A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
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THE CALCULATION OF HEIGHT AND STRUCTURE PARAMETERS OF COMBUSTION ZONE IN COKE-FIRED CUPOLAS

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The calculation formulas of the combustion zone height in the coke-fired cupolas (coke pieces in the form of square based prisms and various dimensions) as well as the structure of combustion zone have been derived in the present work. The structure has been characterized with the following parameters: zone volume, mass of burning coke, number of burning coke pieces and their average dimensions, surface of development of coke pieces, number of their sequences, their volumes and surfaces in sequences, combustion time of coke cartridges, primary height of the filling coke and others.

The presented examples illustrate practical calculations and describe the movement of coke from the melting zone to the combustion one as a continuous process at constant height of the combustion zone. Such an approach completely changes the models of combustion and melting processes proceeding at the interface of combustion and melting zones valid so far.

Keywords: cupolas, coke, zone, combustion, height, structure

W pracy wyprowadza się wzory do obliczania wysokości strefy spalania w żeliwiakach koksowych (kawałki koksu w kształcie graniastosłupów o podstawie kwadratu i różnych wymiarach), oraz struktury strefy, którą charakteryzują następujące wielkości: objętość strefy, masa palącego się koksu; liczba palących się kawałków koksu i ich średnie wymiary, powierzchnia rozwinięcia kawałków koksu, liczba ciągów kawałków koksu, ich objętości i powierzchnie; objętości i powierzchnie kawałków ciągach; czas spalania nabojów koksu; pierwotna wysokość koksu wypełniającego i in.

Zamieszczone przykłady ilustrują praktyczne obliczenia oraz charakteryzują proces przemieszczeń koksu ze strefy topienia do strefy spalania jako proces ciągły, przy zachowaniu stałej wysokości strefy spalania, co całkowicie zmienia dotychczasowe poglądy na temat modelu procesów spalania i topienia, zachodzących na granicy stref spalania i topienia.

General denotations

a – initial length of sides of coke piece base (square), m, $a = 2\,\mu_k\,\tau_c$
b – initial height (length) of coke piece (square based prism), m
$F_{c,k}$ – surface of coke pieces in one sequence of pieces, m^2
$\bar{F}_{k,s}$ – development surface of combustion zone (total surface of burning coke pieces in the zone), m^2
$F_{r,s}$ – surface of cross-section of combustion zone, perpendicular to cupola axis, m^2
$f_{o,k}$ – initial surface of coke pieces, m^2
$f_{k,\tau}$ – surface of burning coke piece after combustion time τ, m^2
$\bar{f}_k = f_{o,k}\,\varphi_f$ – mean integral surface of coke pieces, m^2
H_s – height of combustion zone, m
$k_1 = 2 + \frac{1}{m}$; $k_2 = 1 + \frac{2}{m}$; $k_3 = \frac{1}{m}$
$L_{k,4}$ – volume of air blast consumed in the combustion of coke mass unit, normal conditions, m_p^3/kg_c
$m = \frac{b}{a}$ – slenderness ratio of coke piece,
$\bar{M}_{k,s}$ – mass of burning coke in the zone, kg
n_k – number of coke pieces in the zone,
$n_{c,k}$ (or n) – number of coke pieces in each sequence
P_c – efficiency of blast, normal conditions, m_p^3/s
P_F – relative efficiency of blast, normal conditions $m_p^3/(m^2\cdot s)$
$r_{k,o} = \frac{v_{k,o}}{f_{k,o}}$ – initial module of coke pieces, m
$\bar{r}_k = r_{k,o}\frac{\varphi_v}{\varphi_f}$ – average integral module of coke pieces, m
$V_{c,k}$ – volume of pieces in one sequence, m^3
$\bar{v}_k = v_{o,k}\varphi_v$ – average integral volume of coke pieces, m^3
$v_{o,k}$ – initial volume of coke pieces, m^3
$v_{k,X}$ – volume of burning coke piece in dependence on relative time X, m^3

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$X = \frac{\tau}{\tau_c}$ – relative thickness of burnt layer of coke, in unit fraction

2z – constant difference of linear dimensions of coke pieces in sequences,

$$\delta = \frac{1}{n} = \frac{2z}{a}$$

$\eta_{v,4}$ – degree of combustion in unit fraction

μ_k – linear rate of coke combustion, m/s

ρ_k – density of coke mass, kg/m_k^3

$\rho_{n,k}$ – bulk density of coke, kg$_k$/m^3

τ – time measured from the beginning of combustion process, s

φ_v – average integral volume of coke piece divided by its initial volume, $\varphi_v = \frac{\bar{v}_k}{v_{o,k}}$

φ_f – average integral surface of coke piece divided by its initial surface, $\frac{\bar{f}_k}{f_{o,k}}$

$\varphi_{v,n}$ – factor φ_v dependent on the number of coke pieces in sequence

$\varphi_{f,n}$ – factor φ_f dependent on the number of coke pieces in sequence

1. Introduction

The height of combustion is an important factor in the theory and practice of the coke-fired cupola process. Its height affects the degree of liquid metal superheating and the effective height of cupolas. The combustion zone, together with the melting zone and height of the filling coke have become the object of long-term controversial ideas of the optimal model of the coke process. The participants of the dispute although not having at disposal a rational approach to the calculation of height and structure of melting and combustion zones, have produced simplified models, which have not resulted in the solution of the problem, since they comprise only fragments of the process. In the considered models, such an obvious fact, that burning coke pieces become smaller as they move in the direction of lower zone boundary similarly to melting metal pieces in the melting zone has been neglected.

The prediction of burnt coke parameters allowing for their random distribution in the zone is the basic problem in the elaboration of the calculation method of combustion zone height

The mathematical description of metal piece melting and coke combustion was derived in work [1], in which also their average integral volumes and surfaces together with the accuracy of calculations were given. In work [2] the paradigm was applied to the calculation of height and structure parameters of the melting zone as well as to the description of its efficiency and the movements of metal and coke in the zone. The present work refers to the calculation of height and structure parameters of the combustion zone in coke cupolas and it comprises the following problems:

- the formation of stabilized height of the combustion zone,
- the calculation of combustion zone height for the coke pieces of identical shape and weight,
- the calculation of average integral volumes and surfaces of coke pieces of the square based prism shape,
- the calculation of volumes and surfaces of coke sequences in the combustion zone as well as volumes and surfaces of individual pieces of sequences,
- the calculation of combustion zone height for the coke cartridges containing fractions of different shape and weight,
- the calculation of initial height of the filling coke,
- examples of application of the derived formulas.

The present work is a synthesis and development of the models considered in papers [3÷6].

2. Formation of a stabilized combustion zone height in single-rowed cupolas

The process of coke burning in the cupola shaft will be followed above the level of lower nozzles from the moment, when after the repair of the furnace lining and other preparatory actions (like drying, annealing, glowing up the coke, the measurement and supplement of the initial height of the filling coke, loading the shaft with cartridges of coke, metal and flux) the blast is turned on, for the first 10 minutes, in the amount of 50 % of the target volume. The material structure in the cupola shaft is as follows: above the level of lower nozzles there is a homogenous column of the coke charge, called filling coke, about 1250 mm high lying on a column of cupola crucible coke or on the cupola hearth.

The bulk layers of charge coke cartridges lie on the filling coke column alternately with the flux and metal. The mentioned column of the coke fills up the space in the cupola shaft between the lower level of nozzles and the lowest cartridge of the metal charge. A primary and a secondary height of filling coke can be distinguished in the cupola process.

The primary height of the filling coke concerns the initial period of the cupola process (20÷30 minutes), in which a stabilized combustion zone should form from the primary height of the filling coke. During the formation of stable combustion zone height also a stabilized height of the melting zone should appear which requires a proper selection of weight and shape of metal pieces in the charge cartridges.

The term of secondary height of filling coke refers to the cupola process after the formation of combustion and melting zones. It consists of the height of the combustion zone and the lower part of the melting zone, which contains low percentage of the zone metal. Its height can be controlled from outside through inspection openings, usually by enlarging it with additional coke cartridges. The secondary filling coke height or simply, filling coke height definitely affects the degree of superheating of the smelted cast iron and should be optimal (its optimality is assessed according to the cast iron temperature obtained)

In the following part, a method of calculation of primary filling coke height is derived.

3. Stabilized combustion zone height – coke pieces of identical shape and weight

Model assumptions

– height of combustion zone is stabilized; the metal melting does not occur in the zone, but the drops of melted metal and slag from the melting zone passing the combustion zone undergo superheating, leading to the decrease of temperature of gases, without the change of zone height;
– surfaces limiting the combustion zone height are flat perpendicular to the cupola axis; the lower boundary is at the level of bottom row of nozzles, while the upper one at the level, where burning of coke ends (due to the lack of oxygen). The internal cross-section of the zone, perpendicular to the cupola axis is constant at the whole zone height;
– the burning coke pieces passing to the combustion zone form sequences of diminishing pieces. The number of such sequences is equal to the amount of coke pieces moving simultaneously through the upper zone surface of the volume gained in the melting zone (primary volume diminished as a result of losses of coke coal for the reduction of CO_2); coke pieces burning in the stabilized zone have their volumes and surfaces equal to their average integrals defined in work [1];
– the coke pieces moving towards the combustion zone are of the same volume, shape and physical and chemical properties;
– coke pieces, forming individual sequences of pieces are randomly distributed at the zone height with tendency of smaller and smaller pieces to move towards the lower nozzles. Their bulk density is identical in the whole zone and equal to the bulk density of coke cartridges loaded into the cupola;
– coke pieces burn on their whole surface and in the whole zone height at the same, constant linear combustion rate.

Height of combustion zone

A mass rate of coke burning in the combustion zone can be formulated with the following:

$$m_{k,s} = \mu_k \bar{F}_{k,s}\, \rho_k \tag{1}$$

where:

$m_{k,s}$ – mass rate of coke burning in the combustion zone, kg_c/s

μ_k – linear rate of coke burning (formula given further on), m/s

$\bar{F}_{k,s}$ – development surface of the combustion zone (total surface of burning coke pieces in the zone), m^2

ρ_k – mass density of coke pieces, kg/m_k^3

Surface $\bar{F}_{k,s}$ can be written with formula

$$\bar{F}_{k,s} = n_k \bar{f}_k = \frac{\bar{M}_{k,s}}{\rho_k \bar{v}_k}\bar{f}_k \tag{2}$$

And after further conversion

$$\bar{F}_{k,s} = \frac{H_s F_{r,s}\, \rho_{n,k}}{\rho_k \bar{r}_k} \tag{3}$$

at which:

$$\bar{r}_k = \frac{\bar{v}_k}{\bar{f}_k} \tag{4}$$

where:

n_k – number of coke pieces in the zone

$\bar{f}_k, \bar{v}_k$ – mean integral surface and average integral volume of coke pieces in the zone, respectively, m^2 and m^3

$\bar{M}_{k,s}$ – mass of coke in the zone, kg

$\bar{r}_k$ – average integral module of coke pieces in the zone, m

H_s – height of combustion zone, m

$F_{r,s}$ – surface of cross-section of the combustion zone, perpendicular to the cupola axis, m^2

$\rho_{n,k}$ – bulk density of coke in the zone, kg_k/m^3

After the substitution of (3) into (1)

$$m_{k,s} = \mu_k \frac{H_s F_{r,s} \rho_{n,k}}{\bar{r}_k} \tag{5}$$

H_s can be calculated from (5)

$$H_s = \frac{m_{k,s}\bar{r}_k}{\mu_k F_{r,s} \rho_{n,k}} \tag{6}$$

$m_{k,s}$ may be described with formula

$$m_{k,s} = \frac{P_c}{C_k L_{c,4}} \tag{7}$$

where:

P_c – efficiency of blast delivered to the cupola, normal conditions, m_p^3/s

C_k – relative fraction of coal in the coke, kg_c/kg_k

L_{c4} – volume of air blast consumed in the process for burning elementary mass of coke coal, normal conditions, m_p^3/kg_c

Incorporating (7) into (6)

$$\mathbf{H_s} = \frac{\mathbf{P_F \bar{r}_k}}{\mathbf{C_k L_{c,4} \mu_k \rho_{n,k}}} \tag{8}$$

where: $P_F = \dfrac{P_c}{F_{r,s}}$ – relative amount of blast in the combustion zone, $m_p^3/(m^2 \cdot s)$

The height of combustion zone for charge coke pieces of equal volume, mass and shape can be calculated from formula (8).

Volume L_{c4} is calculated from formula (blast with normal content of oxygen)

$$L_{c4} = 4,45(1 + \eta_{v,4}) \tag{9}$$

at which:

$$\eta_{v,4} = \frac{(CO_2)_{v,4}}{(CO_2)_{v,4} + (CO)_{v,4}} \tag{10}$$

where:

$\eta_{v,4}$ – degree of gas combustion in the process, in unit fraction

$\bar{f}_k$ and $\bar{v}_k$ can be obtained from formulas

$$\bar{f}_k = f_{o,k}\phi_f \tag{11}$$

$$\bar{v}_k = v_{o,k}\phi_v \tag{12}$$

where:

$f_{o,k}$, $v_{o,k}$ – initial surface and volume of coke pieces, m^2 and m^3, respectively

φ_f, φ_v – dimensionless coefficients dependant on shape of coke pieces.

4. Calculation of coefficients φ_v and φ_f for coke pieces in the shape of square based prisms

Taking advantage of recipe from work [1] the formulas to calculate coefficients φ_v and φ_f will be derived for coke pieces in the shape of square based prisms

Coefficient φ_v

The process of burning of singular coke piece in the shape of square based prism at constant linear rate will be now considered. The volume of piece, after burning time τ is equal to

$$v_{k,\tau} = (a - 2\mu_k\tau)^2(b - 2\mu_k\tau) \tag{13}$$

where:

a, b – length of prism base side and its height before the beginning of the process, m

τ – time measured from the moment of the start of coke piece combustion, s

μ_k – linear rate of burning of coke piece, m/s.

Equation (13) is transformed to the form

$$v_{k,\tau} = v_{k,o}\left(1 - \frac{2\mu_k}{a}\tau\right)^2\left(1 - \frac{2\mu_k}{am}\tau\right) \tag{14}$$

where:

$v_{k,o} = a^2b$ – initial volume of coke piece, m^3

$m = \frac{b}{a}$ – fineness of coke piece

a and X can be defined as

$$a = 2\mu_k\tau_c \tag{15}$$

$$X = \frac{\tau}{\tau_c} \tag{16}$$

where:

τ_c – time of complete combustion of coke piece, s

X – relative thickness of burnt layer of coke, in unit fraction

After substituting (15) and (16) into (14) volume $v_{k,X}$ is derived

$$v_{k,X} = v_{k,o}(1 - X)^2\left(1 - \frac{X}{m}\right) \tag{17}$$

where:

$v_{k,X}$ substitutes $v_{k,\tau}$ (volume of burnt coke piece in dependence on relative time X), m^3.

After the multiplication, equation (17) takes form

$$v_{k,X} = v_{k,o}(1 - k_1X + k_2X^2 - k_3X^3) \tag{18}$$

in which:

$$k_1 = 2 + \frac{1}{m}; k_2 = 1 + \frac{2}{m}; k_3 = \frac{1}{m} \tag{19}$$

The following formula is a definition of average integral volume of coke piece after relative time of burning X

$$\bar{v}_{k,X} = \frac{\int_0^X v_{k,X}dX}{\int_0^X dX} \tag{20}$$

After incorporating (18) into (20), the integration of the numerator and the denominator of the obtained equation and the addition of integration limits and then the simplification the following formula is obtained

$$\bar{v}_{k,X} = v_{k,o}\left(1 - k_1\frac{X}{2} + k_2\frac{X^2}{3} - k_3\frac{X^3}{4}\right) \tag{21}$$

From equation (21), the average integral volume of burning coke piece after relative time X is calculated.

After the substitution of X=1 into (21), the formula of calculation of the average integral volume for the total time of burning coke piece is derived

$$\bar{v}_k = v_{k,o}\left(1 - k_1\frac{1}{2} + k_2\frac{1}{3} - k_3\frac{1}{4}\right) \tag{22}$$

where:

$\bar{v}_{k,X} = \bar{v}_k$ – mean integral volume of coke piece, m^3.

After substituting expressions (19) into (22) and the simplification, the formula for $\bar{v}_k$ is as follows

$$\bar{v}_k = v_{k,o}\left(\frac{1}{3} - \frac{1}{12m}\right) \tag{23}$$

Based on (12) and (23), the following formula for the calculation of coefficient ϕ_v is derived

$$\boldsymbol{\varphi_v = \frac{\bar{v}_k}{v_{k,o}} = \frac{1}{3} - \frac{1}{12m}} \tag{24}$$

Dimensionless coefficient ϕ_v is equal to the ratio of average integral volume of coke piece in time of its complete burning to the volume of coke piece before the beginning of the combustion process.

Formula for the calculation of φ_f

The surface of the burning coke piece in the form of square based prism, after time τ, can be written down as follows

$$f_{k,\tau} = 2(a - 2\mu_k\tau)^2 + 4(a - 2\mu_k\tau)(b - 2\mu_k\tau) \tag{25}$$

Eq. (25) is reshaped to the form

$$f_{k,\tau} = 4ab\left(1 - \frac{2\mu_k}{a}\tau\right)\left(1 - \frac{2\mu_k}{b}\tau\right) + 2a^2\left(1 - \frac{2\mu_k}{a}\tau\right) \tag{26}$$

Relations (15) (16) and $m = \frac{a}{b}$ if $f_{k,\tau} = f_{k,X}$ are inserted into (26)

$$f_{k,X} = 4ab(1 - X)\left(1 - \frac{X}{m}\right) + 2a^2(1 - X)^2 \tag{27}$$

After the multiplication, eq. (27) takes form

$$f_{k,X} = 4ab\left(1 - \frac{X}{m} - X + \frac{X^2}{m}\right) + 2a^2(1 + X^2 - 2X) \tag{28}$$

Average integral surface of coke piece after relative time X can be described as

$$\bar{f}_{k,X} = \frac{\int_0^X f_{k,X}dX}{\int_0^X dX} \tag{29}$$

After including (28) into (29), the integration of the numerator and denominator of the obtained relationship, after inserting the integration limits and its simplification, the following equation of average integral surface of coke piece in dependence on value of relative combustion time X can be produced

$$\bar{f}_{k,X} = 4ab\left(1 - \frac{X}{2m} - \frac{X}{2} + \frac{X^2}{3m}\right) + 2a^2\left(1 - X + \frac{X^2}{3}\right) \tag{30}$$

Equation (30) is transformed into the following shape

$$\bar{f}_{k,X} = f_{k,o}\left(1 - \frac{2 + m}{2m + 1}X + \frac{X^2}{2m + 1}\right) \tag{31}$$

at which

$$f_{k,o} = 4ab + 2a^2 = (4m + 2)a^2 \tag{32}$$

where

$f_{k,o}$ – initial surface of burnt piece of coke, m^2

The average integral surface of burning coke piece after relative time X is calculated from eq. (31). After including X=1 into (31) the following formula of the average integral surface for the total combustion time of piece of coke can be derived

$$\bar{f}_k = f_{k,o}\left(1 - \frac{2 + m}{2m + 1} + \frac{1}{2m + 1}\right) \tag{33}$$

where:

$\bar{f}_{k,X} = \bar{f}_k$ for X = 1.

Eq. (33) can be transformed into the form of

$$\bar{f}_k = f_{k,o}\frac{m}{2m + 1} \tag{34}$$

Based on (11) and (34), the following expression for coefficient ϕ_f is established

$$\boldsymbol{\varphi_f = \frac{\bar{f}_k}{f_{k,o}} = \frac{m}{2m + 1}} \tag{35}$$

Dimensionless coefficient φ_f is equal to the ratio of average integral surface of coke piece after it is completely burnt to its surface before the combustion started.

Calculation of average integral modules of coke pieces in the form of square based prisms

Including (23) and (34) into (4) gives

$$\bar{r}_k = \frac{\bar{v}_k}{\bar{f}_k} = \frac{v_{k,o}\varphi_v}{f_{k,o}\varphi_f} = r_{k,o}\frac{\bar{\varphi}_v}{\bar{\varphi}_f} \tag{36}$$

at which:

$$r_{k,o} = \frac{v_{k,o}}{f_{k,o}} \tag{37}$$

where:

$r_{k,o}$ – initial module of coke pieces, m.

Equation (37) can be written for the square based prisms in the following way

$$r_{k,o}=\frac{ma}{2(2m+1)}=\frac{a}{2}\varphi_f \tag{38}$$

where:

a – length of square side of coke piece base, m.

Eq. (35) was included into (38).

Formulas (38) and (24) were substituted into (36) and after the simplification, the $\bar{r}_k$ was obtained

$$\bar{r}_k=\frac{a}{2}\varphi_v=\frac{a}{2}\left(\frac{1}{3}-\frac{1}{12m}\right) \tag{39}$$

Relations (38) and (39) contain new dependencies among $\bar{r}_k$, $r_{k,o}$, φ_v and φ_f.

5. Structure parameters of combustion zone

The morphology of combustion zone can be described with the following parameters: height and volume of zone, mass of burning coke in the zone, number of burning coke pieces and their average dimensions, development surface of coke pieces, number of coke piece sequences, their volumes and surfaces and so on.

The calculations start from the height of zone according to equation (8).

Number of coke pieces in the combustion zone

The number of burning coke pieces in the zone can be calculated based on the mass of coke in the zone and the average integral volume of coke pieces.

The mass of coke $\bar{M}_{k,s}$ (kg) in the zone is calculated from formula

$$\bar{M}_{k,s}=H_sF_{r,s}\rho_{n,k} \tag{40}$$

The number of coke pieces n_k in the zone is obtained from

$$n_k=\frac{H_sF_{r,s}\rho_{n,k}}{\bar{v}_k} \tag{41}$$

Number of sequences of burning coke pieces

The burning coke pieces form sequences, whose number amounts to the number of pieces moving simultaneously to the combustion zone across its cross-section at its upper boundary The number of sequences can be formulated as

$$N_{c,k}=\frac{F_{r,s}}{D_k^2} \tag{42}$$

where, for the square based prisms D_k (equivalent dimension of coke pieces, m) is

$$D_k=\frac{2a+b}{3} \tag{43}$$

Knowing n_k and $N_{c,k}$, the number of coke pieces in each sequence as well as their total volume and surface are calculated from the following expressions

$$n_{c,k}=\frac{n_k}{N_{c,k}} \tag{44}$$

$$V_{c,k}=n_{c,k}\bar{v}_k \tag{45}$$

$$F_{c,k}=n_{c,k}\bar{f}_k \tag{46}$$

where:

$n_{c,k}$ – number of coke pieces in each sequence,

$V_{c,k}$ – volume of pieces in one sequence, m^3

$F_{c,k}$ – surface of pieces in one sequence, m^2.

The $\bar{V}_k$ and $\bar{f}_k$ in formulas (45) and (46), respectively concern the average integrals i.e. calculated from infinite number of burning pieces, while the formulas contain finite numbers of pieces $n_{c,k}$, so their application requires the determination of minimal allowed number $n_{c,k}$.

Minimal number $n_{c,k}$ for equation (45)

In the next step, formulas to calculate a total volume of coke piece sequence, average volume of pieces in a sequence, volume of individual pieces and minimal number $n_{c,k}$ will be derived.

The combustion of coke pieces in a sequence is to be considered. The pieces in the form of square based prisms burn at constant linear rate at all surfaces of pieces. Another assumption is, that in individual sequences dimensions of neighboring pieces differ by value z, obtainable from formula

$$z=\frac{0,5a}{n_{c,k}} \tag{47}$$

where:

z – thickness of burnt layer of coke in each sequence of pieces, m The volume of i-piece in each sequence of pieces may be expressed with formula

$$v_{k,i}=(a-2zi)^2(b-2zi) \tag{48}$$

After extraction of dimensions a and b out of brackets

$$v_{k,i}=v_{k,o}\left(1-\frac{2z}{a}i\right)^2\left(1-\frac{2z}{b}i\right) \tag{49}$$

where

$v_{k,o}=a^2b$.

The following relation is substituted into eq. (49)

$$\delta=\frac{2z}{a} \tag{50}$$

In this way, eq. (49) acquires form

$$v_{k,i}= v_{k,o}(1-\delta i)^2\left(1-\frac{1}{m}\delta i\right) \tag{51}$$

After raising to a square power and the multiplication, eq. (51) obtains shape

$$v_{k,i} = v_{k,o}(1 - k_1\delta i + k_2\delta^2 i^2 - k_3\delta^3 i^3) \tag{52}$$

where:

k_1, k_2 and k_3 are given with formulas (19)

Using (52) the total volumes of piece sequence summing from i =1 to i=$n_{c,k}$ and from i=0 to i=$i_{c,k}$ (n= $n_{c,k}$ for simplification of notations)

$$V'_{c,n}= v_{k,o}\sum_{i=1}^{i=n}\left(1 - k_1\delta i + k_2\delta^2 i^2 - k_3\delta^3 i^3\right) \tag{53}$$

$$V''_{c,n}= v_{k,o}\sum_{i=0}^{i=n}\left(1 - k_1\delta i + k_2\delta^2 i^2 - k_3\delta^3 i^3\right) \tag{54}$$

The sum in equation (53) means, that the first piece of sequence has dimensions lessened by 2z; while the sum in formula (54) indicates that the first piece of sequence has initial size (passing from the melting zone to the combustion one it does not burn).

The total in equation (53) can be calculated writing down its first three terms and the last term in the form:

i = 1 $\quad 1 - k_1\delta\, 1 + k_2\delta^2 1^2 - k_3\delta^3 1^3$ (a)

i = 2 $\quad 1 - k_1\delta\, 2 + k_2\delta^2 2^2 - k_3\delta^3 2^3$ (b)

i = 3 $\quad 1 - k_1\delta\, 3 + k_2\delta^2 3^2 - k_3\delta^3 3^3$ (c)

. . . .

i = n $\quad 1 - k_1\delta\, n + k_2\delta^2 n^2 - k_3\delta^3 n^3$ (d)

After summing up the columns of lines (a)÷(d):

First column:

1 + 1 + 1+ . . . . + 1 = n (e)

Second column:

$$- k_1\delta\,(1 + 2 + 3 + \dots + n) = - k_1\delta\frac{n(n + 1)}{2} \tag{f}$$

third column:

$$k_2\delta^2\,(1^2 + 2^2 + 3^2 + \dots + n^2) = k_2\delta^2\frac{n(2n^2+3n + 1)}{6} \tag{g}$$

fourth column:

$$- k_3\delta^3\,(1^3+2^3+3^3+ \dots + n^3) = - k_3\delta^3\frac{n^2(n^2+2n + 1)}{4} \tag{h}$$

The totals in formulas (f), (g) and (h) are written according to [7] The result of addition of sums of columns (e)-(h) is denoted as R_1

$$R_1= n - k_1\delta\frac{n(n + 1)}{2}+k_2\delta^2\frac{n(2n^2+3n + 1)}{6} - k_3\delta^3\frac{n^2(n^2+2n + 1)}{4} \tag{55}$$

Relations (19) are substituted into (55) and so is the following expression resulting from (47) and (50)

$$\delta = \frac{1}{n} \tag{56}$$

After a simplification and transformation we get

$$R_1 = \frac{(4m - 1)\,n}{12m} - \frac{1}{2} + \frac{2m + 1}{12mn} \tag{57}$$

Substituting (57) into (53); an equation of the total volume of coke pieces in a given sequence of pieces is derived

$$V'_{c,n}= v_{k,o}R_1= v_{k,o}\left(\frac{(4m - 1)n}{12m}-\frac{1}{2}+\frac{2m + 1}{12mn}\right) \tag{58}$$

In order to obtain the formula of average volume of coke pieces, eq. (58) will be divided by n. In this way we get

$$\bar{v}'_{k,n}=\frac{V'_{c,n}}{n}= v_{k,o}\varphi'_{v,n} \tag{59}$$

at which:

$$\varphi'_{v,n}=\frac{4m - 1}{12m}-\frac{1}{2n}+\frac{2m + 1}{12mn^2} \tag{60}$$

Based on eq. (60), the value of coefficient $\varphi'_{v,n}$ is calculated as a function of m, n and the limits of summation in eq. (53).

The total of eq. (54) will be, in turn, calculated starting from denoting it as R_2. It is easy to observe that the total will be larger by 1 than sum R_1, because for i=0 the first term of the sum is 1. Thus, it can be written down that

$$R_2 = R_1 + 1 \tag{61}$$

Substituting (57) into (61)

$$R_2=\frac{(4m - 1)\,n}{12m}+\frac{1}{2}+\frac{2m + 1}{12mn} \tag{62}$$

Inserting (62) into (54)

$$V''_{c,n}= v_{k,o}R_2= v_{k,o}\left(\frac{(4m - 1)n}{12m}+\frac{1}{2}+\frac{2m + 1}{12mn}\right) \tag{63}$$

Let us divide (63) by n

$$\bar{v}''_{k,n}=\frac{V''_{c,n}}{n}= v_{k,o}\varphi''_{v,n} \tag{64}$$

at which:

$$\varphi''_{v,n}=\frac{4m - 1}{12m}+\frac{1}{2n}+\frac{2m + 1}{12mn^2} \tag{65}$$

From eq. (65) the value of coefficient $\varphi''_{v,n}$ will be calculated as a function of m, n and the limits of summation in eq. (54).

Thus, two formulas for volumes of sequences [(58) and (59)] as well as formulas to calculate the average volume of coke pieces in the sequence [(59) and (60)] and [(64) and (65)] are obtained It results from the mentioned formulas, i.e: $V''_{c,n} > V'_{c,n}$; $\bar{v}''_{k,n} > \bar{v}'_{k,n}$; $\varphi''_{v,n} > \varphi'_{v,n}$. Such inequalities yield from different summation limits in formulas (53) and (54).

Let us calculate arithmetic means of formulas (58) and (63) as well as (60) and (65):

$$V_{c,n}=\frac{V'_{c,n}+V''_{c,n}}{2}= v_{k,o}\frac{4m-1}{12m}n+\frac{2m+1}{12mn} \tag{66}$$

$$\varphi_{v,n}=\frac{\varphi'_{v,n}+\varphi''_{v,n}}{2}=\frac{1}{3}-\frac{1}{12m}+\frac{2m+1}{12mn^2} \tag{67}$$

The average volume of coke pieces as a function of n can be written down as

$$\bar{v}_{k,n}= v_{k,o}\varphi_{v,n}= v_{k,o}\frac{1}{3}-\frac{1}{12m}+\frac{2m+1}{12mn^2} \tag{68}$$

As it results from (66), (67) and (68) values $V_{c,n}$, $\varphi_{v,n}$, $\bar{v}_{k,n}$ for a given m, are the functions of coke piece number in sequence n. For $n\to\infty$ formulas (67) and (68) are simplified to the form of formulas (23) and (24), obtained for the average integrals.

Two groups of formulas of average volumes of coke pieces in the zone of combustion and other parameters are obtained. This groups of parameters result from the assumption of average integral volume, that is for $n=\infty$ and another for $n<\infty$ The question is, at which minimal n number the formulas for $n=\infty$ may be applied.

For the quantitative approach to the differences between the results of calculations according to the two groups of formulas let us define coefficient $\in_v$

$$\in_v=\frac{\varphi_{v,n}}{\varphi_v} \tag{69}$$

into which the derived formulas (67) and (24) can be substituted; after a simplification eq. (70) is obtained

$$\in_v = 1+\frac{2m+1}{(4m-1)n^2} \tag{70}$$

As it results from (70), the value of coefficient $\in_v$ decreases with n^2 It is also easy to observe, that $\in_v$ decreases with the increase of m. For instance, for n=5 and m=1 $\in_v$=104, which means that the difference between $\varphi_{v,n}$ and φ_n is only 4%. In the combustion zone the values of n are not lower than 5, while the m value is higher than 1. It allows for the application of formulas (23), (24) and (45) instead of (66), (67) and (68) At the same time the calculation of volume of coke pieces in sequences following the differential formula, which results from averaging formula (53) and (54) is possible.

Calculation of coke piece volumes in sequences

(56) is substituted into (52)

$$v_{k,i}= v_{k,o}\left(1-k_1\frac{i}{n}+k_2\frac{i^2}{n^2}-k_3\frac{i^3}{n^3}\right) \tag{71}$$

The definition of arithmetic mean of volume of coke pieces in sequences is

$$\bar{v}_{k,i}=\frac{v_{k,i}+v_{k,i-1}}{2} \tag{72}$$

Eq. (71) is substituted into (72)

$$\bar{v}_{k,i}= v_{k,o}\left[1-k_1\frac{i-0,5}{n}+k_2\frac{i^2+(1-i)^2}{2n^2}-k_3\frac{i^3+(1-i)^3}{2n^3}\right] \tag{73}$$

From eq. (73) the arithmetic mean of volume of i-piece of coke, can be calculated when number of pieces is n, fineness ratio is m and initial volume $v_{k,o}$.
Example: Data: m=1; n=5; k_1=2+1=3; k_2=3; k_3=1.
The calculation acc. to (73):

$$v_{k,i}=\left[1-3\frac{i-0,5}{5}+3\frac{i^2+(i-1)^2}{2\cdot 25}-1\frac{i^3+(i-1)^3}{2\cdot 5^3}\right]$$

i =1; $v_{k,1}=v_{k,o}$ 0,756 m^3; i = 2, $v_{k,2}=v_{k,o}$ 0,364 m^3; i =3, $v_{k,3}=v_{k,o}$ 0,140 m^3; i = 4; $v_{k,4}=v_{k,o}$ 0,036 m^3; i = 5; $v_{k,5}=v_{k,o}$ $4\cdot 10^{-3}$ m^3
The total volume of sequence is $V_{c,5}$= 13 $v_{k,o}$ m^3
The calculation using formula (45) (φ_v=0,25):

$V_c=v_{k,o}\cdot 0.25\cdot 5=1{,}25\ v_{k,o}$; difference $\frac{1,3-1,25}{1,25}$ = 0,04 or 4%.

Minimal number $n_{c,k}$ for formula (46)

Formulas of total surface of coke pieces in the sequence, mean surface of coke pieces in the sequence, surfaces of individual pieces and minimal number $n_{c,k}$ needed to use formula (46) will be derived in this part.

The surface of i-coke piece in the sequence of pieces square-based prisms will be written down with formula

$$f_{k,i} = 4(a-2zi)(b-2zi)+2(a-2zi)^2 \tag{74}$$

Or after extracting a and b out of brackets

$$f_{k,i}= 4ab\left(1-\frac{2z}{a}i\right)\left(1-\frac{2z}{b}i\right)+2a^2\left(1-\frac{2z}{b}i\right)^2 \tag{75}$$

Substituting relation (50) into (75)

$$f_{k,i}= 4ab(1-\delta i)\left(1-\frac{\delta}{m}i\right)+2a^2(1-\delta i)^2 \tag{76}$$

(76) will be transformed into the form

$$f_{k,i}= f_{k,o}\left(1-2\frac{m+2}{2m+1}\delta i+3\frac{1}{2m+1}\delta^2 i^2\right) \tag{77}$$

at which:

$$f_{k,o} = 4ab + 2a^2 = 2(m+1)a^2 \quad (78)$$

where:

$f_{k,o}$ – initial surface of coke piece m^2

Using (77) the total surface of coke pieces in the sequence can be written down for two summation limits i.e. from i=1 up to i=n and from i=0 to i=n:

$$F'_{c,n} = f_{k,o} \sum_{i=1}^{i=n} (1 - k_4\delta i + k_5\delta^2 i^2) \quad (79)$$

$$F''_{c,n} = f_{k,o} \sum_{i=0}^{i=n} (1 - k_4\delta i + k_5\delta^2 i^2) \quad (80)$$

at which:

$$k_4 = 2 - \frac{m+2}{2m+1}; \qquad k_5 = \frac{3}{2m+1} \quad (81)$$

The sum in eq. (79) will be calculated writing three first terms and the last one of it as:

$i = 1\ 1 - k_4\ \delta\ 1 + k_5\ \delta^2\ 1^2$(a)
$i = 2\ 1 - k_4\ \delta\ 2 + k_5\ \delta^2\ 2^2$(b)
$i = 3\ 1 - k_4\ \delta\ 3 + k_5\ \delta^2\ 3^2$(c)
. . . .
$i = n\ 1 - k_4\ \delta\ n + k_5\ \delta^2\ n^2$(d)

After summing up the columns of expressions (a)÷(d):

column 1:
$1 + 1 + 1 + \ldots + 1 = n$ (e)
column 2:
$-k_4\delta(1 + 2 + 3 + \ldots + n) = -k_4\delta\frac{n(n+1)}{2}$ (f)
column 3:
$k_5\delta^2(1^2+2^2+3^2+\ldots + n^2) = k_5\delta^2\frac{n(2n^2+3n+1)}{6}$ (g)

Totals of individual columns are added and denoted as S_1

$$S_1 = n - k_4\delta\frac{n(n+1)}{2} + k_5\delta^2\frac{n(2n^2+3n+1)}{6} \quad (82)$$

After including relation (56) into (82)

$$S_1 = n - k_4\frac{n+1}{2} + k_5\frac{2n^2+3n+1}{6n} \quad (83)$$

and expressions (81) into (83) we get

$$S_1 = \frac{m}{2m+1}n - \frac{1}{2} + \frac{1}{2n(2m+1)} \quad (84)$$

Substituting (84) into (79), one can produce the equation of total surface of pieces in a sequence

$$F'_{c,n} = f_{k,o}\left[\frac{m}{2m+1}n - \frac{1}{2} + \frac{1}{2n(2m+1)}\right] \quad (85)$$

After dividing both sides of (85) by n, a formula for average surface of coke pieces in a given sequence can be derived.

$$\bar{f}'_{k,n} = \frac{F'_{c,n}}{n} = f_{k,o}\varphi'_{f,n} \quad (86)$$

at which:

$$\varphi'_{f,n} = \frac{m}{2m+1} - \frac{1}{2n} + \frac{1}{2n^2(2m+1)} \quad (87)$$

The sum in eq. (80) will be in turn calculated denoting it as S_2. Since for i=0 the first term is equal to 1, the S_2 sum (from i=0 up to i=n can be written as follows

$$S_2 = S_1 + 1 \quad (88)$$

Eq. (84) will be incorporated into (88)

$$S_2 = \frac{m}{2m+1}n + \frac{1}{2} + \frac{1}{2n(2m+1)} \quad (89)$$

After the substitution of (89) into (80) the desired formula of $F''_{c,n}$ is derived

$$F''_{c,n} = f_{k,o}\left[\frac{m}{2m+1}n + \frac{1}{2} + \frac{1}{2n(2m+1)}\right] \quad (90)$$

After dividing both sides of eq. (90) by n another formula for average surface of coke pieces in a given sequence can be derived

$$\bar{f}''_{k,n} = \frac{F''_{k,n}}{n} = f_{k,o}\varphi''_{k,n} \quad (91)$$

at which:

$$\varphi''_{k,n} = \frac{m}{2m+1} + \frac{1}{2n} + \frac{1}{2n^2(2m+1)} \quad (92)$$

In that way two sets of formulas [(85) and (90)] for a total surface of coke pieces in a sequence were derived. Also two expressions for an average surface of coke pieces in a given sequence as well as coefficients $\varphi_{f,n}$ (87) and (92) were obtained.

The resulting inequalities $V''_{c,n} > V'_{c,n}$; $\bar{V}''_{k,n} > \bar{V}'_{k,n}$; $\varphi''_{f,n} > \varphi''_{f,n}$ are due to different limits of sums in formulas (79) and (80).

Now, arithmetic means of (85) and (90) as well as (87) and (92) are going to be calculated:

$$F_{c,n} = \frac{F'_{c,n} + F''_{c,n}}{2} = f_{k,o}\left[\frac{m}{2m+1}n + \frac{1}{2n(2m+1)}\right] \quad (93)$$

$$\varphi_{f,n} = \frac{\varphi'_{f,n} + \varphi''_{f,n}}{2} = \frac{m}{2m+1} + \frac{1}{2n^2(2m+1)} \quad (94)$$

As it follows from equation (94) coefficient $\varphi_{f,n}$ is a function of n, for a given value of m. The second term (94) contains n^2, which means, that its value decreases quickly when m increases and for n=∞ it is simplified to the form of equation (35).

Based on (94), a mean surface of pieces of sequence $\bar{f}_{c,n}$ can be obtained

$$\bar{f}_{c,n} = f_{k,o}\varphi_{f,n} = f_{k,o}\left[\frac{m}{2m+1} + \frac{1}{2n^2(2m+1)}\right] \quad (95)$$

In order to asses the differences of values $\varphi_{v,n}$ and φ_v, coefficient $\in_f$ will be defined as

$$\in_f = \frac{\varphi_{f,n}}{\varphi_f} \quad (96)$$

After substituting equations (94) and (35) into (96) eq. (97) is derived in the form of

$$\in_f = 1 + \frac{1}{2mn^2} \quad (97)$$

The value of coefficient $\in_f$ diminishes with the increase of n and m. For n=5 and m=1 $\in_f$ = 1,02 and it is lower from the calculated $\in_v$.

Surface of coke pieces in sequences

Relation (56) substituted into (77) gives

$$f_{k,i} = f_{k,o}\left(1 - 2\frac{m+2}{2m+1}\frac{i}{n} + 3\frac{1}{2m+1}\frac{i^2}{n^2}\right) \quad (98)$$

The mean arithmetic surface of coke pieces in sequences can be defined as

$$\bar{f}_{k,i} = \frac{f_{k,i} + f_{k,i-1}}{2} \quad (99)$$

Substituting (98) into (99)

$$\bar{f}_{k,i} = f_{k,o}\left[1 - \frac{m+2}{2m+1}\frac{2i-1}{n} + \frac{3}{2m+1}\frac{i^2 - i + 0.5}{n^2}\right] \quad (100)$$

The calculation of coke pieces in a given sequence of pieces.

Data: n=5, m=1. Calculate $\bar{f}_{k,i}$ and $F_{c,5}$.

Let us use Eq. (100)

$\bar{f}_{k,i} = f_{k,o}\left(1 - \frac{2i-1}{5} + \frac{i^2 - i + 0.5}{25}\right)$; results of calculations $\bar{f}_{k,i}$: $\bar{f}_{k,1}$ =0.82 $f_{k,o}$; $\bar{f}_{k,2}$ =0.50 $f_{k,o}$; $\bar{f}_{k,3}$ =0.26 $f_{k,o}$; $\bar{f}_{k,4}$ =0.10 $f_{k,o}$; $\bar{f}_{k,5}$ =0.02 $f_{k,o}$. The total surface of pieces in the sequence is $F_{c,n}$ = 1,7 $f_{k,o}$.

Combustion time of coke cartridges applying the height of the combustion zone

Combustion time of individual coke piece $\tau_{H,s}$ in the combustion zone is described by equation (101)

$$\tau_{H,s} = \frac{\bar{r}_k}{\mu_k} \quad (101)$$

The following proportion can be written down for the combustion zone

$$\bar{M}_{k,s} : \tau_{H,s} = m_{n,k} : \tau_{s,n,k} \quad (102)$$

where

$m_{n,k}$ – mass of coke cartridge, kg

$\tau_{s,n,k}$ – time of coke cartridge combustion, s.

Time $\tau_{s,n,k}$ is calculated from proportion (102)

$$\tau_{s,n,k} = \frac{\tau_{H,s} m_{n,k}}{\bar{M}_{k,s}} \quad (103)$$

After substituting equations (101), (40) and (8) into (103) a simple formula for time $\tau_{s,n,k}$ is obtained

$$\tau_{s,n,k} = \frac{m_{n,k}}{P_c} L_{k,4} \quad (104)$$

Let us compare time $\tau_{s,n,k}$ with the time of cartridge melting, which can be calculated from the following formula

$$\tau_{t,n,m} = \frac{m_{n,m}}{S_c} \quad (105)$$

where:

$\tau_{t,n,m}$ – melting time of metal cartridge, s

S_c – efficiency of melting, kg_{Fe}/s.

After including the Buzek formula for S_c [8] into (105) one can obtain equality

$$\tau_{t,n,m} = \tau_{s,n,k} \quad (106)$$

Equality (106) proves the continuity of the metal cartridge melting processes and combustion of the coke cartridges.

6. The height of combustion zone, when the coke cartridges contain fractions of different masses and shapes of coke pieces

Model assumptions

Part of earlier assumptions takes the following form:

– the cartridges of charge coke contain contributions of different masses and shapes of coke pieces; mass and volume fractions of individual parts in each cartridge are identical

– the linear rate of coke combustion is the same for all fractions of cartridges

– the coke pieces, which move to the combustion zone form sequences of decreasing pieces of individual fractions; the number of sequences is equal to the number of moving simultaneously coke pieces through the upper cross-section of the zone,

– average volume and surface of coke pieces in each sequence of every fraction is equal to their mean integrals.

Formula of the combustion zone height

The following formula of mass rate of combustion can be valid for any number of coke fractions

$$m_{k,s}=\mu_k\rho_k \sum_{i=1}^{i=n} \bar{F}_{s,i} \qquad (107)$$

where:

$\bar{F}_{s,i}$ – total surface in the zone of mean integral surfaces of i-coke fraction (combustion surface or the development surface of the combustion zone), m^2

n – number of fractions in coke cartridges (1,2,3. . .).

Surface $\bar{F}_{s,i}$ can be expressed with the help of mean integral surfaces and volumes of coke pieces of a given fraction

$$\bar{F}_{s,i}= n_{k,i}\bar{f}_{k,i}=\frac{\bar{V}_{s,i}}{\bar{r}_{k,i}} \qquad (108)$$

at which:

$$\bar{r}_{k,i}=\frac{\bar{v}_{k,i}}{\bar{f}_{k,i}} \qquad (109)$$

$$n_{k,i}=\frac{\bar{V}_{s,i}}{\bar{v}_{k,i}} \qquad (110)$$

where:

$n_{k,i}$ – number of coke pieces of i-fraction,

$\bar{f}_{k,i}$ – mean integral surface of individual coke pieces of i-coke fraction, m^2

$\bar{V}_{s,i}$ – total volume in the zone of mean integral volumes of coke pieces in i-fraction, m^3

$\bar{v}_{k,i}$ – mean integral volume of coke pieces in i-fraction, m^3

$\bar{r}_{k,i}$ – mean integral module of coke pieces in i-fraction, m

Let us substitute (108) into (107)

$$m_{k,s}=\mu_k\rho_k \sum_{i=1}^{i=n} \frac{\bar{V}_{s,i}}{\bar{r}_{k,i}} \qquad (111)$$

The relative contribution of i-fraction to the total volume of all coke fractions can be defined as $\bar{U}_{s,i}$

$$\bar{U}_{s,i}=\frac{\bar{V}_{s,i}}{\bar{V}_s} \qquad (112)$$

where:

$\bar{V}_s$ – total volume of mean integral coke pieces in the combustion zone, m^3

$\bar{V}_{s,i}$ can be calculated from (112) and substituted into (111)

$$m_{k,s}=\mu_k\rho_k\bar{V}_s \sum_{i=1}^{i=n} \frac{\bar{U}_{s,i}}{\bar{r}_{k,i}} \qquad (113)$$

The following equation of mass balance can be written for the combustion zone, which do not contain pieces of the metal charge

$$H_s F_{r,s}\rho_{n,k} = \bar{V}_s\rho_k \qquad (114)$$

$\bar{V}_s$ is calculated from equation (114) and inserted into (113)

$$m_{k,s}=\mu_k\rho_k\frac{H_s F_{r,s}\rho_{n,k}}{\rho_k} \sum_{i=1}^{i=n} \frac{\bar{U}_{s,i}}{\bar{r}_{k,i}} \qquad (115)$$

From eq. (115), the height of combustion zone for the coke cartridges containing fractions of different size coke pieces

$$H_s=\frac{m_{k,s}\bar{r}_z}{\mu_k F_{r,s}\rho_{n,k}} \qquad (116)$$

at which:

$$\bar{r}_z= 1/ \sum_{i=1}^{i=n} \frac{\bar{U}_{s,i}}{\bar{r}_{k,i}} \qquad (117)$$

where:

$\bar{r}_z$ – mean integral module of coke pieces of all fractions, m.

After the substitution of relation (7) into (117) the anticipated height of the combustion zone is obtained

$$\mathbf{H_s=\frac{P_F\bar{r}_z}{\mu_k C_k L_{c,4}\rho_{n,k}}} \qquad (118)$$

Calculation of contributions $\bar{U}_{s,i}$

Contributions $\bar{U}_{s,i}$ in formula (117) should be calculated based on the shares of individual fractions in the cartridges of charge coke and coefficients $\phi_{v,i}$. The formula to calculate $\bar{U}_{s,i}$ for i-fraction can be written down based on the definition given in (112)

$$\bar{U}_{s,i}=\frac{\bar{V}_{s,i}}{\bar{V}_s}=\frac{V_{k,o,i}\varphi_{v,i}}{\sum_{i=1}^{n}(V_{k,o,i}\varphi_{v,i})}=\frac{M_{k,o,i}\varphi_{v,i}}{\sum_{i=1}^{n}(M_{k,o,i}\varphi_{v,i})} \qquad (119)$$

at which:

$$\varphi_{v,i}=\frac{\bar{V}_{s,i}}{V_{k,o,i}}=\frac{\bar{v}_{k,i}}{v_{k,o,i}} \qquad (120)$$

where:

$V_{k,o,i}$ – initial volume of coke, of which volume $\bar{V}_{s,i}$ formed at the upper boundary of the combustion zone, i.e. volume of coke pieces of i-fraction, m^3

$v_{k,o,i}$ – initial volume of coke pieces of i-fraction, m^3

$\varphi_{v,i}$ – ratio of mean integral volume of coke pieces of i-fraction in the zone to their initial volume unit fraction

$M_{k,o,i} = V_{k,o,i}\,\rho_k$ – mass of volume $V_{k,o,i}$, kg.

Let us write (119) for two fractions in the coke cartridge (n=2)

$$\bar{U}_{s,1}=\frac{M_{k,o,1}\varphi_{v,1}}{M_{k,o,1}\varphi_{v,1}+M_{k,o,2}\varphi_{v,2}} \quad (121)$$

where:

$\bar{U}_{s,1}$ – contribution $\bar{U}_{s,1}$ in the first fraction, unit fraction

$M_{k,o,1}$; $M_{k,o,2}$ – initial mass of the first and second coke fractions in the coke cartridges, kg, respectively

$\varphi_{v,1}$; $\varphi_{v,2}$ – coefficient $\varphi_{v,i}$ of the first and second fractions of coke in the coke, respectively.

Primary height of the filling coke

The primary height of the filling coke can be described as the height, of which a combustion zone of stable height should form after the start of the cupola process. When the combustion zone height is known from calculations, the primary height of the filling coke can be calculated from formula

$$H_{k,w}=\frac{H_s}{\varphi_v} \quad (122)$$

where:

$H_{k,w}$ – primary height of the filling coke, m.

In the case of different values of φ_v their arithmetic mean should be used in formula (122).

7. Analysis of the Czyżewski's equation of combustion zone height

Czyżewski in work [9] derived the following formula to calculate the height of combustion zone (denotations of the present paper; SI unit system)

$$H_s=\frac{d_k}{2}\sqrt{\left(\frac{P_F}{3L_k\mu_k\rho_k(1-f_k)}\right)^2-1} \quad (123)$$

where:

d_k – diameter of coke pieces (precisely: base of coke cones formed in the zone from burning coke pieces), m

f_k – relative space among coke pieces, unit fraction.

Paper [9] was a qualifying work for assistant professor. It consisted of two parts; the first one reported the investigation of linear rate of coke combustion and their generalization in the form of a formula and diagram (which was the first report in the world-wide literature on linear rate of coke combustion); the second one contained the derivation of formula (123).

In work [13] containing the summary of the theory of cupola process (elaborated by Buzek and Czyżewski) Czyżewski produced only the following formula for the calculation of H without any introduction or a comment:

$$H_s=\frac{P_F d_k}{26700\mu_k} \quad (124)$$

The data on P_F and μ_k were to be substituted into formula (124) either in m/s or in m/min; d_k in m.

At present the model of combustion zone contained in equation (123) and its probable assumptions will be discussed in attempt to obtain formula (124) from (123).

Czyżewski assumed, when deriving formula (123) that the zone of combustion was filled with cones, formed from burning coke pieces of base diameter d_k with vertices directed upside down and height H_s Such an assumption allows writing the following equation of balance of coke combustion mass rate in the zone

$$\frac{P_c}{L_k}=\mu_k f_s n_s \rho_k \quad (125)$$

f_s – side surface of cone, m^2

n_s – number of cones in the zone

Let us write f_s and n_s in a broader form The side surface can be given as

$$f_s=\frac{\pi}{2}d_k\sqrt{\left(\frac{d_k}{2}\right)^2+H_s^2} \quad (126)$$

According to the assumption, the number of cones at the zone cross section is given by equation

$$n_s'=\frac{F_{r,s}}{d_k^2} \quad (127)$$

Using (127) the relative fraction of free volume in the zone volume f_k can be calculated

$f_k=\frac{F_{r,s}H_s-n_s'V_s}{F_{r,s}H_s}=1-\frac{n_s'V_s}{F_{r,s}H_s}=1-\frac{\pi}{12}=$ = 0.74 (74% of volume is occupied with voids)

where: $V_s=\frac{\pi}{12}d_k^2H_s$ – volume of individual cone, m^3.

The equation (127) gives too low the bulk density of coke in the zone. To increase it, Czyżewski applied the following equation for the calculation of the cone number n_s

$$n_s=\frac{F_{r,s}H_s}{V_s}(1-f_k)=\frac{F_{r,s}}{\frac{\pi}{12}d_k^2}(1-f_k), \quad (128)$$

which gives almost a double number of cones compared to the amount calculated from equation (127)

$\frac{n_s}{n_s'}=\frac{\pi}{12}$ (1-f_k) = 1.91 (for f_k=0.5)

After the substitution of (128) and (126) into (125) and after a transformation, equation (123) is obtained.

Czyżewski expressed the following opinion about formula (123) in work [9]: *“The equation is without* *doubt good as long as its qualitative aspect is considered (underlining comes from the author of the present work). It means that the influence of individual factors on the height of combustion zone is in accordance with practice" and that "... although it was based on quite artificial assumption, that the layer of burning fuel consists of cones, it gives quantities quite compatible with the reality".*

It yields from the presented opinion, that equation (123) was assessed by its author as a quite precise one.

Equation (123) can be written down as

$$H_s=\frac{d_k}{2}\sqrt{\left(\frac{P_F}{3L_k\mu_k\rho_{n,k}}\right)^2-1} \qquad (129)$$

where:

$\rho_{n,k} = \rho_k$ (1- f_k) – bulk density of coke, kg/m^3

Now, the origin of the equation (123) will be studied. After the elimination of 1 from equation (129) one can get

$$H_s=\frac{P_F}{L_k\mu_k\rho_{n,k}}\frac{d_k}{6} \qquad (130)$$

The obtained formula is similar to formula (8) (L_k=C_k L_c); it contains module r_k for the coke pieces in the form of spheres (d_k/6) instead of module $\bar{r}_k$. The calculated H_s height for spheres is higher than the height of spheres calculated according to formula (8). Leaving 1 in formula (128) decreases the differences of height H_s, calculated based on formulas (129) and (8).

In turn, equations (8) and (24) can be compared. Let us substitute eq. (39) into (8)

$$H_s=\frac{P_F}{C_kL_c\mu_k\rho_{n,k}}\frac{a}{2}\left(\frac{1}{3}-\frac{1}{12m}\right) \qquad (131)$$

Equations (131) and (124) can be also compared

$$\frac{P_F}{C_kL_c\mu_k\rho_{n,k}}\frac{a}{2}\frac{1}{3}-\frac{1}{12m}=\frac{P_Fd_k}{26700\mu_k} \qquad (132)$$

After accepting that a=d_k, and after a simplification and transformation the expression to calculate M_X takes the form:

$$M_X=\frac{2C_kL_c\rho_{n,k}}{\frac{1}{3}-\frac{1}{12m}} \qquad (133)$$

where: M_X= M_{Cz} = 26700 – dimensionless number given by Czyżewski

The % differences ΔM_{Cz}= M_X-M_{Cz} can be calculated for: C_k=0,86 kg$_c$/kg$_k$; L_c=8,9 m$_p^3$/kg$_c$ (burning of C into CO_2 – assumption in paper [9]; $\rho_{n,k}$=500, 480 and 440 kg$_k$/m^3; m=1; 1,25 and 1,5. Calculations: m=1; $\rho_{n,k}$= 500 kg$_k$/m^3; ΔM_{Cz}= 14,7%; m=1; $\rho_{n,k}$=440 kg$_k$/m^3; ΔM_{Cz}=0,9%; m=1,25; $\rho_{n,k}$= 500 kg$_k$/m^3; ΔM_{Cz}=7,5%; m=1,25; $\rho_{n,k}$=480 kg$_k$/m^3; ΔM_{Cz}=3,26%; m=1,5; $\rho_{n,k}$= 500 kg$_k$/m^3; ΔM_{Cz}=3,2.

It follows from the calculations that the simplified equation (123) can be applied in industrial calculations.

8. Linear rate of coke combustion

The linear rate of coke combustion is here proposed to be calculated from the following formula of Podrzucki [8]:

$$\mu_k= ew^{0,85}c_o^{1,89}T_d^{0,28} \qquad (134)$$

at which:

$$w = 1,05\frac{P_F}{\left(1-\frac{\rho_{n,k}}{\rho_k}\right)1,4} \qquad (135)$$

where

e – coefficient dependent on the kind of coke; for the foundry coke of first sort e=1,437·10^{-5}

w – velocity of gas flow through the combustion zone, normal conditions, m/s

c_o – concentration of oxygen in the blast, m^3 of oxygen/m^3 of air,

T_d – temperature of blast, K.

9. Calculation of combustion zone height

a) Data: T_d=273 + 25 = 298 K, c_o=0,21 m^3/m^3; P_F= 1,7 m/s; ρ_k=1000 kg/m^3, $\rho_{n,k}$=500 kg/m^3; a=0,1 m, m=1,5.

Calculations: $w= 1,05\frac{1,7}{\left(1-\frac{500}{1000}\right)^{1,4}}$ =4,71 m/s acc. to (124);

μ_k=1,437·10^{-5}4,71^{0,85}0,21^{1,89}298^{0,28}= 1384·10^{-5} m/s acc. to (123)

$L_{c,4}$=4,45(1+0,525)=6,786 m^3/kg$_c$ acc. to (9);

$\bar{r}_k=\frac{0,1}{2}\left(\frac{1}{3}-\frac{1}{12\cdot 1,5}\right)$=0,0139 m acc. to (39); H_s = $\frac{1,7\cdot 0,0139}{0,86\cdot 6,786\cdot 1,384\cdot 10^{-5}\cdot 500}$ = 0,585 m acc. to (8).

b) Data: a=0,06 m; m=1,2. The remaining data like in example a. Calculation: $\bar{r}_k=\frac{0,06}{2}\left(\frac{1}{3}-\frac{1}{12\cdot 1,2}\right)$=7,917·10^{-3} m; H_s=0,33 m.

c) Calculation of H_s acc. to (118) for two coke fractions, 24 kg each.

Calculation: $\varphi_{v,1} = \frac{1}{3}-\frac{1}{12\cdot 1,5}$=0,278 acc. to (24);

$\varphi_{v,2} = \frac{1}{3} - \frac{1}{12 \cdot 1,2}$=0,264 wg(24);

$\bar{U}_{s,1} = \frac{24 \cdot 0,278}{24 \cdot 0,278 + 24 \cdot 0,264}$=0,513 acc. to (121);

$\bar{U}_{s,1} = \frac{24 \cdot 0,264}{24 \cdot 0,278 + 24 \cdot 0,264}$=0,487;$\bar{r}_{k,1}$= 0,0139m;

$\bar{r}_{k,2} = 7,917 \cdot 10^{-3}$ m;

$\bar{r}_z = \frac{1}{\frac{0,513}{0,0139} + \frac{0,487}{7,917 \cdot 10^{-3}}}$=0,0102 m acc. to (117);

$H_s = \frac{1,7 \cdot 0,0102}{0,86 \cdot 6,786 \cdot 1,384 \cdot 10^{-5} \cdot 500}$ = 0,429 m acc. to (118)

10. Conclusions

The work describes the formation of stabilized combustion zone height out of a primary height of the filling coke and derivation of formulas for the calculation of parameters typical for the processes of appearance of the zone structure for pieces of coke in the form of square based prisms, cubes and spheres (m=1). It also contains examples of application of the derived formulas.

The derived formulas can be divided into three groups:
– formulas containing mean integral volumes and surfaces of coke pieces, which can be called the basic formulas of the work;
– formulas resulting from the assumption of differences of finite linear dimensions of coke pieces in individual sequences of coke, which can be called formulas completing the model described in the work
– complementary formulas.

The first group contains formulas (1) to (46), which serve for the calculation of the following parameters of combustion zone: zone height for the coke pieces of identical mass and shape; φ_v and φ_f coefficients for the calculation of mean integral volume and surfaces of coke pieces; average integral modules of coke pieces in the zone ($\bar{r}_m$); the number of burning coke pieces in the zone (n_k); the number of coke pieces sequences ($N_{c,k}$); number of coke pieces in sequences of pieces ($n_{c,k}$); volumes and surfaces of sequences ($V_{c,k}$ and $F_{c,k}$).

Formulas from (47) up to (100) belong to the second group and its role is dual; they serve to calculate the minimal number of coke pieces in sequences of pieces, for which mean integral volumes and surfaces can be still used. They also serve to calculate volumes and surfaces of individual pieces in sequences.

It results from the performed calculations, that the minimal number of pieces in sequences of pieces $n_{c,k}$=5, while the volume and surface differences calculated based on the formulas of the first and second group do not exceed several per cent (4% in the above-mentioned examples).

The third group of formulas i.e formula (101) to (135) stands for the group of complementary formulas and they serve to calculate:
– the combustion time of coke cartridges using the height of the combustion zone,
– the height of combustion zone when the coke cartridges contain fractions of various weights and forms of coke pieces,
– the primary height of the filling coke
– the heights of combustion zone according to Czyżewski formulas,
– the linear rate of coke combustion according to the Podrzucki formula.

A new model of coke cartridge burning in the combustion zone as well as a new model of melting metal cartridges in the melting zone, taking into account paper [2], completely different from the models described in the literature (scarce works e.g. [9-12] was established in the present paper.

The new model indicates the continuity and fluidity of melting and burning processes as well as the movement of the material stack in the cupola shaft.

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REFERENCES

[1] W. Longa: The main paradigm used in the theory of calculation of the height and structure of combustion zone and melting zone in coke-fired cupolas. Polish Academy of Sciences Archives of Metallurgy and Materials. **54**, 4, 1207-1222 (2009).
[2] W. Longa: Calculation of height and structure of melting zone of coke cupolas. Polish Academy of Sciences Archives of Metallurgy and Materials **55**, 3, 669-682 (2010).
[3] W. Longa: Calculation of Combustion Zone Height in a Cupola Working under Stable Conditions. Zeszyty Naukowe AGH. Metalurgia I Odlewnictwo **17**, 2, 69 (1991).
[4] W. Longa, G. Friebe: Berechnung der Höhe einer stabilen Verbrennungszone sowie der Anfangshöhe des Füllkokses im Kupolofen. Giesserei Forschung **4**, 184-191 (1999).
[5] W. Longa: Obliczanie pierwotnej wysokości koksu wypełniającego w żeliwiaku. Przegląd Odlewnictwa **47**, 5, 131-134 (1997).
[6] W. Longa: Matematyczny model ustabilizowanej pracy cieplnej stref żeliwiaków koksowych. Przegląd Odlewnictwa (praca w druku).
[7] H.B. Dwight: Tables of integrals and other mathematical data. New York, 1961.

[8] C. P o d r z u c k i, C. K a l a t a: Metalurgia i odlewnictwo żeliwa. Wyd. Śląsk, wydanie drugie , 1976.

[9] M. C z y ż e w s k i: Najkorzystniejsza wysokość warstwy strefy spalania koksu. Przegląd Górniczo-Hutniczy. 9, 437 (1935).

[10] l.M. M a r i e n b a c h: Intensyfikacija wagranocznogo processa. Maszgiz. Moskwa 1954.

[11] C. P o d r z u c k i: Badanie procesów spalania i zgazowywania koksu w żeliwiaku ze szczególnym uwzględnieniem wpływu ilości dmuchu na skład chemiczny gazów odlotowych. Zeszyty Naukowe AGH w Krakowie. Rozprawy 17. Kraków, 1963.

[12] R. K r z e s z e w s k i: Zależność między procesem spalania koksu w żeliwiaku, jego wydajnością i optymalną ilością dmuchu. Prace Instytutu Odlewnictwa w Krakowie. Rok XVIII. Zeszyt specjalny C/68.

[13] Podstawowe wzory Jerzego Buzka, charakteryzujące pracę żeliwiaka. Przegląd odlewnictwa Nr 4, 1951.

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