącej teorii pracy żeliwiaków koksowych.

**Denotations:**

The comment to the denotations used in the work: products of coal burning are called combustion gas; the denotations with c, g and ε indexes refer to coke-, gas- and coke-gas-fired cupolas, respectively.

$c_{s,c}$ ,  $c_{s,g}$ ,  $c_{s,\varepsilon}$  – mean specific heat of combustion gas in heating zone of coke, gas and coke-gas cupolas, respectively,  $J/(m^3 \cdot K)$

$c_{m,c}$  – mean specific heat of heated metal in heating zone of coke cupola,  $J/(kg_{Fe} \cdot K)$

$c_{m,g}$  – mean specific heat of heated metal in heating zone of gas cupola,  $J/(kg_{Fe} \cdot K)$

$F_r$  – cross section surface of cupola shaft,  $m^2$

$F_H$  – development surface of metal pieces in heating zone (total surface of all heated metal pieces in the zone),  $m^2$

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$H_{p,c}$ ,  $H_{p,g}$ ,  $H_{p,\varepsilon}$  – heating zone height of coke, gas and coke-gas cupolas, respectively, m

$K_c$  – expenditure of charge coke coal,  $\text{kg}_c/100 \text{ kg}_{Fe}$

$K_k$  – expenditure of charge coke,  $\text{kg}_c/100 \text{ kg}_{Fe}$

$K_g$  – expenditure of gas, standard conditions,  $\text{m}_g^3/100 \text{ kg}_{Fe}$

$L_c$  – volume of air blast used up for burning 1 kg coal, standard conditions, heating zone,  $\text{m}_g^3/1 \text{ kg}_{Fe}$

$L_g$  – volume of air blast used up for burning 1  $\text{m}^3$  gas, standard conditions, heating zone,  $\text{m}^3/\text{m}_g^3$

$\dot{m}_c$  – mass burning rate of coke coal,  $\text{kg}_c/\text{s}$

$m_1$ ,  $m_2$  – dimensionless parameters of general equation for calculation of limit temperatures of heating zone

$m_1 = \frac{W_m}{W_s}$ ;  $m_{1,c} = \frac{W_{m,c}}{W_{s,c}}$ ;  $m_{1,g} = \frac{W_{m,g}}{W_{s,g}}$ ;  $m_{1,\varepsilon} = \frac{W_{m,\varepsilon}}{W_{s,\varepsilon}}$ ;

$P_c$ ,  $P_g$  – expenditure of air blast in the coke and gas cupola, respectively, standard conditions,  $\text{m}_p^3/\text{s}$

$P_{F,c}$ ,  $P_{F,g}$ , – relative expenditure of air blast in the coke and gas-fired cupola, respectively, standard conditions,  $\text{m}_p^3/(\text{m}^2 \cdot \text{s})$

$S_c$ ,  $S_g$ ,  $S_\varepsilon$  – efficiency of coke, gas and coke-gas cupolas, respectively,  $\text{kg}_{Fe}/\text{s}$

$S_{F,c}$ ,  $S_{F,g}$ ,  $S_{F,\varepsilon}$  – relative efficiency of coke, gas and coke-gas cupolas, respectively,  $\text{kg}_{Fe}/(\text{m}^2 \cdot \text{s})$

$W_m$ ;  $W_{m,c}$ ;  $W_{m,g}$ ;  $W_{m,\varepsilon}$  – temperature power of metal; first notation is general, respective ones refer to coke, gas-fired and coke-gas-fired cupolas, W/K

$W_s$ ;  $W_{s,c}$ ;  $W_{s,g}$ ;  $W_{s,\varepsilon}$  – temperature power of combustion gas; first notation is general, respective ones refer to coke, gas-fired and coke-gas-fired cupolas, W/K

$V_{s,c}$ ,  $V_{s,g}$  – volume of combustion gas in heating zone of coke and gas-fired cupolas, respectively, standard conditions,  $\text{m}_s^3/\text{kg}_c$

$T_{s,3}$ ,  $T_{s,4}$  – temperature of combustion gas at inlet and outlet of the heating zone,  $^\circ\text{C}$

$T_{m,3}$  – melting temperature of metal,  $^\circ\text{C}$

$T_{m,4}$  – temperature of metal loaded into cupola,  $^\circ\text{C}$

$\alpha_F$  – coefficient of heat exchange between combustion gas and surface of metal pieces,  $\text{W}/(\text{m}^2 \cdot \text{K})$

$\eta_c$  – degree of burning coke coal, referred to heating zone, unit fraction

$\vartheta_{s,3} = T_{s,3} - T_{m,4}$ , K

$\vartheta_{s,4} = T_{s,4} - T_{m,4}$ , K

$\vartheta_{m,3} = T_{m,3} - T_{m,4}$ , K

$\rho_{n,m}$  – bulk density of metal pieces,  $\text{kg}/\text{m}^3$

$\rho_m$  – density of mass of metal pieces,  $\text{kg}/\text{m}^3$

$\rho_{n,k}$  – bulk density of coke,  $\text{kg}/\text{m}^3$

## 1. Introduction

The coke-, coke-gas- and gas-fired cupolas belong to the group of shaft furnaces, in which processes of heat exchange proceed according to the rules of

counter-current heat exchangers. This obvious statement has not been converted so far, into a common, analytical thermal theory of classical (coke), coke-gas-fired as well as gas-fired cupola process. The principles of the theory of heat exchangers elaborated by H. Le Chatelier (in 1924) [1] were further developed by the specialists of thermodynamics and applied in various machines and units [e.g. (1÷5)], except for the cupolas.

The only equation for the calculation of temperature of metal piece charge heated in the heating zone of coke cupolas and iron blast furnaces, found in literature, was derived by B. I. Kitajew [6] and has never been applied neither in practice nor in the theory of cupola process. Its way of derivation, final feature and the interpretation of term *completed and uncompleted heating processes* [7] have not been clear enough for its users, which probably was the reason, that it was practically used twice [6, 8]. Other authors [e.g. 9, 10] did not give the examples of its practical applications, they were contented with citing the formula. The problem of practical application of the Kitajew equation has been discussed elsewhere [11].

The lack of general theoretical description of thermal process in the coke, coke-gas-fired and gas-fired cupolas is not conducive to their design, development and application in the industry.

In the present work, a common analytical approach to thermal processes proceeding in the heating zones of coke, gas and coke-gas- cupolas treated as counter-current heat exchangers (derived by H. Le Chatelier) has been presented. The thermal processes in heating zones constitute an important element of a complex thermal theory of cupolas.

## 2. Basic equation of thermal work in the heating zone of cupolas

The equation, based on which one can calculate the distribution of combustion gas and metal temperatures at the zone height, zone height, heating time of metal charge up to the melting temperature and the physical heat of gases transferred to the ambience is called in the paper the basic equation of thermal work in the heating zone of coke-, coke-gas- and gas-fired cupolas [12, 13]. The equation enables also the calculation of other parameters, which characterize, first of all, the thermal work in the heating zone, and also allows the inference on the thermal work in the melting and combustion zones. Based on the formula, the parameters of optimal cupola process can be also calculated.

*Model assumptions:*

a) Two main relationships of H. Le Chatelier theory of

heat exchangers have been used to derive the basic equation, both concerning the limit and intermediate temperatures of combustion gas and these of metal charge.

The formula for the limiting temperatures is following (Fig. 1):

$$W_s(T_{s,3} - T_{s,4}) = W_m(T_{m,3} - T_{m,4}) \quad (1)$$

or

$$\vartheta_{s,3} - \vartheta_{s,4} = m_1 \vartheta_{m,3} \quad (2)$$

at which:

$$m_1 = \frac{W_m}{W_s} \quad (3)$$

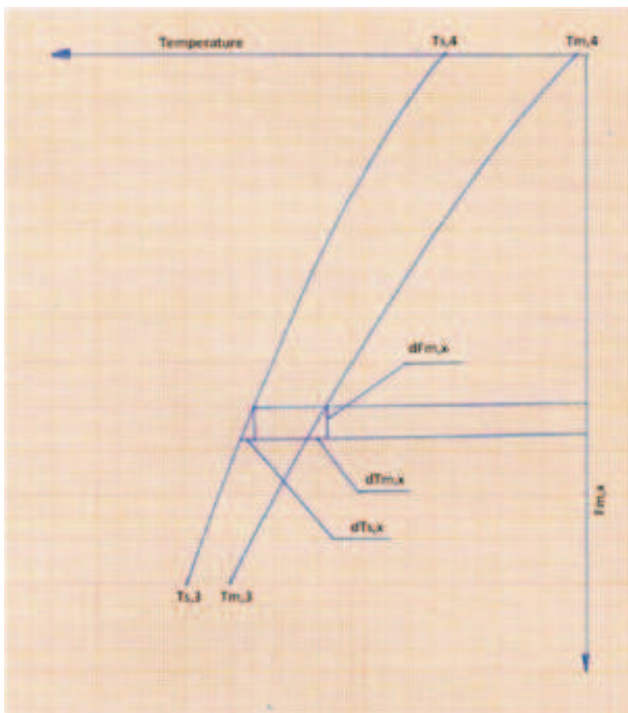


Fig. 1. Scheme of gas and metal temperature course in the heating zone of cupolas (start of coordinate system at upper zone boundary)

where:

$W_s$ ,  $W_m$  – temperature power in the heating zone of combustion gas (heating medium) and metal (heated medium), respectively, referring to the three considered fuel groups of cupolas, W/K (so called water numbers in the past, kcal/(h·°C)) In the present work “temperature power” is proposed i.e. power in watts on 1 K.

$T_{s,3}$ ,  $T_{s,4}$  – limit temperatures of combustion gas at the inlet and outlet of the heating zone, °C

$T_{m,4}$ ,  $T_{m,3}$  – temperature of metal charge when entering and leaving the heating zone °C

$\vartheta_{s,3} = T_{s,3} - T_{m,4}$ ;  $\vartheta_{s,4} = T_{s,4} - T_{m,4}$ ;  $\vartheta_{m,3} = T_{m,3} - T_{m,4}$ , K.

Equation (1) for intermediate temperatures is as follows:

$$W_s(T_{s,x} - T_{s,4}) = W_m(T_{m,x} - T_{m,4}) \quad (4)$$

and

$$W_s(T_{s,3} - T_{s,x}) = W_m(T_{m,3} - T_{m,x}) \quad (5)$$

where:

$T_{s,x}$ ,  $T_{m,x}$  – temperature of combustion gas and the heated metal, respectively in the considered cross section of exchanger at distance  $x$  from the beginning of the coordinate system, °C.

*Remark: Products of burning both coke and gas are called combustion gas in the presented paper.*

b) A column of charge material passes throughout the cupola shaft at a constant rate. The metal pieces have the same starting temperature, melting temperature and volume as well as shape and weight.

c) The metal pieces heat uniformly in their whole mass, i.e. without decreases of temperature between the surface of pieces and their thermal centre, which is equivalent to the assumption about their infinitely high coefficient of heat conduction or a very low intensity of heating

d) The combustion gas generated in the combustion zone moves in the opposite direction to the column of material charge.

e) The coefficient of heat exchange between the combustion gas and the surface of metal charge is constant.

f) the beginning of co-ordinate system is defined to be at the upper surface of the heating zone (Fig. 1). The co-ordinate directed vertically down takes the role of surface axis of heated metal pieces, while the perpendicular one is the temperature axis of combustion gas and metal.

g)  $T_{s,4}$  and  $T_{m,4}$  values of temperatures are given.

#### Derivation of equation

In order to derive the basic equation of thermal work for the heating zone of coke, coke-gas-fired and gas-fired cupolas, a following elementary thermal balance for an arbitrary cross-section of charge material column will be written down

$$W_m d\vartheta_{m,x} = \alpha_F (\vartheta_{s,x} - \vartheta_{m,x}) dF_{m,x} \quad (6)$$

where:

$d\vartheta_{m,x} = d(T_{m,x} - T_{m,4})$  – elementary increase of temperature of heated metal pieces, K

$\vartheta_{m,x} = T_{m,x} - T_{m,4}$ , K

$T_{m,x}$  – temperature of charge pieces at considered level of column of materials  $F_{m,x}$ , °C

$F_{m,x}$  – surface of heated metal charge, started from the beginning of co-ordinate system, m<sup>2</sup>

$\vartheta_{s,x} = T_{s,x} - T_{m,4}$ , K

$T_{s,x}$  – gas temperature at the considered level of material column  $F_{m,x}$ , °C

$\alpha_F$  – coefficient of heat exchange between the gas and the surface of metal pieces in the heating zone,  $W/(m^2 \cdot K)$ .

$dF_{m,x}$  – elementary increment of heated metal surface,  $m^2$

Each side of equation (6) has meaning of power (W).

Variable temperature  $T_{s,x}$  can be eliminated from equation (6) by the application of equation (4);  $\vartheta_{s,x}$  can be calculated from equation (4)

$$\vartheta_{s,x} = m_1 \vartheta_{m,x} + \vartheta_{s,4} \tag{7}$$

Substituting (7) into (6)

$$W_m d\vartheta_{m,x} = \alpha_F (m_1 \vartheta_{m,x} + \vartheta_{s,4} - \vartheta_{m,x}) dF_{m,x} \tag{8}$$

After the rearrangement of (8) the following equation is obtained

$$\frac{d\vartheta_{m,x}}{dF_{m,x}} = \frac{\alpha_F}{W_m} [\vartheta_{s,4} - (1 - m_1) \vartheta_{m,x}] \tag{9}$$

(9) is an ordinary differential equation. It may be qualified as a linear equation or equation with separated variables and written down as follows

$$\frac{d\vartheta_{m,x}}{\frac{\alpha_F}{W_m} \vartheta_{s,4} - \frac{\alpha_F}{W_m} (1 - m_1) \vartheta_{m,x}} = dF_{m,x} \tag{10}$$

or

$$\frac{d\vartheta_{m,x}}{a - b\vartheta_{m,x}} = dF_{m,x} \tag{11}$$

at which:

$$a = \frac{\alpha_F}{W_m} \vartheta_{s,4} \tag{12}$$

$$b = \frac{\alpha_F}{W_m} (1 - m_1) \tag{13}$$

In order to integrate (11) after substitutions:  $y = a - b\vartheta_{m,x}$ ;  $dy = -b d\vartheta_{m,x}$ ; equation (11) takes the shape

$$\frac{dy}{y} = -bdF_{m,x} \tag{14}$$

Integrating (14), we get

$$\ln y = -bF_{m,x} + C \tag{15}$$

Or after the substitution of  $y$

$$\ln(a - b\vartheta_{m,x}) = -bF_{m,x} + C \tag{16}$$

Integration constant  $C$  is found from the following initial condition: for  $F_{m,x} = 0$ ;  $\vartheta_{m,x} = 0$  ( $T_{m,x} = T_{m,4}$ );  $C = \ln a$ .

Substituting  $C$  into (16) and after a rearrangement equation (17) is obtained

$$\ln \frac{a - b\vartheta_{m,x}}{a} = -bF_{m,x} \tag{17}$$

and substituting next  $a$  and  $b$  and rearranging

$$\vartheta_{m,x} = \frac{\vartheta_{(s,4)}}{1 - m_1} \{ 1 - \exp[-m_{2,x}(1 - m_1)] \} \tag{18}$$

at which

$$m_{2,x} = \frac{\alpha_F F_{m,x}}{W_m} \tag{19}$$

The derived equation (18) serves for calculating the metal temperature (heated medium in general) in dependence on the value of development surface  $F_{m,x}$  taken from the beginning of co-ordinate system to the considered cross section of the heating zone.

Distributions of metal and combustion gas temperatures at zone height as a function of independent variable  $F_{m,x}(m_{2,x})$ , boundary temperatures of combustion gas in particular, and next in turn, heights of heating zones, heating times of metal charge pieces up to the melting temperature as well as losses of combustion gas heat transferred to the ambience will be discussed in subsequent parts of the paper.

*Calculation of combustion gas temperature distribution at zone height*

Equation (18) will be rearranged exchanging temperature  $\vartheta_{m,x}$  with  $\vartheta_{s,x}$ ;  $\vartheta_{m,x}$  can be calculated from equation (4)

$$\vartheta_{m,x} = \frac{\vartheta_{(s,x)} - \vartheta_{s,4}}{m_1} \tag{20}$$

Substituting (20) into (18) and after a rearrangement the equation binding temperatures of combustion gas  $\vartheta_{s,4}$  and  $\vartheta_{s,x}$  is derived

$$\vartheta_{s,x} = \frac{\vartheta_{s,4}}{1 - m_1} \{ 1 - m_1 \exp[-m_{2,x}(1 - m_1)] \} \tag{21}$$

In turn,  $\vartheta_{s,4}$  will be substituted with  $\vartheta_{m,3}$  in equation (21). Since the exchanged temperatures are limit ones, a boundary value of  $m_{2,x} = m_2$  will be also incorporated into equation (21), which will transform into the following shape

$$\vartheta_{s,4} = \frac{1 - m_1}{1 - \exp[-m_2(1 - m_1)]} \vartheta_{m,3} \tag{22}$$

Now equation (22) is included into (21) and after a rearrangement, equation (23) to calculate the distribution

of gas temperature at zone height can be obtained as a function of  $m_{2,p,x}$  for a given temperature  $\vartheta_{m,3}$

$$\vartheta_{s,x} = \frac{1 - m_1 \exp[-m_{2,x}(1 - m_1)]}{1 - \exp[-m_2(1 - m_1)]} \vartheta_{m,3} \quad (23)$$

Both presented examples of transformation of equation (18) inform about the principles of conversions. In the first case a simple substitution (20) has been used, which contains the  $\vartheta_{m,x}$  variable dependant on  $\vartheta_{s,x}$  as well as the  $\vartheta_{s,4}$  constant also appearing in the converted equation. In the second example, the  $\vartheta_{s,4}$  temperature as a function of temperature  $\vartheta_{m,3}$  should be incorporated; the function was obtained after including limit value  $m_{2,x}=m_2$  into (18)

#### Calculation of limit temperatures

The boundary temperatures of the heating zone can be calculated from (18), after substitution  $F_{m,x}=F_H$

$$\vartheta_{m,3} = \frac{\vartheta_{(s,4)}}{1 - m_1} \{1 - \exp[-m_2(1 - m_1)]\} \quad (24)$$

at which

$$m_2 = \frac{\alpha_F F_H}{W_m} \quad (25)$$

where:

$F_H$  – development surface of metal pieces in the heating zone (total surface of all heated metal pieces in the zone),  $m^2$ .

The derived equation (24) is a basic general equation for the calculation of boundary temperatures in counter-current heat exchangers of different types including also coke, coke-gas and gas cupolas.

In the case of cupolas, the  $T_{m,4}$  and  $T_{m,3}$  temperatures are known (imposed), while  $T_{s,3}$  and  $T_{s,4}$  are unknown. In order to calculate them, equation (24) should be appropriately rearranged. The kinds of cupolas in dependence on the fuel used are taken into consideration in a suitable notation of formulas to calculate dimensionless factors  $m_1$  and  $m_2$ .

Equation (24) can be converted into equation (22) in the simplest way treating  $\vartheta_{m,3}$  as argument and  $\vartheta_{s,4}$  as a function.

Equation (22) can have two more variants, in which pairs of temperatures  $T_{s,3}$  and  $T_{m,4}$  as well as  $T_{s,3}$  and  $T_{m,3}$  act as arguments. In this way the equation occurs in three variants. Simultaneously, each pair of temperatures bounded with the heating zone can be calculated from three formulas, which gives 12 formulas altogether.

Subsequent mutations of equation (20) can be established through a simple method of changing places by the temperatures, described on examples of equations (20) and (22), as well as a more complicated method, which consists in an elimination of particular temperatures from the equations. For instance, using relation (2), temperature  $\vartheta_{s,4}$  is substituted with  $\vartheta_{s,3}$  in equation (22). To do this, temperature  $\vartheta_{s,4}$  is calculated from eq. (2).

$$\vartheta_{s,4} = \vartheta_{s,3} - m_1 \vartheta_{m,3} \quad (26)$$

Including (26) into (22) the following formula is derived

$$\vartheta_{s,3} = \frac{1 - m_1}{1 - \exp[-m_2(1 - m_1)]} \vartheta_{m,3} + m_1 \vartheta_{m,3} \quad (27)$$

Or, after a rearrangement

$$\vartheta_{s,3} = \frac{1 - m_1 \exp[-m_2(1 - m_1)]}{1 - \exp[-m_2(1 - m_1)]} \vartheta_{m,3} \quad (28)$$

The temperature of combustion gas at the inlet to the heating zone is calculated from eq. (28) as a function of temperatures  $T_{m,3}$  and  $T_{m,4}$  and factors  $m_1$  and  $m_2$ .

Two more formulas will be quoted here without their derivation procedure. They constitute a basic set of formulas to calculate limit temperatures of combustion gas in the heating zone.

$$\vartheta_{s,3} = \frac{1 - m_1 \exp[-m_2(1 - m_1)]}{1 - m_1} \vartheta_{s,4} \quad (29)$$

$$T_{s,3} = \frac{T_{m,3} \{1 - \exp[m_2(1 - m_1)]\} m_1 + T_{s,4} (1 - m_1)}{1 - m_1 \exp[m_2(1 - m_1)]} \quad (30)$$

The temperature of combustion gas at the inlet to the heating zone is calculated from eq. (28), if values of temperatures  $T_{s,4}$ ,  $T_{m,4}$  and factors  $m_1$  and  $m_2$  are given, while the  $T_{s,3}$  can be determined based on eq. (30), provided, values  $T_{m,3}$  and  $T_{s,4}$  as well as  $m_1$  and  $m_2$  are known.

After calculating the particulate temperature, the only one to be established is the fourth temperature, which can be determined from eq.(1) and (2).

The formula of temperature difference of combustion gas and metal at the entry to the zone can be put down using eq. (28)

$\delta(T_{s,3}-T_{m,3}) = \frac{1-m_1 \exp[-m_2(1-m_1)]}{1-\exp[-m_2(1-m_1)]} \vartheta_{m,3} - \vartheta_{m,3}$ , or, after rearrangement

$$\delta(T_{s,3}-T_{m,3}) = \frac{(1 - m_1) \exp[-m_2(1 - m_1)]}{1 - \exp[-m_2(1 - m_1)]} \vartheta_{m,3} \quad (31)$$

The use of the derived formulas for  $m_1 < 1$  and  $m_1 > 1$  is not difficult except for  $m_1=1$ , which is the

boundary case and the resulting value is indeterminate. The problem needs an explanation.

*Limit temperatures for  $m_1 = 1$*

The condition  $m_1=1$  means the equality of temperature powers of combustion gas (as a heating medium) and metal (as a heated medium), i.e.  $W_s=W_m$  (temperature power of combustion gas is transferred to the metal)

To examine the  $m_1=1$  condition, eq. (22) can be written as a function of variable  $z$

$$\vartheta_{s,4} = \frac{(1 - z)}{1 - \exp[-m_2(1 - z)]} \vartheta_{m,3} \quad (32)$$

where:

$$z = m_1.$$

After incorporating  $z=1$  into (32), the result is indeterminate

$$\vartheta_{s,4} = \frac{(1 - 1)}{1 - \exp[-m_2(1 - 1)]} \vartheta_{m,3} = \frac{1 - 1}{1 - 1} \frac{0}{0} \quad (33)$$

because  $\exp(-0)=1$ .

The examination of the limit of the function ratio can be replaced using de l'Hospital rule with the examination of ratio limit of numerator and denominator derivatives; the derivatives are as follows:

Numerator derivative:  $-1 \cdot \vartheta_{m,3}$

Denominator derivative for  $z=1$ :  $-\exp[-m_2(1-x)] \cdot (+m_2) = -\exp(-m_2(1-1)) m_2 = -1 \cdot m_2 = -m_2$

The obtained derivatives are substituted into (32) and the limit can be calculated

$$\left\{ \lim_{m_1 \rightarrow 1} \vartheta_{s,4} = \frac{[1 - m_1]'}{\{1 - \exp[-m_2(1 - m_1)]\}'} = \frac{1}{m_2} \vartheta_{m,3} \right. \quad (34)$$

So equation (22), for  $m_1 \rightarrow 1$ , is simplified to the form

$$\vartheta_{s,4} = \frac{1}{m_2} \vartheta_{m,3} \quad (35)$$

From eq. (35) results the decrease of combustion gas temperature  $\vartheta_{s,4}$ , when value  $m_2$  as well as a linear dependence of temperature  $\vartheta_{m,3}$  on  $\vartheta_{s,4}$  increase.

Equation (35) contains three boundary temperatures:  $T_{s,4}$ ,  $T_{m,3}$ ,  $T_{m,4}$ .

Each of them may be replaced with temperature  $T_{s,3}$  taking advantage of (2) for  $m_1=1$ . Formula (2) for  $m_1=1$  takes the form

$$\vartheta_{s,3} - \vartheta_{s,4} = \vartheta_{m,3} \quad (36)$$

From (36)  $\vartheta_{s,4}$  is calculated and substituted into (32); which becomes

$$\vartheta_{s,3} - \vartheta_{m,3} = \frac{1}{m_2} \vartheta_{m,3} \quad (37)$$

or after a transformation

$$\vartheta_{s,3} = \left(1 + \frac{1}{m_2}\right) \vartheta_{m,3} \quad (38)$$

From (35) and (38) equation (39) can be obtained

$$\vartheta_{s,3} = (1 + m_2) \vartheta_{s,4} \quad (39)$$

Left side of (37) refers to the temperature difference of combustion gas and metal at the upper boundary of heating zone (at the entry of combustion gas into the heating zone)

$$\delta(T_{s,3} - T_{m,3}) = \frac{1}{m_2} \vartheta_{m,3} \quad (40)$$

The temperature difference of combustion gas and metal at the upper boundary of heating zone (at the exit of combustion gas out of the heating zone) can be calculated from the following formula

$$\delta(T_{s,4} - T_{m,4}) = \vartheta_{s,4} \quad (41)$$

or after allowing for (35)

$$\delta(T_{s,4} - T_{m,4}) = \frac{1}{m_2} \vartheta_{m,3} \quad (42)$$

*Limiting temperatures for  $m_2 = \infty$*

In turn for  $m_2 = \infty (m_1 \neq 1)$  formulas (22) and (24) are simplified to the form:

$$\vartheta_{s,4} = (1 - m_1) \vartheta_{m,3} \quad (43)$$

$$\vartheta_{s,4} = (1 - m_1) \vartheta_{s,3} \quad (44)$$

formula (45) results from (43) and (44)

$$\vartheta_{s,3} = \vartheta_{m,3} \quad (45)$$

For  $m_1=1$  relationship (43) simplifies itself to the form  $\vartheta_{s,4}=0$ , i.e.  $T_{s,4}=T_{m,4}$

(the temperature of combustion gas at the upper boundary of the heating zone is equal to the temperature of metal loaded into the furnace); however eq.(44) shows equality of  $\vartheta_{s,3} = \vartheta_{m,3}$ , i.e.  $T_{s,3}=T_{m,3}$  (temperature of combustion gas at the lower boundary of heating zone is the same as the melting temperature of metal); equalities  $T_{s,4}=T_{m,4}$  and  $T_{s,3}=T_{m,3}$  mean, that the temperatures of combustion gas and metal are equal.

For  $m_2 = \infty$ , equation (4) simplifies to the form

$$\delta(T_{s,3} - T_{m,3}) = 0 \quad \text{czyli} \quad T_{s,3} = T_{m,3} \quad (46)$$

It results from (46), that the temperature of combustion gas at the inlet to the heating zone is equal to the temperature of metal melting, while equation (42) simplifies to the form of (47), when  $m_2 = \infty$

$$\delta(T_{s,4} - T_{m,4}) = 0 \quad \text{czyli} \quad T_{s,4} = T_{m,4} \quad (47)$$

It results from (47), that the temperature of combustion gas at the outlet from the heating zone is equal to the temperature of metal loaded into the cupola.

#### *Influence of $m_1$ and $m_2$ on temperatures and*

The derived formulas for the calculation of limiting temperatures for coke gas and coke-gas cupolas find the application not only to the general analysis of the influence of dimensionless factors  $m_1$  and  $m_2$  on values of boundary temperatures, but also they can be used to the general assessment of the thermal condition of the furnace.

Temperatures  $\vartheta_{s,4}$  determine the values of physical heat losses of gases to the ambience, while temperatures

$\vartheta_{s,3}$  deliver information on temperature condition inside the furnace and allow the estimation, whether the physical energy of gases in the combustion and melting zone is satisfactory for the furnace process proceeding.

Table 1 contains the calculations informing about the influence of factors  $m_1$  and  $m_2$  on values of limiting temperatures of combustion gas  $\vartheta_{s,3}$  and  $\vartheta_{s,4}$  for constant value  $\vartheta_{m,3}=1130$  K ( $T_{m,3}=1150^\circ$ ,  $T_{m,4}=20^\circ$  or  $T_{m,3}=1480^\circ$  and  $T_{m,4}=350^\circ$ ). Based on Table 1 the following conclusions can be established:

- temperatures  $\vartheta_{s,4}$  and  $\vartheta_{s,3}$  decrease when values  $m_1$  and  $m_2$  increase from 2 up to  $\infty$ . For  $m_2 = \infty$  the gas temperature at the inlet to the zone is equal to the temperature of metal  $\vartheta_{m,3}$ ;

- for a given value of  $m_2$  and growing  $m_1$  value, temperature values  $\vartheta_{s,4}$  decrease while  $\vartheta_{s,3}$  rise; the changes are due to either decreasing values of temperature power of gases  $W_s$  or the growing temperature power of the heated metal, which results from equation (3)

TABLE 1

Temperatures  $\vartheta_{s,4}$  and  $\vartheta_{s,3}$  in dependence on  $m_1$  i  $m_2$

↓→	$m_1=0,5$		$m_1=0,65$		$m_1=0,75$		$m_1=0,9$		$m_1=1$		$m_1=1,2$	
	$\vartheta_{s,4}$ K	$\vartheta_{s,3}$ K	$\vartheta_{s,4}$ K	$\vartheta_{s,3}$ K	$\vartheta_{s,4}$ K	$\vartheta_{s,3}$ K	$\vartheta_{s,4}$ K	$\vartheta_{s,3}$ K	$\vartheta_{s,4}$ K	$\vartheta_{s,3}$ K	$\vartheta_{s,4}$ K	$\vartheta_{s,3}$ K
<b>2,0</b>	894	1459	786	1520	718	1565	623	1640	565	1695	459	1815
<b>2,5</b>	792	1357	678	1413	608	1455	511	1528	452	1582	348	1704
<b>3,0</b>	727	1292	608	1343	535	1383	436	1453	377	1507	275	1631
<b>3,5</b>	684	1249	560	1294	484	1332	383	1400	323	1453	223	1579
<b>4,0</b>	653	1218	525	1259	447	1294	343	1360	282	1412	184	1540
<b>4,5</b>	632	1197	499	1233	418	1266	312	1329	251	1381	155	1511
<b>5,0</b>	615	1181	479	1213	396	1243	287	1304	226	1356	132	1488
<b>5,5</b>	604	1169	463	1198	378	1226	267	1284	205	1335	113	1469
<b>6,0</b>	595	1160	451	1185	364	1211	250	1267	188	1318	97	1453
<b>7,0</b>	583	1148	433	1167	342	1189	224	1241	161	1291	74	1430
<b>8,0</b>	576	1141	421	1156	327	1174	205	1222	141	1271	57	1413
<b>10,0</b>	569	1134	408	1142	308	1155	179	1196	113	1243	35	1391
$\infty$	565	1130	395	1130	282	1130	113	1130	0	1130	0	1356

$\vartheta_{s,4}=T_{s,4}-T_{m,4}$ ;  $\vartheta_{s,3}=T_{s,3}-T_{m,4}$ ;  $\vartheta_{m,3}=T_{m,3}-T_{m,4}=1150-20=1130$  K

### 3. Thermal work of coke cupolas – calculation of characteristic quantities

#### Calculation of $m_1 = m_{1,c}$

Factor  $m_1$  is a ratio of temperature powers of charge passing throughout the heating zone to that of combustion gas moving counter-currently towards the charge [formula (3)]; for the coke cupolas  $m_1 = m_{1,c}$ .

The temperature power of heated metal for the coke cupolas can be denoted by  $W_{m,c}$  and written down with the formula

$$W_{m,c} = c_{m,c} S_c \quad (48)$$

where:

$c_{m,c}$  – effective specific heat of metal in the heating zone of coke cupolas,  $J/(kg_{Fe} \cdot K)$

$S_c$  – efficiency of heating zone of coke cupolas or efficiency of metal charge heating up to the melting temperature,  $kg_{Fe}/s$ .

*Explanation: a stabilized work of cupola means, that the efficiency of metal heating up to the melting temperature in the heating zone is equal to the efficiency of metal melting in the melting zone as well as the efficiency of superheating the melted metal in the superheating (combustion) zone.*

In turn, temperature power of combustion gas in coke cupolas is denoted by  $W_{s,c}$  and written down with the formula

$$W_{s,c} = V_{s,c} c_{s,c} \dot{m}_c \quad (49)$$

where:

$V_{s,c}$  – volume of combustion gas in the heating zone of coke cupolas, corresponding to burning 1 kg coke coal, normal conditions,  $m^3/kg_c$

$c_{s,c}$  – mean specific heat of combustion gas in the heating zone of coke cupolas, combustion gas pressure 0,1 MPa,  $J/(m^3 \cdot K)$

$\dot{m}_c$  – mass rate of burning coke coal,  $kg_c/s$

$V_{s,c} \cdot \dot{m}_c$  – volume expenditure of combustion gas in the heating zone,  $m^3/s$ .

After substitution of (48) and (49) into (3)

$$m_{1c} = \frac{c_{m,c} S_c}{V_{(s,c)} c_{s,c} \dot{m}_c} = \frac{100 c_{m,c}}{V_{(s,c)} c_{s,c} K_c} \quad (50)$$

where:

$K_c = \frac{\dot{m}_c}{S_c} 100$  – fraction of coke coal in the heating zone,  $kg_c/100 kg_{Fe}$

*Explanation: It is assumed, that the chemical composition of combustion gas in the heating zone is constant (the reaction of  $CO_2$  decomposition does not occur).*

The calculation of  $c_{m,c}$ ,  $V_{s,c}$  and  $c_{s,c}$  requires complementary formulas.

#### Calculation of $c_{m,c}$

The effective specific heat of metal in the heating zone can be calculated through the appropriate enlargement of specific heat of metal. The general formula takes the form

$$c_{m,c} = c_m + \frac{Q_c}{100(T_{m,3} - T_{m,4})} \quad (51)$$

where:

$c_m$  – specific heat of metal, average one for the temperature range of heating zone,  $J/(kg_{Fe} \cdot K)$

$Q_c$  – total loss of heat in the heating zone, without the losses of heat spent on heating of charge metal,  $J/100 kg_{Fe}$

$(T_{m,3} - T_{m,4})$  – increment of metal charge temperature in the heating zone,  $K$ .

The  $Q_c$  sum consists of the following quantities in  $J/100 kg_{Fe}$ :

heat of limestone (calcium carbonate) decomposition, heat of moisture evaporation contained in the coke and limestone, heat transferred to the furnace wall, heat spent on heating the coke, heat spent on heating the limestone to the decomposition temperature. Owing to the limited volume of the paper, the formulas to calculate the mentioned losses will not be given here.

#### Calculation of $V_{s,c}$

Volume  $V_{s,c}$  can be established from formula [14 ]

$$V_{s,c} = \frac{22.4}{12} \left[ 1 + (1 + \eta_{v,c}) \frac{100 - O_v}{2O_v} \right] \quad (52)$$

where:

$O_v$  – content of oxygen in the air blast, vol. %

$\eta_{v,c}$  – degree of burning coke coal in the heating zone, unit fraction.

#### Calculation of $c_{s,c}$

The knowledge of limit temperatures  $T_{s,3}$  and  $T_{s,4}$  is indispensable to calculate mean, specific heat of combustion gas in the heating zone. It can be calculated based on zone heat balances or assumed and later corrected (method of consecutive approximations). Specific heats of limit temperatures of combustion gas  $c_{s,3}$  and  $c_{s,4}$  are calculated for the limit temperatures, followed by the determination of mean specific heat based on formula

$$c_{s,c} = \frac{T_{s,3} c_{s,3} - T_{s,4} c_{s,4}}{T_{s,3} - T_{s,4}} \quad (53)$$



Specific heat of combustion gas for the assumed limit temperatures can be calculated using methods of thermal engineering

#### Calculation of $m_2=m_{2,c}$

Factor  $m_2$  for coke cupolas, defined with formula (25) can be denoted as  $m_{2,c}$ . After substituting formula (48) into (25) one can obtain

$$m_{2,c} = \frac{\alpha_F F_H}{c_{m,c} S_c} \quad (54)$$

The development surface  $F_H$  in formula (54) can be written down for two applications. They are:

- to calculate the height of heating zone, or
- to calculate the heating time of metal charge pieces up to the melting temperature

#### Calculation of the height of heating zone

The development surface for the pieces of metal of identical shape, size and weight can be written down in the following form

$$F_H = n_m f_m = \frac{M_m}{\rho_m v_m} f_m = \frac{M_m}{\rho_m r_m} \quad (55)$$

where:

- $n_m$  – number of metal pieces in the heating zone
- $f_m$  – surface of a metal piece,  $m^2$
- $v_m$  – volume of a metal piece,  $m^3$
- $M_m$  – weight of metal in the heating zone, kg
- $\rho_m$  – density of metal piece mass,  $kg/m^3$
- $r_m$  – modulus of metal pieces, m.

The volume of heating zone is filled with metal, coke and limestone pieces. For simplification of calculations the presence of limestone is neglected, while the balance of bulk volumes for metal and coke is as follows

$$H_{pc} F_c = \frac{M_m}{\rho_{n,m}} + \frac{M_k}{\rho_{n,k}} \quad (56)$$

where:

- $H_{pc}$  – height of heating zone in coke cupolas, m
- $F_c$  – surface of internal cross-section of cupola in the heating zone,  $m^2$
- $M_k$  – weight of coke in the heating zone, kg
- $\rho_{n,m}, \rho_{n,k}$  – bulk density of metal and coke, respectively,  $kg/m^3$

Formula (56) can be transformed into the form of

$$H_{pc} F_r = \frac{M_m}{\rho_{n,m}} K_{\rho,p} \quad (57)$$

at which:

$$K_{\rho,p} = 1 + \frac{K_w \rho_{n,m}}{100 \rho_{n,k}} \quad (58)$$

where:

$K_w = \frac{M_k}{M_m} 100$  – expenditure of charge coke,  $kg_k/100 kg_{Fe}$

$$K_c = K_w C_k$$

$C_k$  – content of coal in coke,  $kg_c/kg_k$

$M_m$  is calculated from (57) and substituted into (55)

$$F_H = \frac{H_{p,c} F_r \rho_{n,m}}{K_{\rho,p} \rho_m r_m} \quad (59)$$

Next (59) is included into (54)

$$m_{2,c} = \frac{\alpha_F H_{p,c} F_r \rho_{n,m}}{K_{\rho,p} \rho_m r_m c_{m,c} S_c} \quad (60)$$

$m_2$  can be calculated from e.g. (22) and denoted as  $m_{2,c}$

$$m_{2,c} = \frac{1}{1 - m_{1,c}} \ln \frac{\theta_{s,4}}{\theta_{s,4} - (1 - m_{1,c}) \theta_{m,3}} \quad (61)$$

(6) is substituted into (61) and  $H_{pc}$  can be calculated

$$H_{pc} = \frac{S_{F,c} r_m K_{\rho,p} c_{m,c} \rho_m}{\alpha_F (1 - m_{1,c}) \rho_{n,m}} \ln \frac{\vartheta_{s,4}}{\vartheta_{s,4} - (1 - m_{1,c}) \vartheta_{m,3}} \quad (62)$$

where:

$S_{F,c} = \frac{S_{F,c}}{F_r}$  – relative efficiency of coke cupolas,  $kg/(m^2 \cdot s)$ .

For a particular case  $m_{1,c} = 1$ , equation (62) is simplified to the form below ( $m_2=m_{2,c}$  is obtained from (35))

$$H_{p,c} = \frac{S_{F,c} r_m K_{\rho,p} c_{m,c} \rho_m \vartheta_{m,3}}{\alpha_F \rho_m \rho_{n,m} \vartheta_{s,4}} \quad (63)$$

The height of heating zone can be obtained from equations (62) and (63). The temperatures of combustion gas  $\vartheta_{s,4}$  ( $T_{s,4}$ ) are given from measurements (contemporary cupolas are equipped with a device for a continuous measurement of outgoing gas temperature, while the cupola efficiency is known from measurements (e.g. based on the number of melted cartridges) or from the Buzek formula [15].

#### Calculation of heating time of metal pieces up to the melting temperature

After the substitution of  $M_m = S_c \cdot \tau_H$  (where:  $\tau_H$  – heating time of metal piece up to the melting temperature, s) into (57)

$$F_H = \frac{S_c \tau_H}{\rho_m r_m} \quad (64)$$

and (64) into (56) and after its simplification, formula (65) is obtained

$$m_{2,c} = \frac{\alpha_F \tau_H}{\rho_m r_m c_{m,c}} \quad (65)$$

Including (65) into (61)  $\tau_H$  can be determined

$$\tau_H = \frac{r_m c_{m,c} \rho_m}{\alpha_F (1 - m_{1,c})} \ln \frac{\vartheta_{s,4}}{\vartheta_{s,4} - (1 - m_{1,c}) \vartheta_{m,3}} \quad (66)$$

Based on (62) and (66), the following formula to calculate rate of displacement of charge material column in the heating zone is established

$$w_p = \frac{H_{p,c}}{\tau_H} = \frac{S_{F,c}}{\rho_{n,m}} K_{\rho,p} \quad (67)$$

#### New analysis of Buzek formula

The formula has a form reported in [15]

$$S_{F,c} = 100 \frac{P_{F,c}}{K_c L_c} \quad (68)$$

at which

$$L_c = 4.45 \frac{21}{O_v} (1 + \eta_{v,c}) \quad (69)$$

$$\eta_{v,c} = \frac{(CO_2)_{v,c}}{(CO_2)_{v,c} + (CO)_{v,c}} \quad (70)$$

where:

$L_c$  – volume of air blast spent to burn 1 kg coal contained in the coke, normal conditions,  $m^3/kg_c$

$O_v$  – content of oxygen in the air blast, vol. %

$\eta_{v,c}$  – degree of coal burning, in unit fraction

$(CO_2)_{v,c}$   $(CO)_{v,c}$  – content of  $CO_2$  and  $CO$ , respectively in the combustion gas of the heating zone, vol. %.

The physical meaning of the Buzek formula, analyzed by a number of authors for tens of years before the era of computers [e.g. 16-18] had not been explained in full. Moreover, one of its interpretations was proved incorrect. Here are the examples, which justify this opinion. Let us start from the erroneous interpretation of limits of the formula use, which will be shown based on a new method of its derivation.

In order to derive the formula, an equality of melting time of metal cartridges and combustion time of coal cartridges in coke ones), i.e.

$$\tau_{n,m} = \tau_{n,c} \quad (71)$$

where:

$\tau_{n,m}$ ,  $\tau_{n,c}$  – melting time of metal cartridges and combustion time of coal cartridges in the coke cartridges, respectively, s.

Equality (71) may be described as

$$\frac{m_{n,m}}{S_c} = \frac{m_{n,c}}{\dot{m}_c} \quad (72)$$

$S_c$  can be calculated from (72)

$$S_c = \frac{m_{n,m}}{m_{n,c}} \dot{m}_c \quad (73)$$

If obvious relations are substituted into (73)  $\frac{m_{n,m}}{m_{n,c}} = \frac{100}{K_c}$ ;  $K_c = \frac{m_{n,c}}{m_{n,m}} 100$ ;  $\dot{m}_c = \frac{P_c}{L_c}$ , and both sides are divided by  $F_r$ , equation (68) can be derived

Since the Buzek formula contains condition (71), which, at the same time, is the condition of delivering an adequate amount of thermal energy to the furnace the author of work [15] must have been mistaken, when he claimed, that the (68) formula “is valid for absolutely all the conditions of cupola working”. The formula of Buzek has a limited range of application determined first of all by admissible range of variation of thermal energy delivered to the furnace, which in the case of traditional coke cupolas is determined by amounts of  $P_{F,c}$  and  $K_c$ . The limits of variation are assessed (approximately) based on experience (practice). Although the amounts of  $P_{F,c}$  and  $K_c$  are changed (in acceptable limits), the work of cupola proceeds according to condition (71), which can be interpreted as “the cupola readjustment to the varying starting parameters”. The accommodation of cupola to the fulfillment of condition (71) depends on running thermal processes and accompanying distribution of heat to: superheating the liquid metal, heating the furnace wall and heating the departing combustion gas. If the total loss of physical heat of departing combustion gas and heat transferred to the furnace wall increases, the amount of heat spent on the increase of superheating degree of liquid metal decreases. However, the changes do not result in the alteration of condition (71) [12].

Unequality  $\tau_{n,m} > \tau_{n,c}$  means, that part of metal does not melt and it becomes a reason that the melting process stops (“freezing” of the furnace), while inequality  $\tau_{n,m} < \tau_{n,c}$  effects in unstable process of melting (breaks in melting)

The next questions, unanswered in literature but important anyhow, are: Which zone of cupola (heating or melting one) does the efficiency  $S_c$  refers to? Is the cupola process a continuous one?

The first question may be answered by analyzing the physical meaning of the denominator of equation (68), (which will be named  $V_d$ ) [19]

$$V_d = K_c L_c = K_{c,x} L_{c,x} = \frac{1 + \eta_{v,x}}{1 + \eta_{v,4}} K_c A (1 + \eta_{v,x}) = K_c L_c \quad (74)$$

where:

$K_{c,x}$   $L_{c,x}$  – product of values  $K_{c,x}$  and  $L_{c,x}$  at a arbitrary level of material column above the level of lower nozzles,  $m^3/100 kg_{Fe}$ .

It follows from (74), that formula (68) applies to each cupola zone, because in a stabilized process, a numerator and a denominator are of the same value for each zone.

In answer to the second question, a subsequent property of the cupola process will be characterized. It will be shown, that the column of charge materials moves inside the cupola shaft in a continuous manner (in spite of the jump-one assumed by the author of [18]) as a result of metal melting, coke burning and the reduction of part of CO<sub>2</sub> as well as in the effect of gravitation. For the whole column of charge materials its weight can be written in a general way

$$w_p = w_t + w_s + w_r \quad (75)$$

where:

$w_t$ ,  $w_s$ ,  $w_r$  – rates of motion of charge column as the result of melting metal, burning coke and the expenditure of coke coal on the reduction of CO<sub>2</sub>, m/s.

The right side of (75) for the melting zone can be expressed as

$$w_t F_r \rho_{n,m} = S_c \quad (76)$$

The formula to calculate  $w_t$  is obtained from (76)

$$w_t = \frac{S_c}{\rho_{n,m} F_r} = \frac{S_{F,c}}{\rho_{n,m}} \quad (77)$$

The following balance can be written down for the melting zone provided CO<sub>2</sub> does not undergo the reduction

$$w_s F_r C_k \rho_{n,k} = \frac{P_c}{L_{c,1}} \quad (78)$$

The formula to calculate  $w_s$  from (78) is as follows

$$w_s = \frac{P_{F,c}}{L_{c,1} C_k \rho_{n,k}} \quad (79)$$

In turn to obtain the formula of rate  $w_r$ , proportion (80) is formed

$$\frac{w_r}{w_s} = \frac{K_c - K_{c,1}}{K_{c,1}} \quad (80)$$

where:

$K_{c,1}$  – weight of coal in charge coke reduced by the loss of coal for the CO<sub>2</sub> reduction, kg<sub>c</sub>/100 kg<sub>Fe</sub>

With the use of (79) and (80) total  $w_{s,r} = w_s + w_r$  is put down

$$w_{s,r} = \frac{P_{F,c}}{L_c C_k \rho_{n,k}} \frac{K_c}{K_{c,1}} \quad (81)$$

Substituting the formula from [19] into (81)

$$K_{c,1} = \frac{1 + \eta_{v,c}}{2} K_c \quad (82)$$

After the simplification the following formula for the calculation of the total rates  $w_{s,r}$

$$w_{s,r} = \frac{P_{F,c}}{L_c C_k \rho_{n,k}} \quad (83)$$

where:

$$L_c = L_{c,1} \frac{1 + \eta_{v,c}}{2} = \frac{8,9}{2} \frac{21}{0_v} (1 + \eta_{v,c}) \text{ wg} \quad (69).$$

After including (77) and (83) into (75) and reshaping it

$$w_p = \frac{S_{F,c}}{\rho_{n,m}} + \frac{P_{F,c}}{L_c \rho_{n,k}} = \frac{S_{F,c}}{\rho_{n,m}} K_{\rho,p} \quad (84)$$

where:

The substitutions:  $\frac{P_{F,c}}{L_c} = \frac{S_{F,c}}{\rho_{100}}$  and (58) were used in (84)

Formula (84) is identical with (67) for the heating zone, which is a subsequent proof, that the efficiency calculated from the Buzek formula may be referred to each cupola zone, and it is in turn also a new interpretation of his formula.

#### Physical heat of outgoing combustion gas

Heat power of outgoing combustion gas  $Q_{s,4}$  (W) is contained in the formula

$$Q_{s,4} = W_{s,c} \quad \vartheta_{s,4} = V_{s,c} c_{s,c} \dot{m}_c \vartheta_{s,4} \quad (85)$$

Formula (49) and assumption  $T_{m4} = T_{ot}$  (where  $T_{ot}$  – ambient temperature) were applied to work out (85).

After dividing (85) by efficiency  $S_c$

$$Q_{s,4}^* = \frac{Q_{s,4}}{S_c} = V_{s,c} c_{s,4} K_c \vartheta_{s,4} \quad (86)$$

Or after substitution of (22)

$$Q_{s,4}^* = V_{s,c} c_{s,4} K_c \frac{1 - m_{1,c}}{1 - \exp[-m_{2,c}(1 - m_{1,c})]} \vartheta_{m,3} \quad (87)$$

where:

$Q_{s,c}^*$  – loss of physical heat of combustion gas to the ambience, J/100 kg<sub>Fe</sub>

$c_{s,4}$  – specific heat of outgoing combustion gas, J/(m<sup>3</sup>·K).

Equation (86) can be utilized, when the temperature of combustion gas  $T_{s,4}$  is known from measurements.

#### **Example 1.**

Calculation of work parameters for coke cupolas.

Data:  $m_{n,m} = 400$  kg<sub>Fe</sub>,  $m_{n,k} = 48$  kg<sub>k</sub>,  $C_k = 0,86$  kg<sub>c</sub>/kg<sub>k</sub>,

$P_{F,c} = 1,6$  m<sup>3</sup>/(m<sup>2</sup>·s),  $F_r = 0,503$  m<sup>2</sup>,  $c_{m,c} = 850$  J/(kg<sub>Fe</sub>·K),

$c_{s,c}=1600 \text{ J/(m}^3\cdot\text{K)}$  (value calculated for temperature range 500–1300°C),  $r_m = 0,015 \text{ m}$ ,  $O_v=21 \text{ vol.}\%$ ,  
 $\rho_m=7000 \text{ kg}_{Fe}/\text{m}^3$ ,  $\rho_{n,m}=2500 \text{ kg}_{Fe}/\text{m}^3$ ,  $\rho_{n,k}=500 \text{ kg}_k/\text{m}^3$ ,  
 $\alpha_F=130 \text{ W/(m}^2\cdot\text{K)}$

$H_{p,c}=3,5 \text{ m}$  (assumed),  $T_{m,3}=1150^\circ\text{C}$ ,  $T_{m,4}=20^\circ\text{C}$ .

Calculations:  $K_w=\frac{48}{400} 100=12 \text{ kg}_k/100 \text{ kg}_{Fe}$ ,

$K_c=0,86\cdot K_w=10,32 \text{ kg}_c/100 \text{ kg}_{Fe}$ ,  $\eta_{v,c}=0,525$  (according to formula of H. Jungbluth)

$L_c=4,45(1+0,525)=6,79 \text{ m}^3/\text{kg}_c$

$S_{F,c} = 100 \cdot \frac{1,6}{10,32 \cdot 6,79} = 2,28 \text{ kg}_{Fe}/(\text{m}^2\cdot\text{s})$ ,

$S_c=2,28\cdot 0,503=1,145 \text{ kg}_{Fe}/\text{s}$ ,

$\tau_{n,m}=\frac{400}{1,145}=349 \text{ s}$ ,  $\tau_{n,c}=\frac{48 \cdot 0,86 \cdot 6,79}{1,6 \cdot 0,503}=349 \text{ s}$ ,

$m_{1,c}=\frac{100 \cdot 850}{7,23 \cdot 1600 \cdot 10,32}=0,712$ ,

$V_c=5,39(1+0,65\cdot 0,525)=7,23 \text{ m}^3/\text{kg}_c$ ,

$K_{\rho,p}=1+\frac{12 \cdot 2500}{100 \cdot 500}=1,6$

$m_{2,c}=\frac{3,5 \cdot 130 \cdot 2500 \cdot 10,32 \cdot 6,79}{100 \cdot 1,6 \cdot 0,015 \cdot 1,6 \cdot 850 \cdot 7000}=3,49$

$\vartheta_{s,4}=\frac{1130}{1-\exp[-4,49(1-0,712)]}=1130=513 \text{ K}$ ,

$T_{s,4}=533^\circ\text{C}$ ,

$\vartheta_{s,3}=513+0,712\cdot 1130=1318 \text{ K}$ ,  $T_{s,3}=1338^\circ\text{C}$ ,

$\tau_H = \frac{0,015 \cdot 850 \cdot 7000}{130(1-0,712)} \ln \frac{513}{513-(1-0,712)1130}=2398 \text{ s}$  (40 min).

In the presented example value  $H_{p,c}$  was given, which enabled the calculation of  $T_{s,4}$ ; if  $T_{s,4}$  is known from measurements, then  $H_{p,c}$  can be calculated.

## Gas-fired cupolas

The following formulas characterizing the work of gas cupolas will be subsequently derived: the formula for calculating the efficiency of the heating zone ( $\text{kg}_{Fe}/\text{s}$ ), the formulas for factors  $m_{1,g}$  and  $m_{2,g}$  appearing in the general equation (20) and its variants as well as the formulas for heating zone height and heating time of metal pieces up to the melting temperature.

### Calculation of efficiency

The formula for the calculation of efficiency of gas cupolas (an analogy to the Buzek formula for the coke cupolas) can be derived, assuming the equality of heating times to the melting temperature of metal charge

cartridges (portion of metal charge,  $\text{kg}_{Fe}$ ) and burning times of gas cartridges (portion of gas,  $\text{m}_{gaz}^3$ )

$$\frac{m_{n,m}}{S_g} = \frac{V_{n,g}}{\dot{V}_{s,g}} \quad (88)$$

at which:

$$\tau_{n,m} = \frac{m_{n,m}}{S_g} \quad (89)$$

$$\tau_{n,g} = \frac{V_{n,g}}{\dot{V}_{s,g}} \quad (90)$$

where:

$S_g$  – efficiency of metal heating up to the melting time in gas cupolas,  $\text{kg}/\text{s}$

$V_{ng}$  – volume of gas cartridge (an assumed volume of gas required for melting of one metal cartridge) normal conditions,  $\text{m}_g^3$

$\dot{V}_{s,g}$  – volumetric rate of gas burning normal conditions,  $\text{m}_g^3/\text{s}$

$\tau_{n,m}$ ,  $\tau_{n,g}$  – time of metal cartridge melting and that of gas cartridge burning, respectively, s

Quantity  $\dot{V}_{s,g}$  is calculated from

$$\dot{V}_{s,g} = \frac{P_g}{L_g} \quad (91)$$

where:

$P_g$  – consumption of air blast in the gas cupolas, normal conditions of pressure and temperature (content of oxygen can be optional),  $\text{m}_g^3/\text{s}$

$L_g$  – volume of air blast used up for burning 1  $\text{m}^3$  of gas, normal conditions,  $\text{m}_p^3/\text{m}_g^3$

After the substitution of (91) into (88) the efficiency can be found

$$S_g = \frac{m_{n,m}}{V_{n,g}} \frac{P_g}{L_g} \quad (92)$$

Equation (92) can be transformed into the shape of the Buzek equation

$$S_g = 100 \frac{P_g}{K_g L_g} \quad (93)$$

at which

$$K_g = \frac{V_{n,g}}{m_{n,m}} 100 \quad (94)$$

where:

$K_g$  – volume of gas needed for 100 kg metal charge,  $\text{m}_g^3/100 \text{ kg}_{Fe}$ .

Equation (93) concerns the working (melting metal) gas cupolas. It does not consider the thermal processes. Moreover, the range of  $P_g$  and  $K_g$  changes should be assessed based on experiments.

A more general form of equation (94) is obtained after dividing its both sides by  $F_r$

$$S_{F,g} = 100 \frac{P_{F,g}}{K_g L_g} \quad (95)$$

where:

$$S_{F,g} = \frac{S_g}{F_r} - \text{kg}_{Fe}/(\text{m}^2 \cdot \text{s}) ; P_{F,g} = \frac{P_g}{F_r} - \text{m}^3/(\text{m}^2 \cdot \text{s}) \text{ or } \text{m}_p/\text{s}.$$

#### Calculation of $m_{1,g}$

The value of  $m_{1,g}$  comes from the ratio of temperature powers of the heated medium (metal) and the heating one (combustion gas), given with a general equation (3), which, for the gas cupolas considered, can be written down as

$$m_{1,g} = \frac{W_{m,g}}{W_{s,g}} \quad (96)$$

where:

$W_{m,g}$ ,  $W_{s,g}$  – temperature power of the heated metal and the heating combustion gas, respectively in the heating zone, W/K.

Let us write the values  $W_{m,g}$ ,  $W_{s,g}$  with the formulas

$$W_{m,g} = S_g c_{m,g}, \quad \text{and} \quad (97)$$

$$W_{s,g} = V_{sg} c_{sg} \frac{P_{c,g}}{L_g} \quad (98)$$

where:

$c_{m,g}$  – effective specific heat of metal charge in the heating zone of gas cupolas, J/(kg<sub>Fe</sub>·K)

$V_{sg}$  – volume of combustion gas appearing after burning 1 m<sup>3</sup> gas, standard conditions, m<sup>3</sup>/m<sup>3</sup>

$c_{sg}$  – mean specific heat of combustion gas in the heating zone, pressure of combustion gas 0,1 MPa, J/(m<sup>3</sup>·K).

After substitution of (97) and (98) into (96)

$$m_{1,g} = \frac{100 c_{m,g}}{V_{s,g} c_{s,g} K_g} \quad (99)$$

where:

$$K_g = \frac{P_{c,g}}{L_g S_{c,g}} 100 = \frac{v_{g,n}}{m_{n,m}} 100, \frac{\text{m}_{\text{gas}}^3}{\text{kg}_{Fe}} 100 \text{ [similar to (66)]}$$

$$\frac{P_{c,g}}{L_g} \left( \frac{\text{m}_{\text{pow}}^3 \text{m}_{\text{gas}}^3}{\text{sm}_{\text{pow}}^3} = \frac{\text{m}_{\text{gas}}^3}{\text{s}} \right); \frac{1}{S_{c,g}} \left( \frac{\text{s}}{\text{kg}_{Fe}} \right).$$

It results from (99), that the increase of  $K_g$  yields the decrease of  $m_{1,g}$ , which in turn effects in the growth of  $\vartheta_{s,4}$  and fall of  $\vartheta_{s,3}$  (Table 1).

#### Calculation of $m_{2,g}$

Equation (98) is substituted into general formula (21)

$$m_{2,g} = \frac{\alpha_F F_H}{W_{m,g}} \quad (100)$$

Formula of  $F_H$  will be derived analogically to (48) obtaining

$$F_H = \frac{M_{m,g}}{\rho_m r_m} \quad (101)$$

where:

$M_{m,g}$  – weight of metal in the heating zone of coke cupolas, kg.

Substituting (97) and (101) into (100) formula of  $m_{2,g}$  is derived

$$m_{2,g} = \frac{\alpha_F M_{m,g}}{\rho_m r_m S_g c_{m,g}} \quad (102)$$

From (102) and (54) two important formulas are obtained, i.e. the formula for calculation of heating zone height and another to calculate heating time of metal pieces charge up to the melting temperature.

#### Formula to calculate the heating zone height

The mass of metal in the heating zone  $M_{m,g}$  can be put down with formula

$$M_{m,g} = H_{p,g} F_r \rho_{n,m} \quad (103)$$

where:

$H_{p,g}$  – heating zone height of gas cupolas (presence of limestone and possible additions are neglected), m

Substituting (103) into (102)

$$m_{2,g} = \frac{\alpha_F H_{p,g} F_r \rho_{n,m}}{\rho_m r_m S_g c_{m,g}} \quad (104)$$

and (104) into (54) the heating zone height can be obtained

$$H_{p,g} = \frac{\rho_m r_m S_{F,g} c_{m,g}}{\alpha_F \rho_{n,m} (1 - m_{1,g})} \ln \frac{\vartheta_{s,4}}{\vartheta_{s,4} - (1 - m_{1,g}) \vartheta_{m,3}} \quad (105)$$

Equation (64) will be incorporated into (105), which defines the efficiency of gas cupolas

$$H_{p,g} = \frac{100 P_{F,g} \rho_m r_m c_{m,g}}{K_g L_g \alpha_F \rho_{n,m} (1 - m_{1,g})} \ln \frac{\vartheta_{g,4}}{\vartheta_{g,4} - (1 - m_{1,g}) \vartheta_{m,3}} \quad (106)$$

Formula to calculate the heating time of metal charge pieces up to the melting temperature

The weight of metal in the heating zone may be used to obtain the formula of heating time of metal pieces up to the melting temperature

$$M_{m,g} = S_g \tau_H \quad (107)$$

where:

$\tau_H$  – heating time of metal pieces up to the melting temperature, s.

Formula (108) is obtained after including (107) into (102) and after a simplification

$$m_{2,g} = \frac{\alpha_F \tau_H}{\rho_m r_m c_{m,g}} \quad (108)$$

Time  $\tau_H$  can be calculated by inserting (108) into (54)

$$\tau_H = \frac{r_m c_{m,g} \rho_m}{\alpha_F (1 - m_{1,g})} \ln \frac{\vartheta_{s,4}}{\vartheta_{s,4} - (1 - m_{1,g}) \vartheta_{m,3}} \quad (109)$$

When comparing the gas and coke cupola processes, the equality of melting times of metal cartridges may be assumed, which results in the following relations

$$\frac{V_{n,g}}{P_g} L_g = \frac{m_{n,c}}{P_c} L_c \quad (110)$$

Eg.  $P_g$  for the given values of other parameters can be obtained from (110)

$$P_g = P_c \frac{V_{n,g} L_g}{m_{n,c} L_c} \quad (111)$$

Analogically to formulas (85), (86) and (87), the formulas of  $Q_{s,4}$  and valid for gas cupolas can be written down

### Example 2.

Calculation of work parameters of gas cupola

Data: gas composition: 100% CH<sub>4</sub> (e.g. natural gas of Carpathian Mountains origin contains 92% methane, 2% other combustible components and 6% nitrogen [21]);  $P_{F,g}=1,6 \text{ m}^3/(\text{m}^2 \cdot \text{s})$ ,  $c_{m,g}=850 \text{ J}/(\text{kg}_{Fe} \cdot \text{K})$ ;  $K_g=8 \text{ m}^3_{gas}/100 \text{ kg}_{Fe}$ ;  $F_r=0,503 \text{ m}^2$ ;  $\alpha_F=130 \text{ W}/(\text{m}^2 \cdot \text{K})$ ;  $\rho_m=7000 \text{ kg}/\text{m}^3$ ;  $r_m=0,015 \text{ m}$ ,  $O_v=21 \text{ vol.}\%$ , calculated mean specific heat for the zone (temperature range from 1300 ° to 600 °):  $c_{s,g}=1687 \text{ J}/(\text{m}^3 \cdot \text{K})$ .

Calculations: Methane burns according to reaction  $\text{CH}_4 + 2 \text{ O}_2 = \text{CO}_2 + 2 \text{ H}_2\text{O}$ ;  $L_g = (2+2 \frac{79}{21}) = 9,52 \text{ m}^3/\text{m}^3$ ;

$$V_{s,g} = 1+2+2 \frac{79}{21} = 10,52 \text{ m}^3/\text{m}^3, P_g=1,6 \frac{8}{10,32} \frac{9,52}{6,79} = 1,74 \text{ m/s},$$

$$S_{F,g} = 100 \frac{1,74}{8 \cdot 9,52} = 2,28 \text{ kg}_{Fe}/(\text{m}^2 \cdot \text{s}); S_g = 2,28 \cdot 0,503 = 1,147 \text{ kg}_{Fe}/\text{s}.$$

$$m_{1,g} = \frac{100 \cdot 850}{10,52 \cdot 1687 \cdot 8} = 0,6; H_{p,g} = \frac{H_{p,c}}{K_{\rho,p}} = \frac{3,5}{1,6} = 219 \text{ m};$$

$$m_{2,g} = \frac{130 \cdot 2,19 \cdot 2500}{7000 \cdot 0,015 \cdot 2,28 \cdot 850} = 3,5;$$

$$\vartheta_{s,4} = \frac{(1 - 0,6) 1130}{1 - \exp[-3,5(1 - 0,6)]} = 600 \text{ K}$$

$$T_{s,4} = 600 + 20 = 620^\circ\text{C}; \vartheta_{s,3} = 600 + 0,6 \cdot 1130 = 1278 \text{ K};$$

$$T_{s,3} = 1298^\circ\text{C};$$

$$\tau_H = \frac{0,015 \cdot 850 \cdot 7000}{130(1 - 0,6)} \ln \frac{600}{600 - (1 - 0,6)1130} = 2402 \text{ s} \quad (40 \text{ min}).$$

The  $H_{p,g}$  assumed height was respectively lowered compared to the one from example 1. It enabled the calculation of temperature  $T_{s,4}$ . If  $T_{s,4}$  is known (e.g. from measurements),  $H_{p,g}$  can be calculated from equation (74) or (79)

$$\text{Further calculations: } \tau_{n,m} = \frac{400}{1,147} = 348 \text{ s};$$

$$\tau_{n,g} = \frac{4 \cdot 8}{1,74 \cdot 0,503} = 9,52 = 348 \text{ s};$$

$$M_{m,g} = 2,19 \cdot 0,503 \cdot 2500 = 2754 \text{ kg}_{Fe}; S_g = \frac{2754}{1,147} = 2401 \text{ s} \quad (\text{check}).$$

The energy comparison of examples 1 and 2:

- Lower combustion heat of natural gas from Carpathian Mountains equals 34,715 MJ/m<sub>g</sub><sup>3</sup> [21 ],
- energy delivered to the cupola in the result of gas combustion is 8·34,715=277,72 MJ/100 kg<sub>Fe</sub>
- effective heat of burning of coke coal (decreased by losses for the reaction of CO<sub>2</sub> reduction) is 33,66 (0,3+0,7·0,525)10,32=231,9 MJ/100 kg<sub>Fe</sub>
- the difference is (277,72-231,9)=45,82 MJ/100 kg<sub>Fe</sub>
- the volume of combustion gas in the coke process is equal to 5,39(1+0,65·0,525)10,32=74,5 m<sub>s</sub><sup>3</sup>/100 kg<sub>Fe</sub>;
- while the volume of combustion gas in the gas process is equal to 10,52·8=84,2 m<sub>s</sub><sup>3</sup>/100 kg<sub>Fe</sub>
- volumes of combustion gas components in the coke process are: CO=9,15 m<sub>s</sub><sup>3</sup>/100 kg<sub>Fe</sub>; CO<sub>2</sub>=10,11 m<sub>s</sub><sup>3</sup>/100 kg<sub>Fe</sub>; N<sub>2</sub>=52,24 m<sub>s</sub><sup>3</sup>/100 kg<sub>Fe</sub>
- volumes of combustion gas components in the gas process are:

$$\text{CO}_2 = 0,095 \cdot 84,2 = 8 \text{ m}^3/100 \text{ kg}_{Fe}; \text{H}_2\text{O} = 0,19 \cdot 84,2 = 16 \text{ m}^3/100 \text{ kg}_{Fe};$$

$$\text{N}_2 = 0,715 \cdot 84,2 = 60,2 \text{ m}^3/100 \text{ kg}_{Fe}$$

## Coke-gas-fired cupolas

Basic formulas for the coke-gas cupolas are derived in the analogical sequence as for the gas cupolas.

### Efficiency of coke-gas cupolas

The equality of times of metal cartridge melting, coal (coke) cartridge burning and gas cartridge burning is assumed for coke-gas cupolas

$$\tau_{n,m} = \tau_{n,c} = \tau_{n,g} \quad (112)$$

Or in a wider form

$$\frac{m_{n,m}}{S_\varepsilon} = \frac{m_{n,c}}{P_c} L_c = \frac{v_{n,g}}{P_g} L_g \quad (113)$$

Equation (111) results from (113). It allows the calculation of value  $P_g$  which meets the condition of  $\tau_{n,c} = \tau_{n,g}$

Since in the same time a coal cartridge and a gas one melt one metal cartridge each, their efficiency is a sum

$$2S_\varepsilon = S_c + S_g \quad (114)$$

where:

$S_\varepsilon$  – efficiency of coke-gas cupola,  $\text{kg}_{Fe}/\text{s}$ .

Equations (68) and (93) included into (114) give

$$S_\varepsilon = \frac{100}{2} \left( \frac{P_c}{K_c L_c} + \frac{P_g}{K_g L_g} \right) \quad (115)$$

After dividing both sides of (115) by the cupola shaft cross section

$$S_{F,\varepsilon} = \frac{100}{2} \left( \frac{P_{F,c}}{K_c L_c} + \frac{P_{F,g}}{K_g L_g} \right) \quad (116)$$

where:

$S_{F,\varepsilon}$  – relative efficiency of coke-gas-fired cupolas,  $\text{kg}_{Fe}/(\text{m}^2 \cdot \text{s})$ .

#### Calculation of $m_{1,\varepsilon}$

Using a general definition (3), value  $m_1$  for the coke-gas cupolas will be denoted as  $m_{1,\varepsilon}$  and will form the equation

$$m_{1,\varepsilon} = \frac{W_{m,\varepsilon}}{W_{s,\varepsilon}} \quad (117)$$

where:

$W_{m,\varepsilon}$  – temperature power absorbed by the metal charge, W/K

$W_{s,\varepsilon}$  – temperature power of combustion gas in the heating zone of coke-gas-fired cupolas, W/K.

For the coke-gas-fired cupolas it is assumed, that the temperature power of combustion gas in the heating zone

is equal to the total of temperature powers of combustion gases produced as the result of burning the coke and gas.

$$W_{s\varepsilon} = (V_{s,c} \dot{m}_c + V_{s,g} \dot{v}_g) c_{s\varepsilon} \quad (118)$$

where:

$c_{s,\varepsilon}$  – mean specific heat of the mixture of combustion gases in the heating zone, released on burning the coal and gas,  $\text{J}/(\text{m}^3 \cdot \text{K})$ .

In turn, the temperature power absorbed by the heated metal in coke-gas-fired cupolas will be put down as

$$W_{m,\varepsilon} = c_{m,\varepsilon} S_\varepsilon \quad (119)$$

where:

$c_{m,\varepsilon}$  – effective specific heat of metal charge of coke-gas cupolas,  $\text{J}/(\text{kg}_{Fe} \cdot \text{K})$ .

After incorporating (118) and (119) into (117) the following equation is obtained

$$m_{1,\varepsilon} = \frac{c_{m,\varepsilon}}{\left( V_{s,c} \frac{\dot{m}_c}{S_\varepsilon} + V_{s,g} \frac{\dot{v}_g}{S_\varepsilon} \right) c_{s,\varepsilon}} \quad (120)$$

or

$$m_{1,\varepsilon} = \frac{100 c_{m,\varepsilon}}{\left( V_{s,c} K_{c,\varepsilon} + V_{s,g} K_{g,\varepsilon} \right) c_{s,\varepsilon}} \quad (121)$$

where:

$K_{c,\varepsilon}$  – consumption of coke coal in coke-gas-fired cupolas,  $\text{kg}_c/100 \text{ kg}_{Fe}$

$K_{g,\varepsilon}$  – consumption of gas in coke-gas-fired cupolas,  $\text{m}_g^3/100 \text{ kg}_{Fe}$

#### Calculation of parameter $m_{2,\varepsilon}$

For the coke-gas-fired cupolas, value  $m_2$  can be denoted as  $m_{2,\varepsilon}$  and it can be formulated as (122) based on (21) and (119)

$$m_{2,\varepsilon} = \frac{\alpha_F F_H}{c_{m,\varepsilon} S_\varepsilon} \quad (122)$$

Formula  $F_H$  is analogical with (48)

$$F_H = \frac{M_{m,\varepsilon}}{\rho_m \Gamma_m} \quad (123)$$

where:

$M_{m,\varepsilon}$  – weight of metal in the heating zone of coke-gas-fired cupolas,  $\text{kg}_{Fe}$

Substituting (123) into (122)

$$m_{2,\varepsilon} = \frac{\alpha_F M_{m,\varepsilon}}{\rho_m \Gamma_m c_{m,\varepsilon} S_\varepsilon} \quad (124)$$

Coupling equations (124) and (54), the formula to calculate the heating zone height or the heating time of metal charge to the melting temperature of coke-gas-fired cupolas may be obtained.

Formula to calculate the heating zone height

The weight of metal in the heating zone  $M_{m,\varepsilon}$  takes the form of

$$M_{m,\varepsilon} = \frac{H_{p,\varepsilon} F_r \rho_{n,m}}{K_{\rho,\varepsilon}} \quad (125)$$

where:

$H_{p,\varepsilon}$  – heating zone height of coke-gas-fired cupolas, m

$K_{\rho,\varepsilon}$  – ratio of heating zone volume containing metal and coke to bulk volume of metal

Including (125) into (124)

$$m_{2,\varepsilon} = \frac{\alpha_F H_{p,\varepsilon} \rho_{n,m}}{\rho_m \Gamma_m c_{m,\varepsilon} K_{\rho,\varepsilon} S_{F,\varepsilon}} \quad (126)$$

After the substitution of (126) into (54)  $H_{p,\varepsilon}$  may be established

$$H_{p,\varepsilon} = \frac{\rho_m \Gamma_m K_{\rho,\varepsilon} S_{F,\varepsilon} c_{m,\varepsilon}}{\alpha_F \rho_{n,m} (1 - m_{1,\varepsilon})} \ln \frac{\vartheta_{s,4}}{\vartheta_{s,4} - (1 - m_{1,\varepsilon}) \vartheta_{m,3}} \quad (127)$$

Expression (116) can be included into (127)

Formula for calculation of heating time of metal charge to the melting temperature

The weight of metal  $M_{m,\varepsilon}$  in (124) will be put down as the formula

$$M_{m,\varepsilon} = S_\varepsilon \tau_H \quad (128)$$

After simplifying

$$m_{2,\varepsilon} = \frac{\alpha_F \tau_H}{\rho_m \Gamma_m c_{m,\varepsilon}} \quad (129)$$

$\tau_H$  is calculated substituting (129) into (54)

$$\tau_H = \frac{\Gamma_m c_{m,\varepsilon} \rho_m}{\alpha_F (1 - m_{1,\varepsilon})} \ln \frac{\vartheta_{s,4}}{\vartheta_{s,4} - (1 - m_{1,\varepsilon}) \vartheta_{m,3}} \quad (130)$$

Identically to relations (85), (86) and (87), the formulas of  $Q_{s,4}$  and are obtained for the coke-gas cupolas.

**Example 3.**

Data:  $P_{F,c}=1,6$  m/s,  $F_r=0,503$  m<sup>2</sup>,  $P_c=0,8$  m<sup>3</sup>/s,  $K_{c\varepsilon}=7$  kg<sub>c</sub>/100 kg<sub>Fe</sub>,  $K_{g,\varepsilon}=3$  m<sup>3</sup>/100 kg<sub>Fe</sub>,  $\eta_{v,c}=0,7$  (from the formula of H. Jungbluth),  $L_c=756$  m<sup>3</sup>/kg<sub>c</sub>,  $L_g=9,52$  m<sup>3</sup>/m<sup>3</sup> (methane),  $m_{n,m}=400$  kg<sub>Fe</sub>,  $m_{n,c}=4 \cdot 7 = 28$

kg<sub>c</sub>/m<sub>n,m</sub>,  $v_{n,g}=4 \cdot 3=12$  m<sup>3</sup>/m<sub>n,m</sub>,  $\alpha_F=130$  W/(m<sup>2</sup>·K),  $V_{s,c}=7,84$  m<sup>3</sup>/kg<sub>c</sub>,  $V_{s,g}=10,52$  m<sup>3</sup>/m<sub>g</sub><sup>3</sup>,  $c_{m,\varepsilon}=850$  J/(kg<sub>Fe</sub>·K),  $c_{s,\varepsilon}=1640$  J/(m<sup>3</sup>·K),  $\rho_{n,m}=2500$  kg/m<sup>3</sup>;  $\rho_m=7000$  kg/m<sup>3</sup>,  $\rho_{n,k}=500$  kg<sub>k</sub>/m<sup>3</sup>,  $C_k=0,86$  kg<sub>c</sub>/kg<sub>k</sub>,  $c_{s,\varepsilon,4}=1400$  J/(m<sup>3</sup>·K)

$$H_{p,\varepsilon}=3,5 \text{ m}, \vartheta_{m,3}=1130 \text{ K.}$$

Calculations:  $P_g=0,8 \frac{3 \cdot 9,52}{7 \cdot 7,56} = 0,432$  m<sub>p</sub><sup>3</sup>/s acc. to (113),

$$S_\varepsilon = \frac{100}{2} \frac{0,8}{7 \cdot 7,56} + \frac{0,432}{3 \cdot 9,52} = 1,51 \text{ kg}_{Fe}/s,$$

$$\tau_{n,m} = \frac{400}{1,51} = 265 \text{ s}, \tau_{n,c} = \frac{(4 \cdot 7)}{0,8} \cdot 7,56 = 265 \text{ s},$$

$$\tau_{n,g} = \frac{4 \cdot 3}{0,432} \cdot 9,52 = 265 \text{ s}, m_{1,\varepsilon} = \frac{100 \cdot 850}{(7,84 \cdot 7 + 10,52 \cdot 3) 1640} =$$

$$0,6, K_{\rho,\varepsilon}=1,41,$$

$$m_{2,\varepsilon} = \frac{130 \cdot 3,5 \cdot 2500}{7000 \cdot 0,015 \cdot 850 \cdot 1,41 \cdot 3} = 3,$$

$$\vartheta_{s,4} = \frac{1 - 0,6}{1 - \exp[-3(1 - 0,6)]} 1130 = 647 \text{ K},$$

$$\vartheta_{s,3} = 647 + 0,6 \cdot 1130 = 1325 \text{ K.}$$

Energy calculations:

– chemical heat of charge coke coal:  $33,66 \cdot 7 = 235,6$  MJ/100 kg<sub>Fe</sub>

– combustion heat of gas:  $3 \cdot 34,715 = 104,15$  MJ/100 kg<sub>Fe</sub>

– heat delivered to the cupola:  $235,6 + 104,15 = 340$  MJ/100 kg<sub>Fe</sub>

– heat of coke coal after taking into account the losses for the CO<sub>2</sub> reduction reaction:

$$33,66(0,3 + 0,7 \cdot 0,7) 7 = 186,1 \text{ MJ/100 kg}_{Fe}$$

– disposable heat of cupola:  $186,1 + 104,15 = 290,25$  MJ/100 kg<sub>Fe</sub>

– loss of physical heat in outgoing combustion gas:

$$(7,84 \cdot 7 + 10,52 \cdot 3) 1400 \cdot 647 = 78 \text{ MJ/100 kg}_{Fe}$$

Energy balance of the coke cupola. (Example 1).

– chemical energy of charge coke coal:  $33,66 \cdot 12 \cdot 0,86 = 347,4$  MJ/100 kg<sub>Fe</sub>

– heat of coke coal after taking into account the losses for the CO<sub>2</sub> reduction: (disposable heat of cupola)

$$33,66(0,3 + 0,7 \cdot 0,525) 10,32 = 232 \text{ MJ/100 kg}_{Fe}$$

– physical heat loss in outgoing combustion gas:

$$7,23 \cdot 10,32 \cdot 1400 \cdot 533 = 55,7 \text{ MJ/100 kg}_{Fe}$$

Comparison of energy balances (Example 1 and 3)

– the delivered amounts of heat are similar: 340 and 347,4 MJ/100 kg<sub>Fe</sub>

– the differences of disposable heat amount to  $290,25 - 232 = 58,25$  MJ/100 kg<sub>Fe</sub> to coke-gas-fired cupola advantage

– heat losses with outgoing combustion gas are larger in the coke-gas-fired cupolas by  $78 - 55,7 = 22,3$  MJ/100 kg<sub>Fe</sub>



– excess of disposable heat differences for coke-gas and coke cupolas, after the incorporation of the excess of outgoing heat losses is:  $58,25-22,3=34$  MJ/100 kg<sub>Fe</sub> whose part increases the degree of liquid metal superheating in the coke-gas-fired cupolas with respect to that obtained in gas cupolas [12].

#### 4. Conclusions

Three groups of tasks have been solved in the work:

– the derivation, based on principles of Le Chatelier theory, of general equation of combustion gas and metal temperature in the heating zone of coke, coke-gas and gas-fired cupolas as a function of two dimensionless parameters  $m_1$  and  $m_2$  dependant on several factors of the process; general assessment of the influence of dimensionless parameters  $m_1$  and  $m_2$  on limit temperatures of combustion gas,

– elaboration of the formulas for the calculation of dimensionless parameters  $m_1$  and  $m_2$  separately for coke, coke-gas and gas cupolas, which, together with the basic equation, constitute the mathematical description of thermal work of the mentioned three fuel groups of cupolas and was designed for the calculation of the basic parameters of their work;

– working out the calculation examples of factors characterizing the thermal work of coke, gas and coke-gas cupolas.

In reference to earlier works of the author [e.g. 12, 13], it can be stated, that limit temperatures of combustion gas in the heating zone affect the thermal process in the combustion and melting zones, decisive of the liquid cast iron temperature.

The paper has been the first, in literature, attempt of analytic description of the thermal work of coke, coke-gas and gas-fired cupolas. It has also explained some controversial problems of the theory of coke cupolas process.

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